

Five Dimensional FRW Radiating Cosmological Model in Presence of Bulk Viscous Fluid in Scalar Tensor Theory of Gravitation

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Abstract

In this paper using the Saez-Ballester scalar-tensor theory of gravitation, we examined a 5-dimensional FRW cosmic space-time in a source of bulk viscous fluid in the article. To examine determinate solutions of the field equations, we used a power law between a scalar field and the universe's scale factor. Our research considers radiating flat, closed, and open models. The physical and kinematical properties of the models were explored in each scenario. In this study, we show that our model expands and is free of initial singularities, as well as that our models decelerate in a conventional manner.

Keywords: Bulk viscous model, Radiating models, Scalar-Tensor theory, FRW models

Introduction

We know that the matter distribution in our universe is appropriately illustrated by a perfect fluid due to the vast scale distribution of galaxies. For a practical study of the problem, a state of material dissemination other than perfect fluid is required. When neutrino decoupling occurs, Misner [1] characterized the matter as a viscous fluid in the early phases of the universe. Riess *et al.* [2] and Perlmutter *et al.* [3] explored the function of bulk viscosity in the recent scenario of accelerated expansion of the universe, which is known as the inflationary phase in cosmology. As a result, there has been a lot of interest in studying cosmological models with bulk viscosity in recent years. Pavon *et al.* [4], Mohanty and Pradhan [5], Pimentel [6], Rao *et al.* [7], and Naidu *et al.* [8] have all looked into general relativity and modified theories of gravity. Banerjee *et al.* [9] have also defined Bianchi type-I cosmological models with viscous fluid in higher dimensional space time. Mohanty *et al.* [10] have investigated higher dimensional string cosmological model with bulk viscous fluid in Lyra manifold. Naidu *et al.* [11] have defined 5 dimensional Kaluza-Klein bulk viscous models in modified theories of gravitation. In a simple method, Saez and Ballester [12] investigated a theory in which the metric is paired with a dimensionless scalar field. This combination provides a sufficient explanation for the weak fields. Because of the scalar field's dimensionlessness, an antigravity system emerges. In the non-flat FRW model, this theory suggests a possible solution to define the missing matter problem. In this idea, Saez examined the original singularity and extension universe, as well as the fact that an antigravity system exists at either the beginning of the extension era or before it. Many authors (Reddy and Rao [13], Mohanty and Sahu [14], Reddy and Naidu [15], Singh *et al.* [16], Pradhan *et al.* [17], Reddy *et al.* [18], Katore and Shaikh [19], Rao *et al.* [20], Ram *et al.* [21], Yadav *et al.* [22], Santhi and Sobhanbabub [23]) have studied Saez-Ballester scalar-tensor in different cosmological models. Recently Mishra and Dua [24], Naidu *et al.* [25] had investigated Saez-Ballester Scalar tensor using different cosmological models.

The field equations for combined scalar and tensor fields, proposed by Saez-Ballester are;

$$R_{ij} - \frac{1}{2}g_{ij}R - \omega\phi^n \left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k} \right) = -8\pi T_{ij} \quad (1)$$

where the scalar field ϕ satisfies the equation

$$2\phi^n \phi'^i_{,i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0 \quad (2)$$

and

$$T_{;j}^{ij} = 0. \quad (3)$$

Eq. (3) is a consequence of the field Eqs. (1) and (2), ω and n are constants. Comma and semicolon represent partial and covariant differentiation respectively.

There has been huge attract in the investigation of higher dimensional space-time in current years due to the evidence that the cosmos at its beginning period of expansion of the universe that might have had a higher dimensional epoch. This evidence had attracted several authors (Venkateswarlu and Kumar [26], Khadekar and Avachar [27], Bahrehbakhsh *et al.* [28], Biswal *et al.* [29], Venkateswarlu *et al.* [30] Oli [31], Ramprasad *et al.* [32], Rao *et at.* [33], Aygun *et al.* [34], Caglar *et al.* [35], Caglar and Aygun [36], Singh and Singh [37]) to study to the field of higher dimensions. We know that at beginning period of times before the universe has undergone compactification transitions the results of the field equations in general relativity and in scalar-tensor theories in higher dimensional space-time are of substantial purpose probably. Marciano [38] has advised that the investigational conclusion of the theoretical constants with varying time might develop the information of additional dimensions. Currently, Gomez *et al.* [39], Trivedi and Bhabor [40], Das and Bharali [41] have studied 5 dimensional FRW models using different type of scalar tensor.

In this study, we define 5-dimensional FRW radiating models in the presence of bulk viscous cosmological models in the scalar-tensor theory of gravitation, as a result of the previous investigations and discussion. We discussed about bulk viscous fluid, Saez-Ballester, and FRW models in Section 1. The Section 2, contains metric and field equations. In Section 3, we look at Saez-Ballester cosmological models with an equation of state that is equivalent to disordered radiation in general relativity. The physical explanation of the models is covered in Section 4, and the conclusions are given in Section 5.

Metric and field equations

Here we assume the 5-dimensional FRW metric in the following form;

$$ds^2 = dt^2 - a^2(t) \left[\frac{dr^2}{(1-kr^2)} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) + (1 - kr^2)d\psi^2 \right] \quad (4)$$

From Eq. (4), we get the non-vanishing components of Einstein tensor in the following way;

$$G_1^1 = G_2^2 = G_3^3 = G_4^4 = 3 \frac{a''}{a} + 3 \frac{a'^2}{a^2} + 3 \frac{k}{a^2} \quad (5)$$

and

$$G_5^5 = 6 \frac{a'^2}{a^2} + 6 \frac{k}{a^2} \quad (6)$$

Here a prime symbol means differentiation with respect to time t . Also $k = +1, -1, 0$ denotes *closed, open and flat* models respectively.

For bulk viscous fluid the energy momentum tensor is given by in the following way;

$$T_{ij} = (\rho + \bar{p})u_i u_j - g_{ij} \bar{p} \quad i, j = 1, 2, 3, 4, 5 \quad (7)$$

Together with,

$$u_i u_i = 1 \quad \text{and} \quad u_i u_j = 0 \quad (8)$$

Therefore,

$$T = \rho - 4\bar{p} \quad (9)$$

The total pressure with the proper pressure containing bulk viscosity coefficient ζ and Hubble expansion parameter H are defined in the following way;

$$\bar{p} = p - 3\zeta H = \varepsilon \rho, \quad \text{where} \quad p = \varepsilon_0 \rho, \quad \text{and} \quad \varepsilon = \varepsilon_0 - \beta \quad (10)$$

By applying co moving coordinates and with the help of Eqs. (3) and (5) - (9), the Saez-Ballester field Eqs. (1) and (2) for the metric (4), becomes in the following way;

$$6 \frac{a'^2}{a^2} + 6 \frac{k}{a^2} - \frac{\omega \phi'^2}{2 \phi^2} = -8\pi\rho \quad (11)$$

$$3 \frac{a''}{a} + 3 \frac{a'^2}{a^2} + 3 \frac{k}{a^2} + \frac{\omega \phi'^2}{2 \phi^2} = 8\pi\bar{p} \quad (12)$$

$$\frac{\phi''}{\phi} + 4 \frac{a' \phi'}{a \phi} + \frac{n \phi'^2}{2 \phi^2} = 0 \quad (13)$$

$$\rho' + 4 \frac{a'}{a} (\rho + \bar{p}) = 0 \quad (14)$$

We know that,
Hubble parameter H is

$$H = \frac{a'}{a} \quad (15)$$

and the deceleration parameter q is

$$q = -\frac{(H'+H^2)}{H^2} \quad (16)$$

Solutions and the models

In this paper we apply the condition given by Eq. (10). We also apply the relation between scalar field ϕ and the scale factor of the universe $a(t)$ which was defined by Pimental [42]; Johri and Kalyani [43].

$$\phi = \phi_0 a^n \quad (17)$$

where $n > 0$ are constants.

Here we find the solutions of the field Eqs. (11) - (13) for $k = +1, -1, 0$ (i.e closed, open and flat model) respectively.

Case (i): For $k = 1$, (i. e. Closed model)

Using Eq. (17) in the field Eqs. (11) - (13) we get the scale factor in the following form;

$$a(t) = \left[\left\{ \frac{\left(\frac{n^2}{2} + n + 4 \right)}{n \phi_0 \left(\frac{n+2}{2} \right)} \right\} (a_0 t + t_0) \right]^{\frac{1}{\left(\frac{n^2}{2} + n + 4 \right)}} \quad (18)$$

Now by choosing $a_0 = 1$ and $t_0 = 0$ and using (18), the metric (4) becomes in the following form;

$$ds^2 = dt^2 - \left[\left\{ \frac{\left(\frac{n^2}{2} + n + 4 \right)}{n \phi_0 \left(\frac{n+2}{2} \right)} \right\} t \right]^{\frac{2}{\left(\frac{n^2}{2} + n + 4 \right)}} \left[\frac{dr^2}{(1-r^2)} + r^2 (d\Theta^2 + \sin^2 \Theta d\Phi^2) + (1-r^2) d\psi^2 \right] \quad (19)$$

where the scalar field is

$$\phi = \phi_0 \left[\left\{ \frac{\left(\frac{n^2}{2} + n + 4 \right)}{n \phi_0 \left(\frac{n+2}{2} \right)} \right\} t \right]^{\frac{n}{\left(\frac{n^2}{2} + n + 4 \right)}} \quad (20)$$

The Eq. (19) denotes a 5-dimensional FRW bulk viscous radiating model.
The Spatial volume is

$$V = a^4 = \left[\left(\frac{\left(\frac{n^2}{2} + n + 4 \right)}{n\phi_0 \left(\frac{n+2}{2} \right)} \right) t \right]^{\frac{4}{\left(\frac{n^2}{2} + n + 4 \right)}} \quad (21)$$

Hubble's parameter H is

$$H = \left(\frac{1}{\frac{n^2}{2} + n + 4} \right) \frac{1}{t} \quad (22)$$

The energy density ρ is

$$8\pi\rho = \left\{ \frac{(\omega n^2 - 12)}{2t^2 \left(\frac{n^2}{2} + n + 4 \right)^2} \right\} - 6 \left\{ \left(\frac{\frac{n^2}{2} + n + 4}{n\phi_0 \left(\frac{n+2}{2} \right)} \right) t \right\}^{-\frac{2}{\left(\frac{n^2}{2} + n + 4 \right)}} \quad (23)$$

The isotropic pressure p is

$$8\pi p = \epsilon_0 \left[\left\{ \frac{(\omega n^2 - 12)}{2t^2 \left(\frac{n^2}{2} + n + 4 \right)^2} \right\} - 6 \left\{ \left(\frac{\frac{n^2}{2} + n + 4}{n\phi_0 \left(\frac{n+2}{2} \right)} \right) t \right\}^{-\frac{2}{\left(\frac{n^2}{2} + n + 4 \right)}} \right] \quad (24)$$

The coefficient of bulk ζ viscosity is

$$8\pi\zeta = \left\{ \frac{\left(\frac{n^2}{2} + n + 4 \right) (\epsilon - \epsilon_0) t}{3} \right\} \left[\left\{ \frac{(\omega n^2 - 12)}{2t^2 \left(\frac{n^2}{2} + n + 4 \right)^2} \right\} - 6 \left\{ \left(\frac{\frac{n^2}{2} + n + 4}{n\phi_0 \left(\frac{n+2}{2} \right)} \right) t \right\}^{-\frac{2}{\left(\frac{n^2}{2} + n + 4 \right)}} \right] \quad (25)$$

Case (ii): For $k = -1$, (i. e. Open model)

Here the model is open and is given by

$$ds^2 = dt^2 - \left[\left(\frac{\left(\frac{n^2}{2} + n + 4 \right)}{n\phi_0 \left(\frac{n+2}{2} \right)} \right) t \right]^{\frac{2}{\left(\frac{n^2}{2} + n + 4 \right)}} \left[\frac{dr^2}{(1+r^2)} + r^2(d\theta^2 + \sin^2\theta d\phi^2) + (1+r^2)dp^2 \right] \quad (26)$$

Along with the energy density ρ

$$8\pi\rho = \left\{ \frac{(\omega n^2 - 12)}{2t^2 \left(\frac{n^2}{2} + n + 4 \right)^2} \right\} + 6 \left\{ \left(\frac{\frac{n^2}{2} + n + 4}{n\phi_0 \left(\frac{n+2}{2} \right)} \right) t \right\}^{-\frac{2}{\left(\frac{n^2}{2} + n + 4 \right)}} \quad (27)$$

Isotropic pressure p is

$$8\pi p = \epsilon_0 \left[\left\{ \frac{(\omega n^2 - 12)}{2t^2 \left(\frac{n^2}{2} + n + 4 \right)^2} \right\} + 6 \left\{ \left(\frac{\frac{n^2}{2} + n + 4}{n\phi_0 \left(\frac{n+2}{2} \right)} \right) t \right\}^{-\frac{2}{\left(\frac{n^2}{2} + n + 4 \right)}} \right] \quad (28)$$

The coefficient of bulk viscosity ζ is

$$8\pi\zeta = \left\{ \frac{\left(\frac{n^2}{2} + n + 4 \right) (\epsilon - \epsilon_0) t}{3} \right\} \left[\left\{ \frac{(\omega n^2 - 12)}{2t^2 \left(\frac{n^2}{2} + n + 4 \right)^2} \right\} + 6 \left\{ \left(\frac{\frac{n^2}{2} + n + 4}{n\phi_0 \left(\frac{n+2}{2} \right)} \right) t \right\}^{-\frac{2}{\left(\frac{n^2}{2} + n + 4 \right)}} \right] \quad (29)$$

In this case the scalar field, spatial volume and Hubble parameter are given by Eqs. (20) - (22), respectively.

Case (iii): For $k = 0$, (i. e. Flat model)

In this case the model is flat and define by as follows;

$$ds^2 = dt^2 - \left[\frac{\left(\frac{n^2}{2} + n + 4 \right)}{n\phi_0 \left(\frac{n^2}{2} \right)} \right] t^{\left(\frac{n^2}{2} + n + 4 \right)} [dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2) + d\psi^2] \quad (30)$$

The energy density ρ is

$$8\pi\rho = \left\{ \frac{(\omega n^2 - 12)}{2t^2 \left(\frac{n^2}{2} + n + 4 \right)^2} \right\} \quad (31)$$

Pressure p is

$$8\pi p = \epsilon_0 \left[\left\{ \frac{(\omega n^2 - 12)}{2t^2 \left(\frac{n^2}{2} + n + 4 \right)^2} \right\} \right] \quad (32)$$

The coefficient of bulk viscosity ζ is

$$8\pi\zeta = \left\{ \frac{\left(\frac{n^2}{2} + n + 4 \right) (\epsilon - \epsilon_0) t}{3} \right\} \left[\left\{ \frac{(\omega n^2 - 12)}{2t^2 \left(\frac{n^2}{2} + n + 4 \right)^2} \right\} \right] \quad (33)$$

Here also the scalar field, spatial volume and Hubble parameter are given by Eqs. (20) - (22) respectively.

The deceleration parameter q for all the 3 cases (i. e. $k = +1, -1, 0$) is

$$q = \left(\frac{n^2}{2} + n + 3 \right) \quad (34)$$

Here the models decelerate in the standard way for all the above 3 cases (since we know that if $q > 0$ the universe decelerates in the standard way and accelerates when $q < 0$).

We plot all the following graphs with $\pi = 3.14$, $n = 1$, $\phi_0 = .001$, $\omega = 500$, $\epsilon_0 = -1$, $\epsilon = \frac{1}{3}$.

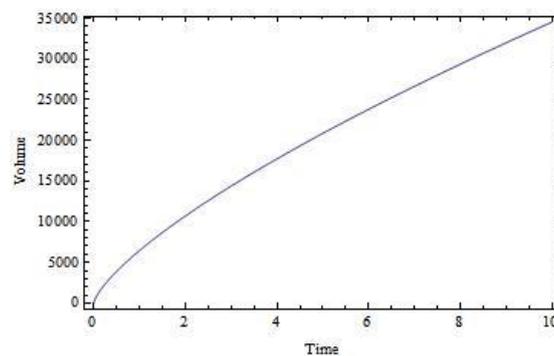


Figure 1 Volume V vs. time ($\pi = 3.14$, $n = 1$, $\phi_0 = 0.001$).

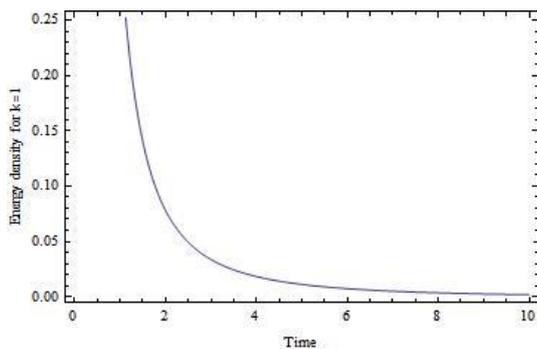


Figure 2 Energy density ρ vs. time for $k = 1$ ($\pi = 3.14, n = 1, \omega = 500, \phi_0 = 0.001$).

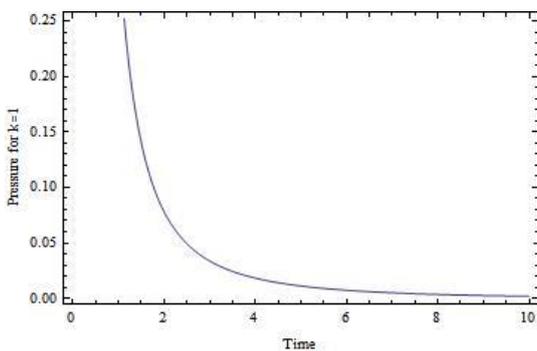


Figure 3 Pressure p vs. time for $k = 1$ ($\pi = 3.14, n = 1, \omega = 500, \phi_0 = 0.001$).

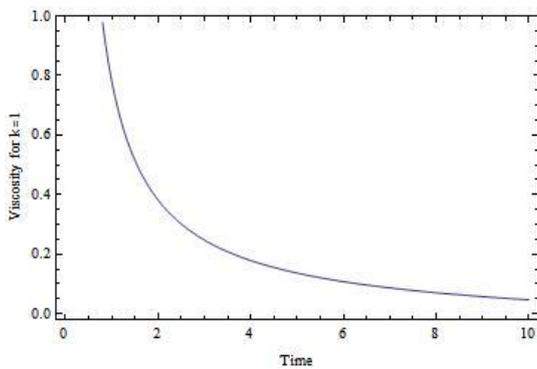


Figure 4 Viscosity ζ vs. time for $k = 1$ ($\pi = 3.14, n = 1, \omega = 500, \phi_0 = 0.001, \varepsilon_0 = -1, \varepsilon = \frac{1}{3}$).

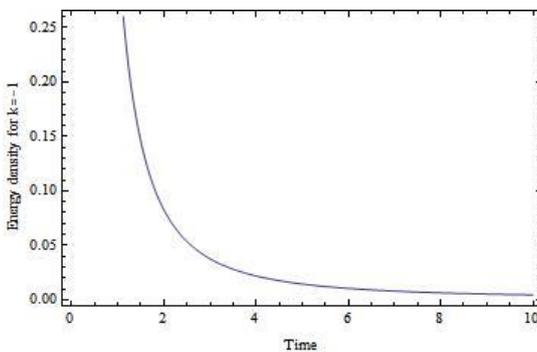


Figure 5 Energy density ρ vs. time for $k = -1$ ($\pi = 3.14, n = 1, \omega = 500, \phi_0 = 0.001$).

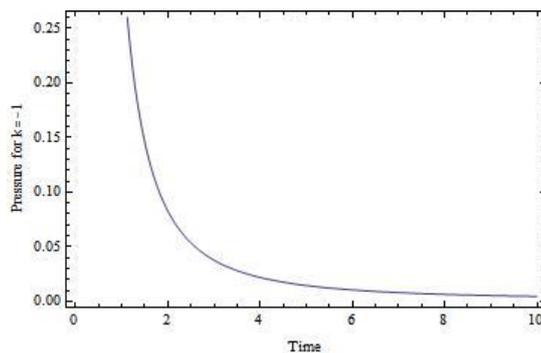


Figure 6 Pressure p vs. time for $k = -1$ ($\pi = 3.14, n = 1, \omega = 500, \phi_0 = 0.001$).

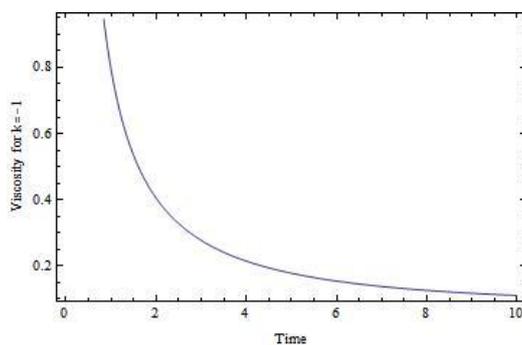


Figure 7 Viscosity ζ vs. time for $k = -1$ ($\pi = 3.14, n = 1, \omega = 500, \phi_0 = 0.001, \varepsilon_0 = -1, \varepsilon = \frac{1}{3}$).

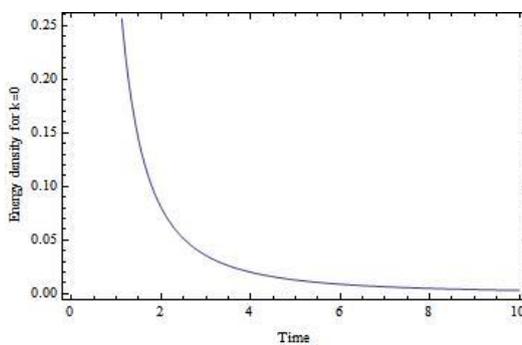


Figure 8 Energy density ρ vs. time for $k = 0$ ($\pi = 3.14, n = 1, \omega = 500, \phi_0 = 0.001$).

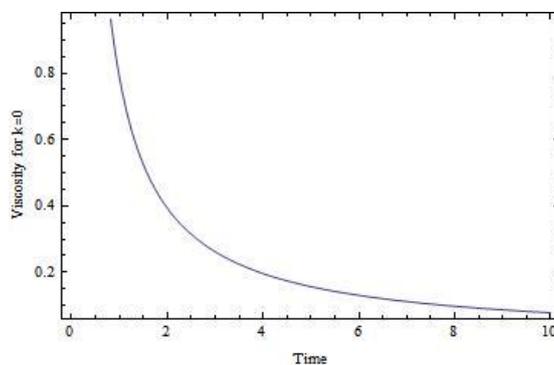


Figure 9 Pressure p vs. time for $k = 0$ ($\pi = 3.14, n = 1, \omega = 500, \phi_0 = 0.001$).

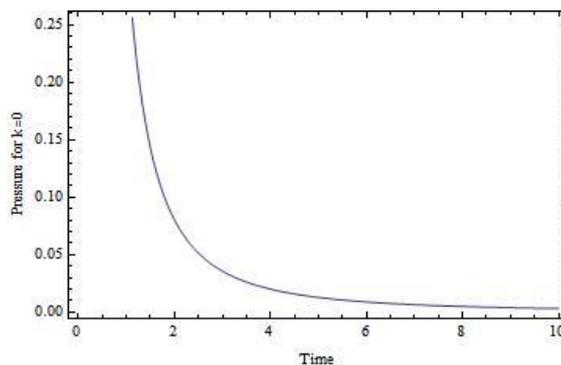


Figure 10 Viscosity ζ vs. time for $k = 0$ ($\pi = 3.14, n = 1, \omega = 500, \phi_0 = 0.001, \varepsilon_0 = -1, \varepsilon = \frac{1}{3}$).

Physical interpretation

In this paper the energy density, pressure and coefficient of bulk viscosity diverge at $t = 0$ and decrease with time for both in closed and open models (**Figures 2 - 7**). The energy density, the pressure and bulk viscosity decrease with time and will vanish for infinitely large time t in the flat model (**Figures 8 - 10**). All of them are diverge at the initial epoch. For all the models the spatial volume is same and increase with time but tends to infinity for infinitely large time (**Figure 1**). Eq. (22) is the average Hubble's parameter for all the models and will diverge at the initial epoch and will approach infinity as t becomes infinitely large. Also, Eqs. (19), (26) and (30) denotes FRW 5 dimensional radiating closed, open and flat models in Saez-Ballester theory respectively. For all the models the deceleration parameter is $q = (\frac{n^2}{2} + n + 3)$. Hence the models represented by Eqs. (19), (26) and (30) in 5 dimensions decelerates in the standard way. (Since we Know that if $n > 0$ the universe decelerates in the standard way and when $n < 0$ the universe accelerates). In this paper the models defined, in 5 dimensions FRW, are quite distinct from the Lyra geometry 5 dimensional models and the Kaluza-Klein 5 dimensional Models defined by many researchers.

Conclusions

Using the Saez-Ballester theory and a 5-dimensional FRW space-time as a source of bulk viscous fluid, we have obtained cosmological models that can be assumed to be equivalent radiating models in closed, open, and flat space-times. We have obtained models [i.e. Eqs. (19), (26) and (30)] that are expanding and free of initial singularity for all cases [i.e. Case I Case (ii), and Case (iii)]. The deceleration parameter found here decelerates in a conventional manner. The findings of this research aid our understanding of Saez-Ballester cosmology in 5 dimensions soon before compactification transition.

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