

## Convective Flow of Radiative Maxwell Nanofluid with Variable Thermo-Physical Properties

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### Abstract

In this article, analysis has been executed to find the impact of radiation, varied thermal conductivity and varied viscosity on the steady 2 dimensional flows of nanoparticles existing Maxwell fluid past a flat surface that can be stretched. The partial differential equations which govern this flow are modified by using similarity transformations in order to form ordinary differential equations. Employing `bvp4c` of MATLAB software the transformed equations are solved and the results for velocity, temperature and species concentration are depicted through graphs for varying parametric values. Comparisons with previous published data of analytical methods are carried out, thereby validating the present numerical results. It has been observed that thermal conductivity as well as viscosity enhances the temperature of the fluid and nanoparticles species concentration. Also the nanoparticles existence in the fluid slows down the fluid motion. In various engineering processes and nanoscience technology, the inferences of this present study can find its applications.

**Keywords:** Thermal radiation, Variable thermal conductivity, Maxwell fluid, Nanoparticles, Variable viscosity

### Nomenclature

- $u$  -  $x$  component of velocity
- $v$  -  $y$  component of velocity
- $\rho_f$  - base fluid density
- $q_r$  - heat flux due to radiation
- $\nu = \frac{\mu}{\rho_f}$  - dimensional fluid kinematic viscosity
- $\mu$  - variable fluid dynamic viscosity
- $\tau$  - ratio of the effective heat capacity of the nanoparticle material and the heat capacity of the ordinary fluid
- $\kappa_0$  - relaxation time of the upper-convected Maxwell fluid
- $C_p$  - specific heat
- $D_B$  - Brownian diffusion coefficient
- $D_T$  - thermophoresis diffusion coefficient
- $\lambda$  - variable thermal conductivity
- $C$  - nanoparticles volume fraction
- $T_\infty$  - ambient fluid temperature
- $T$  - fluid temperature
- $\eta$  - similarity variable
- $\psi$  - stream function
- $\theta$  - dimensionless temperature
- $\phi$  - dimensionless concentration
- $v$  - dimensionless reference temperature corresponding to viscosity
- $\beta$  - Maxwell parameter known as Deborah number
- $\varepsilon$  - dimensionless reference temperature corresponding to thermal conductivity
- $R$  - radiation parameter
- $Pr$  - Prandtl number
- $Nt$  - thermophoresis parameter

$Nb$  - Brownian motion parameter

$Le$  - Lewis number.

## Introduction

In recent times, many researchers are attracted towards the study of viscous flows on account of their wide range of industrial and engineering applications in glass fibre production, polymer production, plastic sheets extrusion, biological and condensation processes. If the stress of viscoelastic fluid is removed suddenly then the strain of the fluid doesn't disappear immediately but relaxes quite slowly. This is a significant property of the viscoelastic fluid. The Maxwell model being the simplest model of viscoelastic fluids has small dimensionless relaxation time. Recently many investigations dealing with the Maxwell fluids are carried out in [1-5]. In 1867, a theoretical model was proposed by James Clerk Maxwell in order to examine the impact of parameters involved in heterogeneous solid particles such as electrical conductivity. This model infused the studies of thermal conductivity of liquids on addition of solid particles. All of these studies are carried out for particles with size millimetre or micrometre. Furthermore, particles with high concentration are needed to acquire significant enhancements in thermal conductivities of the suspensions. Nowadays, upper-convected Maxwell fluid flow has become an area of great interest for many researchers. Bai *et al.* [6]; Mohamadali and Ashrafi [7]; Gireesha *et al.* [8] have investigated Maxwell modelled fluid concerned with various physical conditions such as thermal stratification, chemical reactions, Newtonian heating and viscous dissipation, passed stretching surfaces under different conditions. Flows of viscous fluid, Maxwell fluid over a permeable surface which is considered to shrink under various conditions with the help of different models were analyzed by eminent researchers in their works [9-13]. Recently, using successive linearization method, Motsa *et al.* [14] inspected the flow of Maxwell model taking the geometry of flow as shrinking sheet near a stagnation point.

Choi and Eastman [15] coined the term Nanofluids which represent a new nanotechnology based fluids. These fluids have augmented thermal properties, being higher than that of both the own hosting fluid as well as the conventional fluid particle suspensions. Nanoparticles generally have higher thermal, magnetic, electrical, optical and mechanical properties. The model proposed by Kuznetsov and Nield [16] has been extended by Khan and Pop [17] in their work recently, in which under constant surface temperature the flow of fluid with nanoparticles passing beyond a stretching sheet was investigated. It can be inferred that, instead of an isothermal condition their analysis can be generalized to convective boundary condition. Previously many problems have been investigated using the isothermal or the isoflux boundary conditions. Several researchers revisited those problems and used the convective boundary condition. Some of those papers are authored by Bataller [18]; Ishak [19]; Ahmad and Pop [20]; Makinde and Olanrewaju [21]; Makinde and Aziz [22]. Later, Makinde and Aziz [23] have studied "the flow of a nanofluid past a linearly stretching sheet numerically using the convective boundary conditions". They concluded that "intensifying each of the thermophoresis and Brownian motion, the local temperature rises which as a result thickens the thermal boundary layer". In this problem, the convection Biot number favours the concentration of the nanoparticles. Turkyilmazoglu [24-29] has done a lot of studies on the flow of nanofluids and ferrofluids. He used the spectral method to analyze the flow behaviour under different geometrical conditions.

All the above investigations were carried out under the consideration of constant fluid physical properties whereas in practical problems the physical properties occur with variable characteristics. "One of these properties is the thermal conductivity, which is supposed to vary linearly with the temperature" [30]. The impact of change in thermal conductivity on fluid flow was studied by Chiam [31]; Chiam [32]. In a non-isothermal system, thermal radiation is an important factor which controls the heat transfer. Nasir *et al.* [33] in their work have studied 3D MHD flow across a stretching sheet and inspected the variations caused by the thermal radiations on such flow. Raptis and Perdakis [34] have scrutinized the radiation effect on flows of a viscoelastic liquid considering the physical properties to be constant. Reddy [35] used the "fourth order Runge Kutta method and the shooting method to analyze the influence of heat radiation on MHD flow of a nanofluid through the permeable stretching of a flat surface". Ramesh and Gireesha [36] have made an analysis on the Maxwellian flow with nanoparticles passed a stretched surface taking convective boundary conditions and reported the heat sink/source effects along with the comparative study. Ibrahim and Negera [37] have investigated "the stagnation point flows and slip effects of MHD Maxwell nanofluid past a stretched sheet under the occurrence of chemical reaction and concluded that magnetic field intensifies the temperature and concentration of the nanofluid under such

condition". Not any of the aforesaid works have discussed the combined effects of thermal radiation and variable thermal conductivity on flow of Maxwell fluid with the presence of nanoparticles.

In the entire above investigations the effects of nanoparticles with variable thermal conductivity are seen to be disregarded in the analysis of the problem of upper-convected Maxwell fluid flow in the presence of nanoparticles under influence of the thermal radiation. Thus the objective of the present paper is to examine the effects of variation in thermal conductivity, radiation and variable viscosity on the upper-convected Maxwell fluid flow past a stretching surface along with nanoparticles present in the fluid via programming in MATLAB. Therefore, the inclusion of the effects of variable thermal conductivity, radiation and variable viscosity in the flow of Maxwell fluid along with the nanoparticles make this study a novel one.

### Mathematical formulation

A 2 dimensional steady flow of nanoparticles existing Maxwell fluid in the region  $y > 0$  past a stretching surface placed at  $y = 0$  with point fixed at  $x = 0$  is considered. The fluid is confined to flow in the positive region of  $y$  axis and the sheet is taken as the plane  $y = 0$ . The origin is fixed and along  $x$  axis 2 equal but opposite forces are applied, causing a stretch or shrinkage of the flat sheet.

The flow-governing equations [38,23] of Maxwell fluid with variable viscosity under these assumptions are given by

$$\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} = 0, \quad (1)$$

$$\left(u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y}\right) u = \frac{\partial}{\partial y} \left(v \frac{\partial u}{\partial y}\right) - \kappa_0 \left(u^2 \frac{\partial^2}{\partial x^2} + 2uv \frac{\partial^2}{\partial x \partial y} + v^2 \frac{\partial^2}{\partial y^2}\right) u. \quad (2)$$

Here the novel term is the Maxwell fluid viscosity which is considered to be variable. Its study is relevant in present scenario of isothermal conditions. The governing heat transport [38] and species concentration [17] equations with variable thermal conductivity and nanoparticles being present in the fluid flow under thermal radiation are given by

$$\left(u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y}\right) T = \frac{1}{\rho_f c_p} \frac{\partial}{\partial y} \left(\lambda \frac{\partial T}{\partial y}\right) + \tau \left\{ D_B \left(\frac{\partial C}{\partial y} \cdot \frac{\partial T}{\partial y}\right) + \left(\frac{D_T}{T_\infty}\right) \left(\frac{\partial T}{\partial y}\right)^2 \right\} - \frac{1}{\rho_f c_p} \frac{\partial q_r}{\partial y}, \quad (3)$$

$$\left(u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y}\right) C = D_B \frac{\partial^2 C}{\partial y^2} + \left(\frac{D_T}{T_\infty}\right) \left(\frac{\partial^2 T}{\partial y^2}\right). \quad (4)$$

Associated boundary conditions

$$\left. \begin{aligned} u = U_W(x), v = 0, -\lambda \frac{\partial T}{\partial y} = h_f(T_W - T), C = C_W \text{ at } y = 0, \\ u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } y \rightarrow \infty, \end{aligned} \right\} \quad (5)$$

where  $U_W(x) = cx$  is the stretching sheet velocity with  $c$  being the rate at which the sheet is stretched, at wall the nanoparticles fraction is  $C_W$  and at free surface the volume fraction of the nanoparticles is  $C_\infty$ . The surface of the sheet is convectively heated by a hot fluid at temperature  $T_W$  with heat transfer coefficient  $h_f$ .

In order to convert the above mentioned coupled nonlinear PDEs to a coupled set of nonlinear ODEs, the following similarity transformations are introduced:

$$\eta = \sqrt{\frac{c}{\nu_\infty}} y, \quad \psi = \sqrt{c \nu_\infty} x f, \quad \theta = \frac{T - T_\infty}{T_W - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_W - C_\infty}. \quad (6)$$

From the Rosseland approximation, the heat flux due to radiation  $q_r$  can be modelled [39] as

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y}, \quad (7)$$

where  $k^*$  is the coefficient of mean absorption and  $\sigma^*$  is Stefan Boltzmann constant. Expanding  $T^4$  in Taylor's series about  $T_\infty$  following expression is obtained:

$$T^4 = T_\infty^4 + 4T_\infty^3(T - T_\infty) + 6T_\infty^2(T - T_\infty)^2 + \dots \quad (8)$$

Neglecting all the terms higher than first degree of  $(T - T_\infty)$ , Eq. (8) reduces to

$$T^4 = -3T_\infty^4 + 4T_\infty^3 T. \quad (9)$$

Using (9), Eq. (7) can be written as:

$$q_r = -\frac{16\sigma^*}{3k^*} T_\infty^3 \frac{\partial T}{\partial y}. \quad (10)$$

The dynamic viscosity of Maxwell fluid is assumed to be an exponential decreasing function of temperature given by

$$\mu = \mu_\infty e^{-a(T - T_\infty)}, \quad (11)$$

where  $\mu_\infty$  is the viscosity of the ambient fluid and  $a$  is constant depending on the reference state of the fluid. Similarly, the Maxwell nanofluid thermal conductivity is given by

$$\lambda = \lambda_\infty e^{-b(T - T_\infty)}, \quad (12)$$

where  $\lambda_\infty$  is the thermal conductivity of the ambient fluid and  $b$  is the constant which depends upon the reference state of the fluid.

Here, the equation of continuity (1) is satisfied by the velocity components. Now, using Eqs. (6), (10) - (12) in Eqs. (2) - (4), the following equations of momentum given by Eq. (13), energy given by Eq. (14) and nanoparticles volume fraction given by eq. (15) are obtained as:

$$(1 - \beta e^{v\theta} f^2) f''' - v\theta' f'' + e^{v\theta} (f f'' - f'^2 + 2\beta f f' f'') = 0, \quad (13)$$

$$(1 + e^{\varepsilon\theta} R) \theta'' - \varepsilon \theta'^2 + \text{Pr} e^{\varepsilon\theta} (Nb \theta' \phi' + Nt \theta'^2 + f \theta') = 0, \quad (14)$$

$$\phi'' + \text{Le} \text{Pr} f \phi' + \frac{Nt}{Nb} \theta'' = 0, \quad (15)$$

where the non-dimensional parameters used are defined as:

$$v = a(T_W - T_\infty), \quad \beta = \kappa_0 c, \quad \varepsilon = b(T_W - T_\infty), \quad R = \frac{16 \sigma^* T_\infty^3}{3k^* \lambda_\infty}, \quad \text{Pr} = \frac{v_\infty \rho_f c_p}{\lambda_\infty}, \quad Nt = \frac{\tau D_T}{v_\infty T_\infty} (T_W - T_\infty),$$

$$Nb = \frac{\tau D_B}{v_\infty} (C_W - C_\infty), \quad \text{Le} = \frac{\lambda_\infty}{\rho c_p D_B}.$$

The non-dimensional form of the boundary conditions (5) can be written as:

$$f = 0, f' = 1, \theta' = -\text{Bi} \cdot e^{\varepsilon\theta(0)} \cdot (1 - \theta(0)), \phi = 1 \quad \text{at} \quad \eta = 0,$$

$$f' \rightarrow 0, \theta \rightarrow 0, \phi \rightarrow 0 \quad \text{as} \quad \eta \rightarrow \infty, \quad (16)$$

where  $\text{Bi} = \sqrt{\frac{v_\infty h_f}{c \lambda_\infty}}$  is the Biot number.

Some of the important physical quantities of interest such as local Nusselt number  $Nu_x$  and the Sherwood number  $Sh_x$  are defined as

$$Nu_x = \frac{x q_W}{\lambda(T_W - T_\infty)}, \quad Sh_x = \frac{x q_m}{D_B(C_W - C_\infty)}.$$

here  $q_W = -\lambda \left( \frac{\partial T}{\partial y} \right)_{y=0} + q_r$  is the surface heat flux and  $q_m = -D_B \left( \frac{\partial C}{\partial y} \right)_{y=0}$ .

These physical quantities under the similarity transformations (6) can be written approximately as:

$$Nu_x Re_x^{-1/2} = -(1 + e^{\varepsilon\theta(0)}R)\theta'(0), \quad Sh_x Re_x^{-1/2} = -\phi'(0).$$

**Results and discussion**

The system of Eqs. (13) - (15) along with the conditions (16) are coupled nonlinear differential equations. The solution of this nonlinear system cannot be obtained in the closed form analytically. Thus to deduce the closed form solution of the given system of nonlinear equations, numerical method for developing the codes in MATLAB software bvp4c is used. The numerical results for fluid motion, temperature and nanoparticles concentration profiles of variations in material parameters are carried out and displayed graphically in **Figures 1 - 18**. In plotting the figures the parameters are valuated as  $Pr = 3$ ,  $0 \leq \beta \leq 1$ ,  $0.1 \leq Nb \leq 2$ ,  $0.15 \leq Nt \leq 0.6$ ,  $0 \leq \varepsilon \leq 2$ ,  $0 \leq \nu \leq 3$ ,  $0 \leq R \leq 2$ ,  $Le = 2$  and  $0.1 \leq Bi \leq 1$ .

Tables 1 - 3 exhibit the fact that the currently obtained numerical results agreed perfectly with previously reported results of Khan and Pop [17] under the absence of thermal radiation and variable thermo-physical properties, thus giving the affirmation of the accuracy of the present numerical procedure.

**Table 1** Comparison table for thermophoresis parameter taking  $Pr = 10$ ,  $Le = 1$ ,  $Bi = \infty$ ,  $R = 0$ ,  $\varepsilon = 0 = \nu$  and  $Nb = 0.1$ .

Nt	$-\theta'(0)$ [17]	$-\phi'(0)$ [17]	$-\theta'(0)$ Present	$-\phi'(0)$ Present
0.2	0.6932	2.2740	0.6932	2.2740
0.3	0.5201	2.5286	0.5201	2.5286
0.4	0.4026	2.7952	0.4026	2.7951
0.5	0.3211	3.0351	0.3210	3.0351

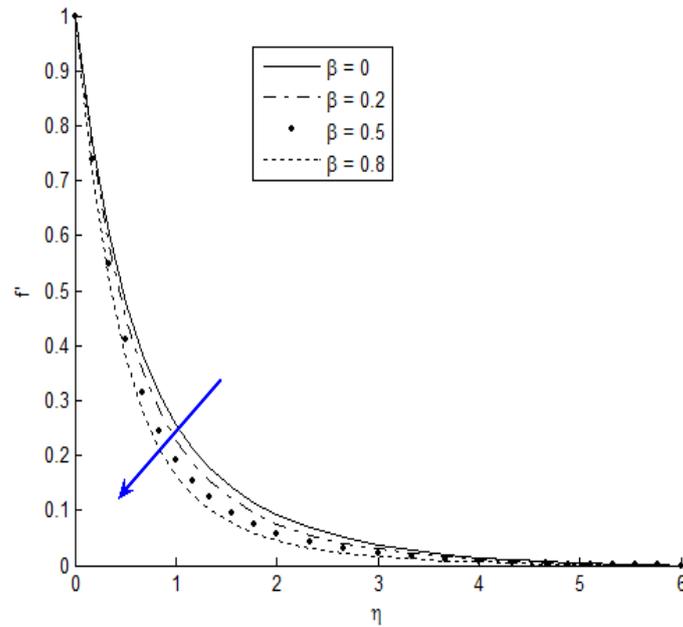
**Table 2** Comparison table for Brownian motion parameter taking  $Pr = 10$ ,  $Le = 1$ ,  $Bi = \infty$ ,  $R = 0$ ,  $\varepsilon = 0 = \nu$  and  $Nt = 0.1$ .

Nb	$-\theta'(0)$ [17]	$-\phi'(0)$ [17]	$-\theta'(0)$ Present	$-\phi'(0)$ Present
0.1	0.9524	2.1294	0.9524	2.1295
0.2	0.5056	2.3819	0.5056	2.3819
0.3	0.2522	2.4100	0.2521	2.4100
0.4	0.1194	2.3997	0.1194	2.3997

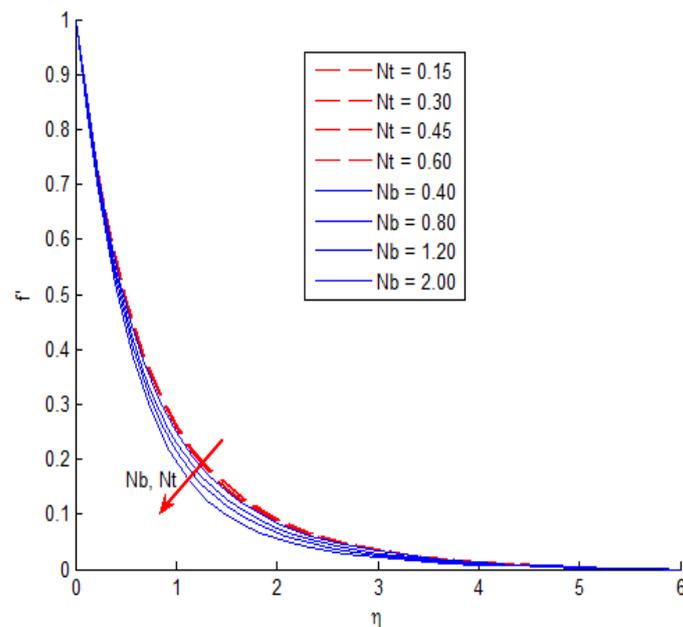
**Table 3** Comparison table for Prandtl number taking  $Bi = 1,000$ ,  $R = 0$ ,  $\varepsilon = 0 = \nu$  and  $Nb = 0 = Nt$ .

Pr	$-\theta'(0)$ [17]	Present results
0.07	0.0663	0.0663
0.20	0.1691	0.1691
0.70	0.4539	0.4539
2.00	0.9113	0.9113
7.00	1.8954	1.8951

**Figures 1** and **2** show the velocity distributions for variations in Deborah number  $\beta$  and nanofluid parameters( $Nb, Nt$ ), respectively. Due to the elastic nature of the Maxwell fluid to restore its deformation during flow, the Maxwell parameter restricts the fluid motion as depicted in **Figure 1**. Thus the hydrodynamic boundary layer thickness gets reduced under the impact of Deborah number. Also the presence of nanoparticles in the Maxwell fluid restricts the fluid motion which thereby reduces the fluid velocity. Thus from the figures it is clear that all these parameters reduce the hydrodynamic boundary layer thickness which as a result slows down the motion of the fluid.



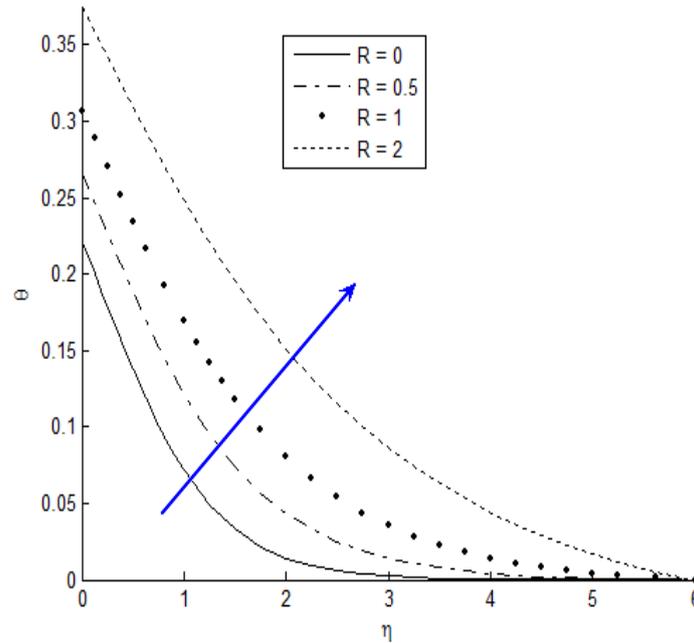
**Figure 1** Deborah number effects on velocity.



**Figure 2** Nanoparticles effect on velocity.

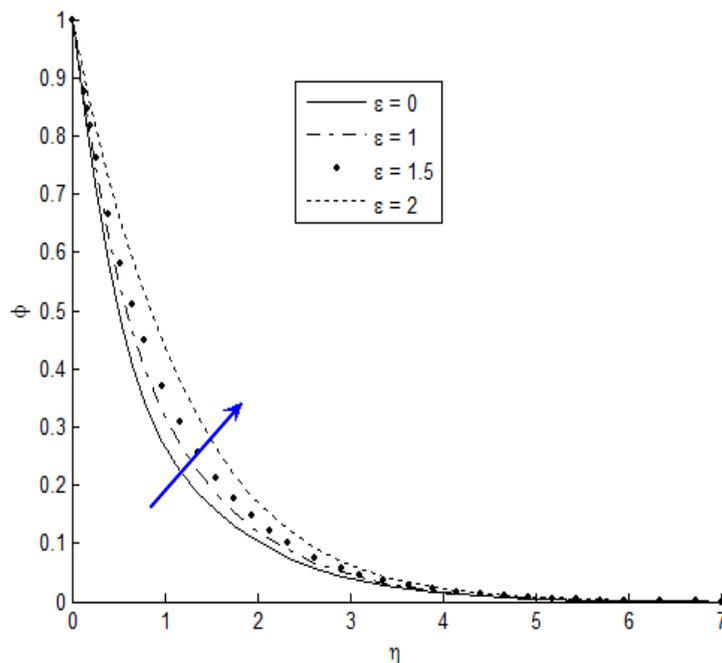
It is evident from the **Figures 3 - 12** that the thermal boundary layer thickens with the rise in the controlling parameters. The effects of these parameters on the temperature profiles are distinguishable only in the region near the stretching sheet although the curves merge at a large distance away from the surface. Unlike the temperature profiles, the nanoparticles concentration profiles are slightly affected by these parameters. Hike in the nanoparticles volume fraction is observed with the upsurge of fluid viscosity, thermal conductivity and thermophoresis parameter but the effect gets reversed with Brownian motion parameter.

**Figure 3** reveals that the thermal radiation increases the temperature profile. This arises owing to the reason that an enhancement in thermal radiation  $R$  has an inclination to increase the conduction effects in addition to the rise in temperature at each point away from the surface. Therefore higher value of radiation parameter implies higher surface heat flux.

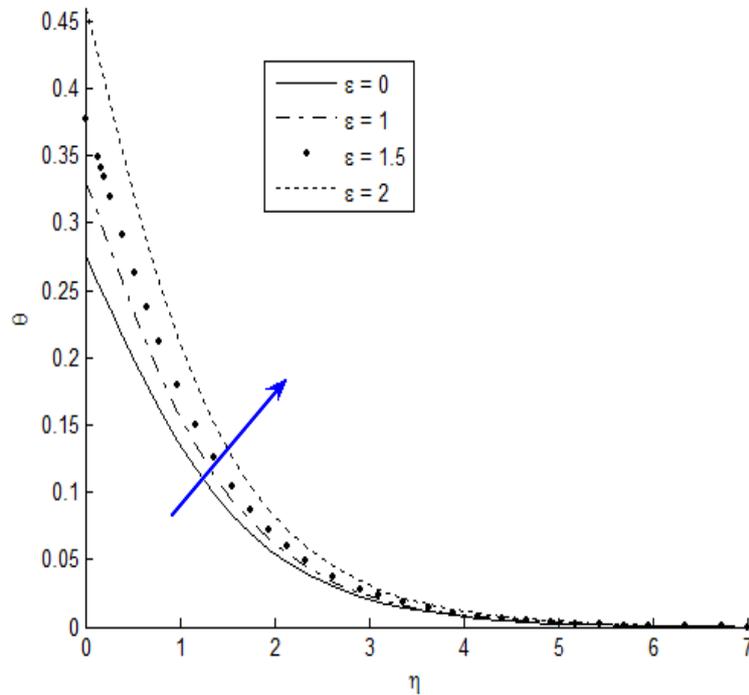


**Figure 3** Radiation effects on fluid temperature.

As the thermal conductivity  $\varepsilon$  of the fluid rises, the nanoparticles present in the fluid get aggregated near the surface which as a result increases the nanoparticles concentration and fluid temperature thereby thickening the related boundary layer as clearly visible in **Figures 4** and **5**.

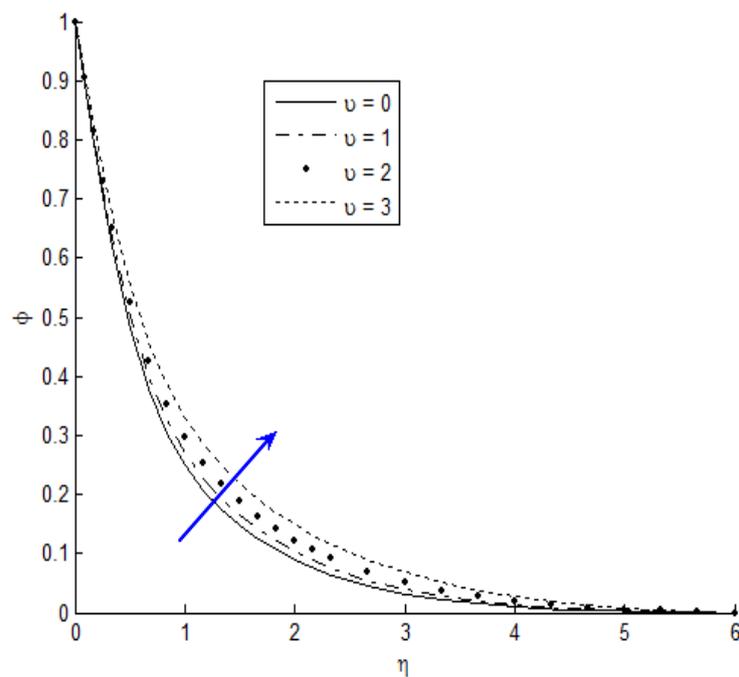


**Figure 4** Thermal conductivity effects on nanoparticles concentration.

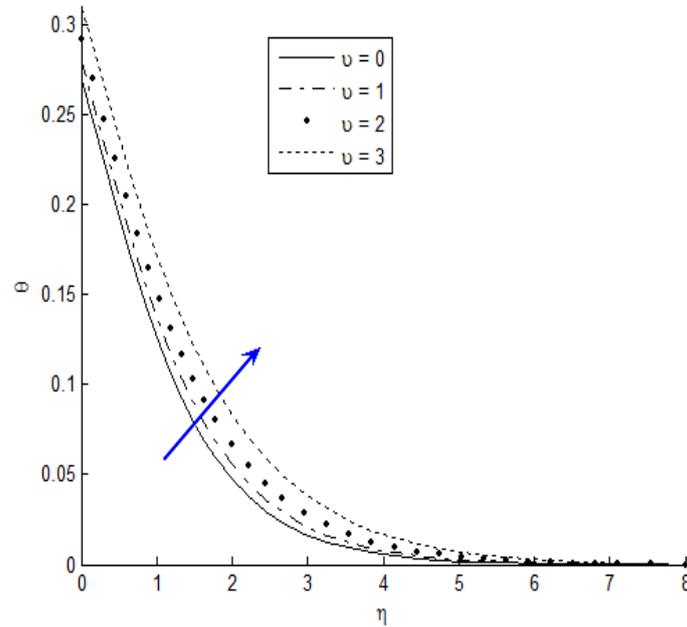


**Figure 5** Thermal conductivity effects on fluid temperature.

It is observable from the **Figures 6 and 7** that greater fluid viscosity  $\nu$  leads to the increment of both the fluid temperature and nanoparticles concentration. Fluid viscosity is that property of the fluid which restricts the fluid motion. Upsurge in the fluid viscosity allows the nanoparticles to move closer thereby enhancing both the nanoparticles volume fraction and thermal boundary layer thickness.



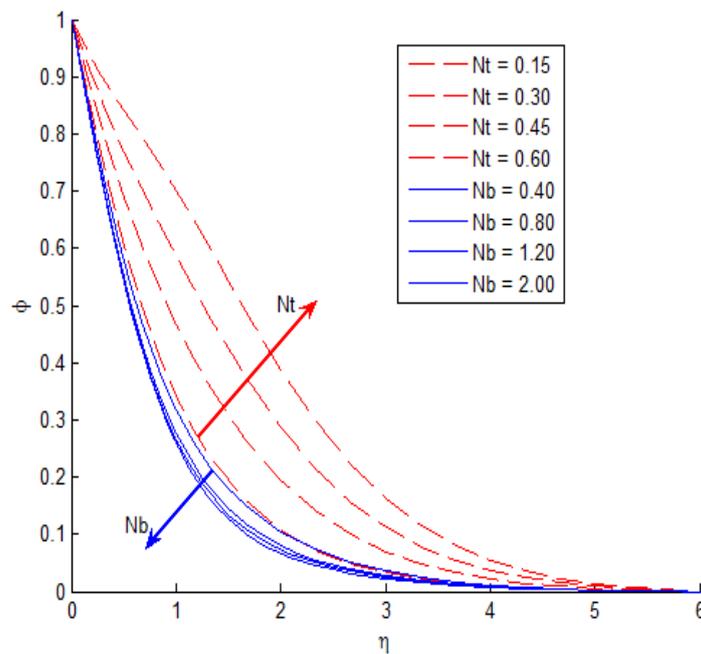
**Figure 6** Viscosity effects on nanoparticles concentration.



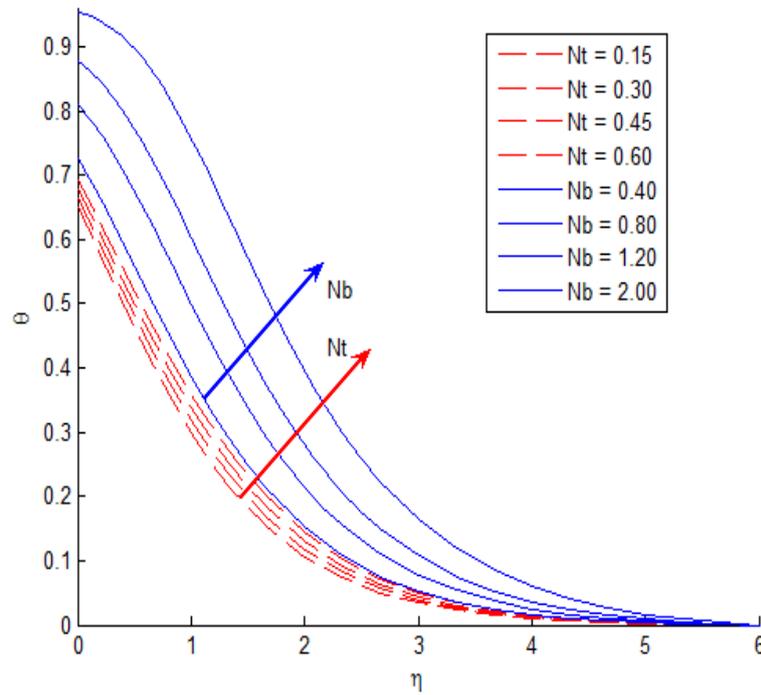
**Figure 7** Viscosity effects on fluid temperature.

The thermophoresis parameter  $Nt$  generates a thermophoretic force which leads to the fast movement of particles away from the sheet with hotter surface to the colder one. As a result the concentration of the nanoparticles and the fluid temperature intensify with the rising values of  $Nt$  thereby thickening both the thermal and concentration boundary layer as shown in **Figures 8** and **9**.

Brownian motion is an irregular motion which is generated by the Brownian motion parameter. Such motion increases the kinetic energy of the particles and the collision between the particles increases. As a result a reduction in nanoparticle volume fraction is observed with the enhancement of  $Nb$  as evident from **Figure 8**. Also increment in kinetic energy due to the irregular motion leads to the thickening of the thermal boundary layer as depicted in **Figure 9**.

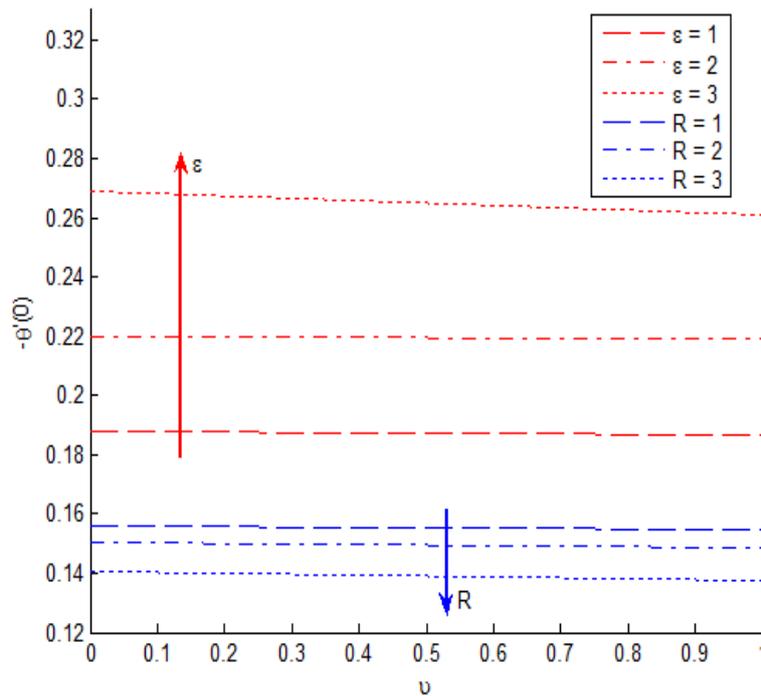


**Figure 8** Nanofluid parameters effect on nanoparticles concentration.

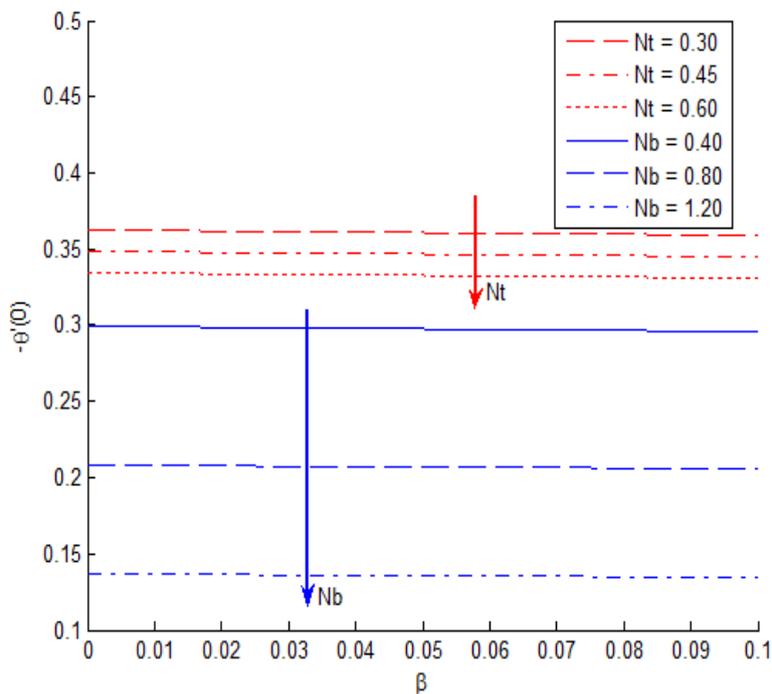


**Figure 9** Nanofluid parameters effect on fluid temperature.

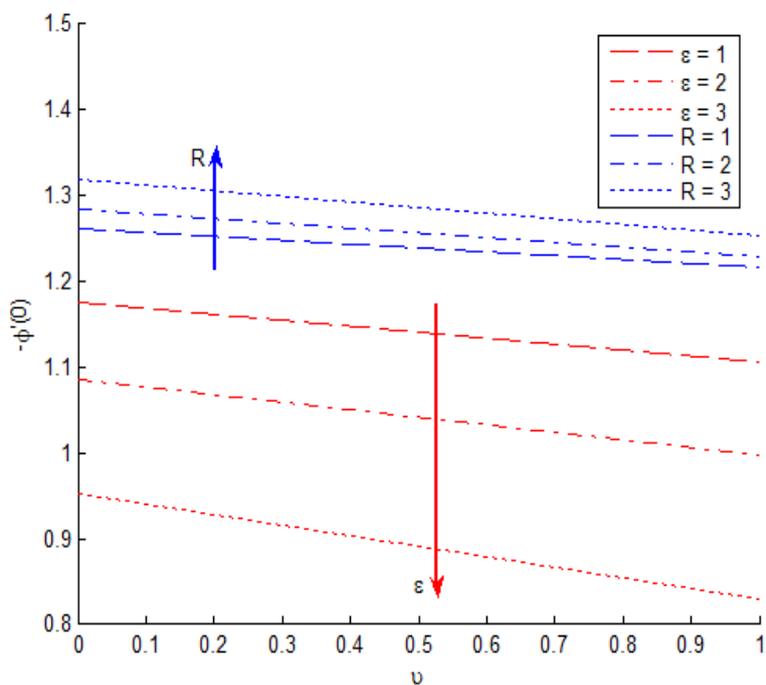
**Figures 10 and 11** depict the effects of the parameters on the local Nusselt number. It is noticed that the rate of heat transfer is favoured by thermal conductivity but opposed by variable viscosity, Maxwell, radiation and nanofluid parameters. The variations on local Sherwood number due to the controlling parameters are shown in **Figures 12 and 13**. The rate of mass transfer is increased by radiation parameter and Brownian motion parameter but reduced by the remaining parameters.



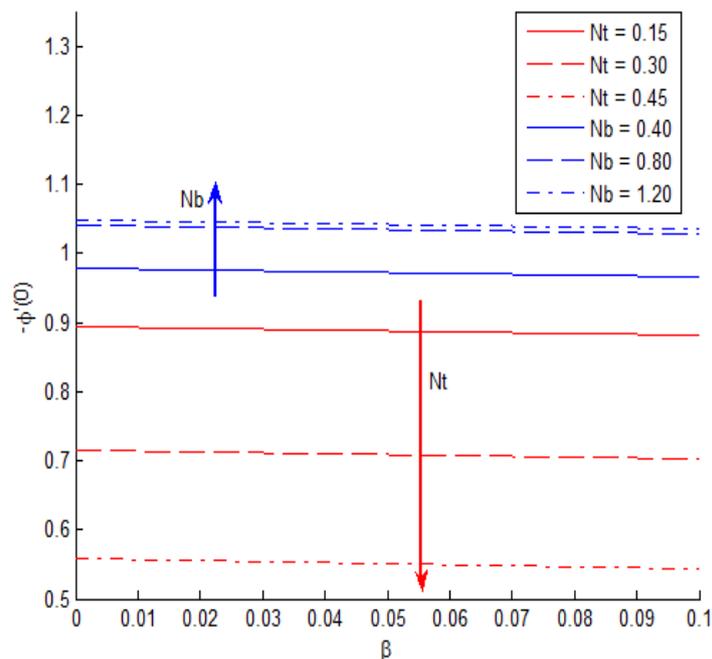
**Figure 10** Nusselt number variations with viscosity, thermal conductivity and radiation parameters.



**Figure 11** Nusselt number variations with Deborah number and nanofluid parameters.



**Figure 12** Sherwood number variation with viscosity, thermal conductivity and radiation parameters.



**Figure 13** Sherwood number variation with Deborah number and nanofluid parameters.

## Conclusions

In the presence of nanoparticles, the flow of Maxwell fluid under the influence of variable thermal conductivity, thermal radiation and variable fluid viscosity are investigated past a convectively heated shrinking/stretching surface. The outcomes of the present analysis can be outlined as follows: (1) The fluid velocity, fluid temperature and species concentration decrease monotonically from the maximum value at the surface to the minimum value at the end of the boundary layer. (2) The controlling parameters reduce the fluid velocity decelerating the fluid motion. (3) The controlling parameters thicken the thermal boundary layer and enhance the fluid temperature. (4) The nanoparticles volume fraction reduces with the Brownian motion parameter but intensifies with the other controlling parameters. (5) The thermal conductivity boosts the local Nusselt number thereby accelerating the rate of heat transfer. (6) The thermal radiation and Brownian motion parameters increase the reduced Sherwood number favouring the rate of mass transfer.

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