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Vibrations of Phase-Lags on Electro-Magneto Nonlocal Elastic Solid with Voids in Generalized Thermoelastic Cylinder/Disk

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Abstract

The stress-strain-temperature relations, strain-displacement relations and governing equations have been addressed for electro-magneto transversely isotropic nonlocal elastic hollow cylinder with voids in the reference of 3-phase-lag effect of heat conduction. The strength of the magnetic field proceeds in the direction of the z-axis of the hollow cylinder/disk. The simultaneous differential equations have been eliminated by applying elimination technique to obtain unknown field functions such as dilatation, equilibrated voids volume fraction, temperature, displacement and stresses. Free vibration analysis has been explored by applying stress free and thermally insulated/isothermal boundaries. Analytical results are verified by employing numerically analyzed results for unknown field functions and presented graphically for the vibrations of stress free field functions such as thermoelastric damping, frequencies and frequency-shift. The results have been authenticated by analyzing analytical and numerical results with existing literature with earlier published work. The study of present paper based on 3-phase-lag model of generalized thermoelasticity may receive better approach to allow voids and relaxation time parameters, which have many applications in the field of science, technology and engineering. The study may also be useful in the area of seismology for mining and drilling in the earth's crust.

Keywords: Transversely isotropic elastic material, Three-phase-lag model, Nonlocal elasticmaterial, Electro-magneto thermoelasticity, Voids

Introduction

Free vibrations of cylindrical and spherical structures are frequently used as structural components and its vibration characteristics are observed for practical design. The interactions between strain and electromagnetic fields with cylindrical structures have been assumed with its numerous applications in the field of science and technology. The development of magneto thermoelasticity with nonlocal elastic materials in presence of voids induces us to study numerous problems of continuum mechanics, geophysics and porous material disks. While studying different theories of thermoelasticity, Boit [1] predicted about infinite wave fronts that governing equations of thermoelasticity are parabolic-hyperbolic mixed type, but due to infinite speed of thermal waves, the phenomenon is not accepted. The classical thermoelasticity theories have been renamed as generalized thermoelasticity and converted to different models which supersede parabolic thermoelasticity to hyperbolic thermoelasticity. The propagation of infinite speed of velocity of thermal signals disagree the facts in classical theories of thermoelasticity. During last 3decades' non-classical theories generates finite speed of transportation of heat in elastic solids have been developed to remove the paradox. Thus contradiction of infinite velocity of transmission was eliminated by Lord and Shulman (LS) [2] in modification of Fourier law of heat conduction model. Green and Lindsay (GL) [3] explored the theory of entropy by introducing relaxation time parameters t_0 and t_1 (thermal and mechanical) that consider a finite speed of transmission of heat. The existence of theories of thermoelasticity having finite speed of thermal signals was studied by Chandrashekhariah [4] to occupy a hyperbolic heat transport equation which occur wave-type second sound effects. Latterly, Chandrashekhariah [5] and Tzou [6] proposed a dual-phase-lag (DPL) model of heat conduction to include phonon-electron interactions, microscopic interactions etc. in time and space. A new model named 3-phase-lag (TPL) model with the support of theory of generalized thermoelasticity was introduced by Roychoudhuri [7]. In this model, the 3 different phase-lags have been introduced which obey classical Fourier's law. The uniqueness and well-posed conditions of TPL theory of generalized thermoelasticity was verified by Quintanilla [8]. The TPL model based on viscoelastic solid of spherical shell due to step input of temperature in the reference of generalized thermoelasticity was studied by Kar and Kanoria [9]. Magana *et al*. [10] examined the stability in TPL model of heat conduction in 2 temperature theory of thermoelasticity. Abo-Dahab and Biswas [11] investigated the effect of TPL model on refraction and reflection of solid liquid interface with initial stress and magnetic field of thermoelastic wave to develop amplitude ratio of P waves (primary waves: they are always the first to arrive) and SV waves (shear waves: A shear wave that is polarized so that its particle motion and direction of propagation occur in a vertical plane). Ezzat and Karamany [12] studied the theory of fractional order with the assumption of Fourier law under 3-phase-lag (TPL) model of generalized thermoelasticity. Lots of research work based on Green-Naghdi model, 3-phase-lag (TPL) and dual-phase-lag (DPL) models of generalized thermoelasticity have been done by some research groups, for instance Sharma *et al*. [13-14], Atwa [15], Sharma *et al*. [16], Deswal and Kalkal [17], etc.

The nonlocal theories of elasticity, thermoelasticity and electro-magneto thermoelastic solids, the governing equations and laws of equilibrium were reviewed by Eringen [18-20]. This theory states that the applied stress at a point ***x*** of continuous body not only depends on the point of strain, but it has been organized by strains at every other region surrounding points of translational motion. Eringen [21] investigated polar field like models and surface waves in the reference nonlocal elasticity theory. The uniqueness of this theory in the preview of continuum mechanics was studied by Wang and Dhaliwal [22]. Zenkour and Abouelregal [23] proposed a beam based model of nonlocal thermoelasticity by considering thermal conductivity with DPL model of generalized thermoelasticity. Thermoelastic response vibrations in which thermal loadings are applied in the context of the nonlocal theory of generalized thermo-elasticity was investigated by Yu *et al*. [24]. The Rayleigh surface wave propagation of semi-infinite solid with magnetic field in nonlocal elasticity was studied by Roy *et al*. [25]. The nonlocal elasticity theory was established thoroughly in the context of continuum mechanics, by some researchers such as Eringen **[**26**]**, Bachher and Sarkar [27], Sharma *et al*. [28,29] etc.

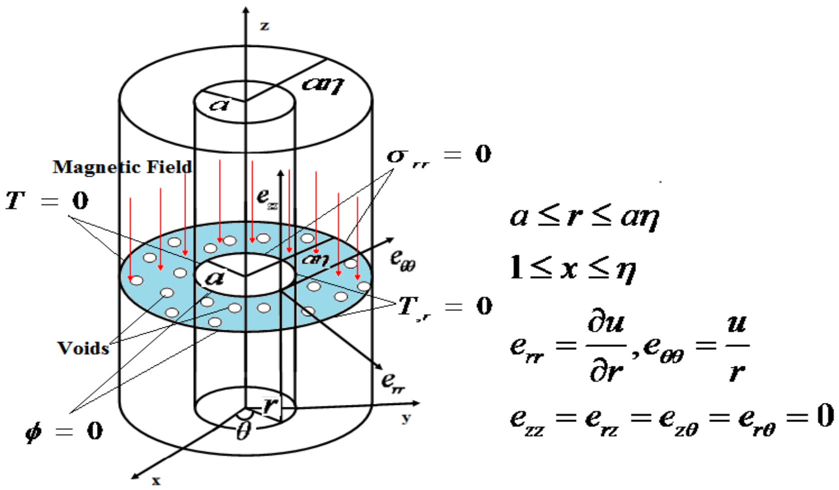
The voids theory with regard to non-linear theory of elasticity was given by Nunziato and Cowin [30]. Later on, Cowin and Nunziato [31] formulated the linear version of voids solid theory in respect of elasticity. Puri and Cowin [32] investigated the harmonic plane waves in the behavior of linear elasticity in the theory of 2 dilatational waves with voids. Iesan [33,34] represented some theories based on the properties of materials with elasticity and thermoelasticity in respect of voids in detail. This theory has added some new features on porous materials, which allows a void (porous) body to reduce and enlarge the body overall volume without body forces. Chandrashekhariah [35] studied the elastic half space surface waves with voids in the context of continuum mechanics. Sharma *et al*. [36] studied the vibration analysis of 3 dimensional cylindrical penal with voids material and represented the field functions analytically. Ezzat and Youssef [37] applied dual Fourier and Laplace transform technique to investigate the magneto thermoelasticity in conducting media analytically and presented the field functions graphically. The analysis of thermal shock applied on generalized magneto thermoelastic circular, annular cylinder with inner surface is considered stress free was studied by Abo-Dahab and Abbas [38]. Das *et al*. [39] used finite element method to study the effect of TPL model on transversely isotropic magneto thermoelastic cylinder with thermal heat source boundary conditions. Abo-Dahab and Biswas [40] studied the Rayleigh wave propagation with rotation in transversely isotropic thermoelastic medium. The effect of rotation on voids electro-magneto thermoelasticity with theory of energy dissipation was studied by Othman and Hilal [41]. The TPL model of magneto thermoelastic solid solution for thin moving rod of thermal distributions with memory dependent thermoelasticity was investigated by Mondal and Kanoria [42]. Sharma *et al*. [43,44] explored free vibrations of thermoelastic cylinder and sphere with voids material in the reference of generalized thermoelasticity. Keles and Tutuncu [45] used Laplace transformation to examine the vibration analysis of inhomogeneous elastic cylinder/sphere and presented the analysis for free and forced vibrations. Lots of research work based on vibrations of wave propagation in the reference of homogeneous/non-homogeneous materials was done by many authors and researchers such as Sharma *et al*. [46], Youssef [47], Sharma *et al*. [48,49], etc.

The developments in the field of traction free vibrations for electro-magneto thermoelastic spherical/cylindrical disks with voids and its use in material engineering, continuum mechanics, also the relevance of high-temperature electronics, drilling and mining in the earth's crust appliances, lightweight armors, etc. has compelled to study vibrations of natural frequencies, quality factor, frequency shift of such materials. Very few authors have used such models to traction free vibrations on disks based on cylindrical/spherical structures. This persuades the authors to analyze free vibrations with the model of 3-phase-lag (TPL) model of electro-magneto generalized thermoelastic cylinder with voids. Therefore, the main aim of current paper is to study the free vibrations in TPL model of transversely isotropic generalized electro-magneto nonlocal thermoelastic hollow cylinder/disk with voids. The time harmonics technique has been employed to govern equations and constitutive relations. The elimination method is used to evaluate unknown field functions by using considered boundary conditions. To verify the effects of TPL model of generalized thermoelasticity, the analytical results have been shown graphically in absence/presence of magnetic field and authenticated with existing literature.

Materials and methods

The basic fundamental equations and mathematical model

We propose a transversely isotropic nonlocal magneto-thermoelastic hollow cylinder with voids material of TPL model has been presented in the reference of generalized thermoelasticity .The inner and outer radii of hollow cylinder are assumed as $R_I = a$, $R_O = a\eta$ with the domain $a \le r \le a\eta$ and the surfaces are considered free from internal and external mechanical/thermal loads .The hollow cylinder is considered perfectly conductive and initially at undisturbed state with uniform temperature T_0 .The strength of magnetic field $\boldsymbol{H}$ and cylindrical coordinates (r, θ, z) proceeds in z direction of the axis . The field components are displacement vector $\boldsymbol{u} = (u_r, u_\theta, u_z)$ where $u_\theta = 0$, $u_z = 0$, $u_r = u(r,t)$, concentration of voids volume fraction $\varphi = \varphi(r,t)$ and temperature component $T = T(r,t)$. Following Cowin and Nunziato]31[, Das *et al*] .39 [and Dhaliwal and Singh]53[, the Maxwell's equations in the absence of charge density and displacement current, with the impact of electromagnetic field the equation of motion, equation of voids equilibrated volume fraction, and heat conduction equation without body forces and heat sources are given as;



Geometry of the problem

Strain-displacement relations

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \tag{1}$$

where e_{ij}; $(i, j = r, \theta, \phi)$ are strain components; $\boldsymbol{u} = (u_r, 0, 0)$ is displacement vector.

Local-nonlocal stress relations

$$\left(1 - \zeta^2\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\right)\sigma_{ij} = \sigma_{ij}^L \quad (i, j = r, \theta). \tag{2}$$

Here the quantities having superscript "L" stands for the local medium; $\zeta = e_0 a_0$ is non local parameter, where a_0 is internal characteristic length and e_0 is material constant; σ_{ij}; $(i, j = r, \theta)$ are stress components; also $\sigma_{ij} = \sigma_{ij}^L$.

Constitutive relations

$$\left(1-\zeta^2\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)\right)\sigma_{ij} = c_{ij}e_{kl} + b_{ij}\varphi - \beta_{ij}T, \tag{3}$$

Here β_{ij}; $(i, j = r, \theta)$ is the components of thermal modulii where $\beta_r = \beta_\theta = (c_{11} + c_{12})\alpha_1 + c_{13}\alpha_3$; $\alpha_1 = \alpha_3 = \alpha_T$ is coefficient of linear thermal expansion (Dhaliwal and Singh [53]), T is assumed as increase in the reference temperature T_0 of the medium, $b_{ij} = b$ is the voids parameter, φ is the void volume fraction.

Modified fourier's law

By introducing 3 phase-lags, namely thermal displacement gradient t_v; heat flux t_q and temperature gradient t_T, the classical Fourier law $\vec{q} = -K\vec{\nabla}T$ has been modified as given below;

$$\vec{q}_i(P,\ t+t_q) = -\left(K\vec{\nabla}T(P,\ t+t_T) + K^*\vec{\nabla}v(P, t+t_v)\right), \tag{4}$$

where K, K^* and $\vec{\nabla}v$ are thermal conductivity, additional material constant of characteristic theory, thermal displacement gradient; q_i are the heat flux vector components.

The entropy strain-temperature-voids relations

$$\rho S = \frac{\rho C_e}{T_0}T + \beta_{ij}e_{ij} + M\varphi, \tag{5}$$

where ρ is mass density; S is entropy per unit mass, M is thermo-void coupling parameter, C_e is specific heat at constant strain.

The equilibrated force balance equation

$$\rho\chi\left(1-\zeta^2\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)\right)\frac{\partial^2\varphi}{\partial t^2} - h_{i,j} = -b_{ij}e_{ij} - \left(\xi_1 + \xi_2\frac{\partial}{\partial t}\right)\varphi + MT. \tag{6}$$

The relation between volume fraction gradient and equilibrated stress vector

$$h_i = \alpha_{ij}\varphi_{,i}, \tag{7}$$

where χ is the equilibrated inertia, ξ_1, ξ_2 are the material constants of voids, h_i is equilibrated stress vector, $\alpha_{ij} = \alpha$ is the void parameters.

The energy equation

$$\rho \frac{\partial S}{\partial t} T_0 = -q_{i,i} \,, \tag{8}$$

where S is entropy per unit mass.

Equation of small motion in tensor form

$$\sigma_{ij,j} + F_i = \rho \left(1 - \zeta^2 \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} \right) \right) \frac{\partial^2 u}{\partial t^2} \,. \tag{9}$$

Here $\boldsymbol{F}_i ; (i = r, \theta, z)$ are the components of body force $\boldsymbol{F} = (\boldsymbol{J} \times \boldsymbol{B})$. If ν is Poisson ratio and E is Young's modulus, then elastic constants are;

$$c_{11} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \;\; , \;\; c_{12} = \frac{E\nu}{(1+\nu)(1-2\nu)} \,.$$

The maxwell's equations

The Maxwell's equations have been generated by electro-magnetic field in the absence of charge density and displacement current as;

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \;,\;\; \nabla \times \boldsymbol{H} = \boldsymbol{J} \,,\;\; \boldsymbol{B} = \mu_e \boldsymbol{H} \;\;,\;\; \nabla . \boldsymbol{B} = 0 \,, \tag{10}$$

The generalized ohm's law in continua of deformation is

$$\boldsymbol{J} = \sigma(\boldsymbol{E} + \frac{\partial u}{\partial t} \times \boldsymbol{B}) \,, \tag{11}$$

Here $\boldsymbol{J}$ is current density, which is neglected due to small effect of temperature gradient. The strength of magnetic field $\boldsymbol{H} = \boldsymbol{H}_0 + h$, where $\boldsymbol{H}_0 = (0, 0, H_0)$; h is perturbation of magnetic field which is very small due to the product of u and h and their derivatives might be neglected due to linearization of basic equations. Therefore, from Eqs. (1) - (11), constitutive relations, the governing field equations are given as;

$$\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} \right) - \frac{\rho}{c_{11}} \left(1 - \zeta^2 \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} \right) \right) \frac{\partial^2 u}{\partial t^2} + \frac{b}{c_{11}}\frac{\partial \varphi}{\partial r} - \frac{\beta_r}{c_{11}}\frac{\partial T}{\partial r} + \frac{1}{c_{11}} \boldsymbol{F}_{,r} = 0 \,, \tag{12}$$

$$-be + \alpha \nabla^2 \varphi - \left(\xi_1 + \xi_2 \frac{\partial}{\partial t} \right) \varphi + \rho\chi \left(1 - \zeta^2 \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} \right) \right) \frac{\partial^2 \varphi}{\partial t^2} + MT = 0 \,, \tag{13}$$

$$\begin{aligned} &\left(\frac{\partial^2}{\partial t^2} + t_q \frac{\partial^3}{\partial t^3} + \frac{t_q^2}{2}\frac{\partial^4}{\partial t^4} \right) \left(\rho C_e T + T_0 \left(\beta_r \frac{\partial u}{\partial r} + \beta_\theta \frac{u}{r} \right) + MT_0 \varphi \right) = \\ &\qquad \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r} \right) \left(K \left(\frac{\partial}{\partial t} + t_T \frac{\partial^2}{\partial t^2} \right) + K^* \left(1 + t_\nu \frac{\partial}{\partial t} \right) \right) , \end{aligned} \tag{14}$$

$$\left.\begin{aligned}\left(1-\zeta^2\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)\right)\sigma_{rr}&=c_{11}\frac{\partial u}{\partial r}+c_{12}\frac{u}{r}+b\varphi-\beta_r T\\ \left(1-\zeta^2\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)\right)\sigma_{\theta\theta}&=c_{12}\frac{\partial u}{\partial r}+c_{11}\frac{u}{r}+b\varphi-\beta_\theta T\end{aligned}\right\}, \tag{15}$$

If the free vibration analysis is restricted to the transversely isotropic thermoelastic cylinder in radial direction, then using Eqs. (10) - (11) the Lorentz force i.e. $F_r=(\boldsymbol{J}\times\boldsymbol{B})_{,r}$ in radial direction (Das *et al*. [39]) we obtained;

$$F_r=\mu_e H_0^2\left(\frac{u}{r}+\frac{\partial u}{\partial r}\right),\quad F_\theta=0\quad,\quad F_z=0, \tag{16}$$

Substituting values of Lorentz force from Eq. (16) in Eq. (12), we get;

$$\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}\right)-\frac{\rho(1-\zeta^2\nabla^2)}{c_{11}}\frac{\partial^2 u}{\partial t^2}+\frac{b}{c_{11}}\frac{\partial\varphi}{\partial r}-\frac{\beta_r}{c_{11}}\frac{\partial T}{\partial r}+\frac{\mu_e H_0^2}{c_{11}}\frac{\partial}{\partial r}\left(\frac{\partial u}{\partial r}+\frac{u}{r}\right)=0, \tag{17}$$

Taking divergence to both sides of Eq. (17) and rearranging it, we obtained;

$$\left(1+\frac{\mu_e H_0^2}{c_{11}}\right)\nabla^2\left(\frac{1}{r}\frac{\partial}{\partial r}(ru)\right)-\frac{\rho}{c_{11}}(1-\zeta^2\nabla^2)\frac{\partial^2 e}{\partial t^2}+\frac{b}{c_{11}}\nabla^2\varphi-\frac{\beta_r}{c_{11}}\nabla^2 T=0, \tag{18}$$

Where; $\nabla^2=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)$.

The transversely isotropic TPL model generalized nonlocal magneto-thermoelastic hollow cylinder with voids material has been presumed to be undisturbed and at rest initially. Therefore, initial conditions are;

$$\frac{\partial u(r,0)}{\partial t}=\frac{\partial\varphi(r,0)}{\partial t}=\frac{\partial T(r,0)}{\partial t}=0,\ u(r,0)=\varphi(r,0)=T(r,0)=0,\quad \text{at}\quad r=a\ ,\ a\eta, \tag{19}$$

The TPL model of generalized nonlocal magneto-thermoelastic hollow cylinder with voids is applied to stress free and equilibrated void volume fraction, thermally insulated/isothermal boundary conditions of domain $a\le r\le a\eta$. Hence, mathematically, we have;

Set 1: $\frac{\partial T}{\partial r}=0\ ,\ \sigma_{rr}=0\ ,\ \varphi=0\ ;\ r=a\ ,\ r=a\eta,$ (20)

Set 2: $T=0\ ,\ \sigma_{rr}=0\ ,\ \varphi=0\ ;\ r=a\ ,\ r=a\eta,$ (21)

Solution of the mathematical model

We set up the following non-dimensional parameters;

$$\left.\begin{array}{l}(\tau_{XX}\,,\,\tau_{\theta\theta})=\dfrac{1}{c_{11}}(\sigma_{rr}\,,\,\sigma_{\theta\theta}),\ (U\,,\,X\,,\,\zeta_0)=\dfrac{1}{a}(u\,,\,r\,,\,\zeta),(\tau\,,\tau_T\,,\tau_q\,,\tau_v)=\dfrac{c}{a}(t,\,t_T\,,\,t_q\,,\,t_v),\theta=\dfrac{T}{T_0},\\ \overline{\beta}_R=\dfrac{\beta_rT_0}{c_{11}},\ \ \overline{\beta}_\theta=\dfrac{\beta_\theta T_0}{c_{11}},\ \ \overline{b}^*=\dfrac{a^2\overline{b}}{\chi\Omega^{*2}},\overline{b}=\dfrac{b}{c_{11}},c_0=\dfrac{c_{12}}{c_{11}},\ c=\sqrt{\dfrac{c_{11}}{\rho}},\ \ \phi=\dfrac{\chi\Omega^{*2}}{a^2}\varphi,\ \ \overline{\xi}=\dfrac{c}{a}\dfrac{\xi_2}{\xi_1},\end{array}\right\}. \tag{22}$$

Using non-dimensional quantities as proposed in Eq. (22) in Eqs. (13) - (15) and (18), we attain following equations in non-dimensional form;

$$\left.\begin{array}{l}\tau_{XX}=\tau^L_{XX}=\dfrac{\partial U}{\partial X}+c_0\dfrac{U}{X}+\overline{b}^*\phi-\overline{\beta}_R\theta\\ \tau_{\theta\theta}=\tau^L_{\theta\theta}=c_0\dfrac{\partial U}{\partial X}+\dfrac{U}{X}+\overline{b}^*\phi-\overline{\beta}_\theta\theta\end{array}\right\}, \tag{23}$$

$$R_h\nabla^2_Xe+\overline{b}^*\nabla^2_X\phi-\overline{\beta}_R\nabla^2_X\theta=(1-\zeta_0^2\nabla^2_X)\frac{\partial^2e}{\partial\tau^2}, \tag{24}$$

$$-a_2e+\nabla^2_X\phi-a_1\left(1+\overline{\xi}\frac{\partial}{\partial\tau}\right)\phi+a_3\theta=(1-\zeta_0^2\nabla^2_X)\frac{1}{\delta_1^2}\frac{\partial^2\phi}{\partial\tau^2}, \tag{25}$$

$$\left(\frac{\partial^2}{\partial\tau^2}+\tau_q\frac{\partial^3}{\partial\tau^3}+\frac{\tau_q^2}{2}\frac{\partial^4}{\partial\tau^4}\right)\left(\Omega^*\theta+a_4e+a_5\phi\right)=\left(\left(\frac{\partial}{\partial\tau}+\tau_T\frac{\partial^2}{\partial\tau^2}\right)+\overline{K}\left(1+\tau_v\frac{\partial}{\partial\tau}\right)\right)\left\{\frac{1}{X}\frac{\partial}{\partial X}\left(X\frac{\partial\theta}{\partial X}\right)\right\}, \tag{26}$$

Where;

$$a_1=\frac{\xi_1a^2}{\alpha},a_2=\frac{b\chi\Omega^{*2}}{\alpha},\ a_3=\frac{M\chi\Omega^{*2}T_0}{\alpha},\ a_4=\frac{\varepsilon_T\Omega^*}{\overline{\beta}_R},\ a_5=\frac{Mca^3}{K\chi\Omega^{*2}},\ \omega^*=\frac{c_{11}C_e}{K},R_h=1+\frac{\mu_eH_0^2}{c_{11}},$$

$$\overline{K}=\frac{aK^*}{cK},\ \Omega^*=\frac{a\omega^*}{c},\ \varepsilon_T=\frac{T_0\beta_r^2}{\rho C_ec_{11}},\ \delta_1^2=\frac{\alpha}{\chi c_{11}},\ e=\frac{1}{X}\left(\frac{\partial}{\partial X}(XU)\right),\nabla^2_X=\frac{1}{X}\frac{\partial}{\partial X}\left(X\frac{\partial}{\partial X}\right).$$

Now we introduce the following time harmonics as proposed by Pierce [51];

$$\left(\overline{e}\quad\overline{\phi}\quad\overline{\theta}\right)=(e\quad\phi\quad\theta)\exp(i\Omega\tau). \tag{27}$$

Here $\Omega=\omega a/c$ denotes circular frequency. Using proposed time harmonics from Eq. (27) in Eqs. (23) - (26), we get;

$$\left\{\begin{array}{l}\tau_{XX}=\overline{e}+\dfrac{c_0-1}{X}\overline{U}+\overline{b}^*\overline{\phi}-\overline{\beta}_R\overline{\theta}\\ \tau_{\theta\theta}=c_0\overline{e}+\left(\dfrac{1-c_0}{X}\right)\overline{U}+\overline{b}^*\overline{\phi}-\overline{\beta}_\theta\overline{\theta}\end{array}\right., \tag{28}$$

$$\begin{cases}\left((R_h-\zeta_0^2\Omega^2)\nabla_X^2+\Omega^2\right)\overline{e}+\overline{b}^*\nabla_X^2\overline{\phi}-\overline{\beta}_R\nabla_X^2\overline{\theta}=0\\ -a_2\overline{e}+\left(a_1^*\nabla_X^2+\dfrac{(a_1 i\Omega\overline{\xi}^*\delta_1^2+\Omega^2)}{\delta_1^2}\right)\overline{\phi}+a_3\overline{\theta}=0,\\ \Omega^2\Omega^*\tau_q^*\overline{e}+\Omega^2\tau_q^*a_4\overline{\phi}+\left(a_2^*\nabla_X^2-\Omega^2\tau_q^*a_5\right)\overline{\theta}=0\end{cases} \tag{29}$$

Where;

$$a_1^*=\frac{\delta_1^2-\zeta_0^2\Omega^2}{\delta_1^2},\ a_2^*=(\Omega^2\tau_T^*-\overline{K}i\Omega\tau_v^*),\ \overline{\xi}^*=i\Omega^{-1}-\overline{\xi},$$

$$\tau_q^*=\left(\Omega^{-2}+i\Omega^{-1}\tau_q-\frac{\tau_q^2}{2}\right),\ \tau_T^*=i\Omega^{-1}-\tau_T,\ \tau_v^*=i\Omega^{-1}-\tau_v.$$

For the solution of Eq. (29), we have non-trivial solution given as;

$$(\nabla_X^6-L_1\nabla_X^4+L_2\nabla_X^2-L_3)(\overline{e},\ \overline{\phi},\ \overline{\theta})=0, \tag{30}$$

Where;

$$L_1=\left(-\frac{\Omega^2}{R_h-\zeta_0^2\Omega^2}-\frac{a_1 i\Omega\overline{\xi}^*\delta_1^2+\Omega^2}{\delta_1^2a_1^*}+\frac{\Omega^2\tau_q^*a_5}{a_2^*}+\frac{\overline{b}^*a_2}{(R_h-\zeta_0^2\Omega^2)a_1^*}-\frac{\overline{\beta}_R\Omega^2\Omega^*\tau_q^*}{(R_h-\zeta_0^2\Omega^2)a_2^*}\right),$$

$$L_2=\left(\begin{aligned}&\frac{\Omega^2(a_1 i\Omega\overline{\xi}^*\delta_1^2+\Omega^2)}{(R_h-\zeta_0^2\Omega^2)\delta_1^2a_1^*}-\frac{(a_1 i\Omega\overline{\xi}^*\delta_1^2+\Omega^2)\Omega^2\tau_q^*a_5}{\delta_1^2a_1^*a_2^*}-\frac{\Omega^2\Omega^2\tau_q^*a_5}{(R_h-\zeta_0^2\Omega^2)a_2^*}-\frac{a_3\Omega^2\tau_q^*a_4}{a_1^*a_2^*}+\\&\frac{\overline{b}^*a_2\Omega^2\tau_q^*a_5}{(R_h-\zeta_0^2\Omega^2)a_1^*a_2^*}+\frac{\overline{b}^*a_3\Omega^2\Omega^*\tau_q^*}{(R_h-\zeta_0^2\Omega^2)a_1^*a_2^*}+\frac{\overline{\beta}_Ra_2\Omega^2\tau_q^*a_4}{(R_h-\zeta_0^2\Omega^2)a_1^*a_2^*}+\frac{\overline{\beta}_R\Omega^2\Omega^*\tau_q^*(a_1 i\Omega\overline{\xi}^*\delta_1^2+\Omega^2)}{(R_h-\zeta_0^2\Omega^2)a_1^*a_2^*\delta_1^2}\end{aligned}\right),$$

$$L_3=\left(\frac{\Omega^4(a_1 i\Omega\overline{\xi}^*\delta_1^2+\Omega^2)\tau_q^*a_5}{(R_h-\zeta_0^2\Omega^2)\delta_1^2a_1^*a_2^*}+\frac{\Omega^4\tau_q^*a_3a_5}{(R_h-\zeta_0^2\Omega^2)a_1^*a_2^*}\right).$$

Since the solution of Eq. (30) is bounded for $X\to\infty$, therefore its roots must be positive real parts, i.e. $\mathrm{Re}(k_i)\geq 0,\ \forall\ i=1,2,3)$. Therefore, the roots $k_i;\ i=1,2,3$ of Eq. (30) are;

$$k_1=\sqrt{\frac{1}{3}\left(2A\sin B+L_1\right)},\ k_2=\sqrt{\frac{1}{3}\left(L_1-A(\sqrt{3}\cos B+\sin B)\right)},\ k_3=\sqrt{\frac{1}{3}\left(L_1+A(\sqrt{3}\cos B-\sin B)\right)},$$

Where;

$$A=\sqrt{L_1^2-3L_2},\ C=-\frac{2L_1^3-9L_1L_2+27L_3}{2L_1^3},\ B=\frac{1}{3}\sin^{-1}(C).$$

Hence, after solving the characteristic Eq. (30), and on applying elimination technique, the complete solution obtained as;

$$\left(\overline{\theta} \quad \overline{e} \quad \overline{\phi}\right) = \sum_{i=1}^{3}\left(1 \quad R_i \quad S_i\right)\left(P_i J_0(k_i X) + Q_i Y_0(k_i X)\right), \tag{31}$$

Where;

$$R_i = R_{1i} / R_{2i} \ , \ S_i = -S_{1i} / S_{2i} \ ; \ i = 1, 2, 3,$$

$$R_{1i} = k_i^2\left(\frac{\overline{\beta}_R a_2}{(R_h - \zeta_0^2\Omega^2)a_1^*} - \frac{a_3}{a_1^*}\right) - \frac{\Omega^2 a_3}{(R_h - \zeta_0^2\Omega^2)a_1^*},$$

$$R_{2i} = k_i^4\left(\frac{\overline{\beta}_R}{R_h - \zeta_0^2\Omega^2}\right) + k_i^2\left(\frac{\overline{\beta}_R(a_1 i\Omega\overline{\xi}^*\delta_1^2 + \Omega^2)}{(R_h - \zeta_0^2\Omega^2)\delta_1^2 a_1^*} + \frac{\overline{b}^* a_3}{(R_h - \zeta_0^2\Omega^2)a_1^*}\right),$$

$$S_{1i} = k_i^2\left(\frac{\Omega^2\Omega^*\tau_q^*}{a_2^*}\right) + \left(\frac{(a_1 i\Omega\overline{\xi}^*\delta_1^2 + \Omega^2)\Omega^2\Omega^*\tau_q^*}{\delta_1^2 a_1^* a_2^*} + \frac{a_2\Omega^2\tau_q^* a_4}{a_1^* a_2^*}\right),$$

$$S_{2i} = k_i^4 + \left(\frac{(a_1 i\Omega\overline{\xi}^*\delta_1^2 + \Omega^2)}{\delta_1^2 a_1^*} - \frac{\Omega^2\tau_q^* a_5}{a_2^*}\right)k_i^2 - \left(\frac{\Omega^2\tau_q^* a_5(a_1 i\Omega\overline{\xi}^*\delta_1^2 + \Omega^2)}{\delta_1^2 a_1^* a_2^*} + \frac{a_3\Omega^2\tau_q^* a_4}{a_1^* a_2^*}\right).$$

Here P_i, Q_i; $i = 1, 2, 3$ are arbitrary constants that depend on Ω only. J_0 and Y_0 are Bessel functions of First and Second kinds of order zero respectively. Resolving cubical dilation $(\overline{e})$ from Eq. (31) for displacement $\overline{U}$, we obtain;

$$\overline{U} = \sum_{i=1}^{3}\frac{1}{k_i}R_i\left(P_i J_1(k_i X) - Q_i Y_1(k_i X)\right). \tag{32}$$

The temperature gradient has been obtained on differentiating the first part of Eq. (31) with respect to X , we obtain;

$$\frac{\partial\overline{\theta}}{\partial X} = \sum_{i=1}^{3}k_i\left[P_i J_1\left(k_i X\right) - Q_i Y_1\left(k_i X\right)\right], \tag{33}$$

On substitution of $\overline{e}$, $\overline{U}$, $\overline{\phi}$, $\overline{\theta}$ from Eqs. (31) - (32) in Eq. **(**28**)**, we get ;

$$\tau_{XX} = \sum_{i=1}^{3}\left(P_i\left\{\mathrm{H}_i J_0(k_i X) + \left(\frac{c_0 - 1}{k_i X}\right)R_i J_1(k_i X)\right\} + Q_i\left\{\mathrm{H}_i Y_0(k_i X) - \left(\frac{c_0 - 1}{k_i X}\right)R_i Y_1(k_i X)\right\}\right), \tag{34}$$

$$\tau_{\theta\theta} = \sum_{i=1}^{3}\left(P_i\left\{\mathrm{H}_i^* J_0(k_i X) - \left(\frac{c_0 - 1}{k_i X}\right)R_i J_1(k_i X)\right\} + Q_i\left\{\mathrm{H}_i^* Y_0(k_i X) + \left(\frac{c_0 - 1}{k_i X}\right)R_i Y_1(k_i X)\right\}\right), \tag{35}$$

Where;

$$\mathrm{H}_i = R_i + S_i\overline{b}^* - \overline{\beta}_R, \ \ \mathrm{H}_i^* = c_0 R_i + S_i\overline{b}^* - \overline{\beta}_R, \quad i = 1, 2, 3.$$

Frequency relations

In this section, for the analysis of free vibrations, the frequency equations have been obtained for Eqs. (31) - (34) for boundary conditions given in Eqs. (20) - (21), at inner and outer radii $X = 1$ and $X = \eta$. On simplification these equations, we obtain a system of homogenous equations given below;

$$(\Pi_{ij})_{6\times 6}\,(\mathrm{H})_{6\times 1} = 0 \;\; ; (i, j = 1,\ 2,\ \ldots, 6)\,, \tag{36}$$

Where;

$$\mathrm{H} = (P_1,\ P_2,\ P_3,\ Q_1,\ Q_2,\ Q_3)^T .$$

On solving Eq. (36), 6 linear homogeneous equations have been obtained with 6 unknowns. Therefore, for non-trivial solution of Eq. (36), we must have;

$$\left|\ \Pi_{ij}\right| = 0\ ; \quad i, j = 1,\ 2, ---, 6\,, \tag{37}$$

Here, the constants of Π_{ij} ; $i, j = 1,\ 2, ---,\ 6$ are defined for thermally insulated boundary conditions in set I and isothermal boundary conditions in set II given below;

Set 1:The constant parameters of Π_{ij} ; $i, j = 1,\ 2, ---,\ 6$ are;

$$\left.\begin{array}{l}
\Pi_{1j} = \mathrm{H}_i J_0(k_i) + \left((c_0 - 1)/k_i\right) R_i J_1(k_i); \quad i, j = 1\,\mathrm{to}\,3; \\
\Pi_{3j} = S_i J_0\left(k_i\right); \quad \Pi_{5j} = k_i J_1\left(k_i\right); \quad i, j = 1\mathrm{to}\,3; \\
\Pi_{1j} = \mathrm{H}_i Y_0(k_i) - \left((c_0 - 1)/k_i\right) R_i Y_1(k_i);\ i = 1\,\mathrm{to}3,\ j = 4\,\mathrm{to}\,6 \\
\Pi_{3j} = S_i Y_0\left(k_i\right); \quad \Pi_{5j} = -k_i Y_1\left(k_i\right); i = 1\,\mathrm{to}\,3;\ j = 4\,\mathrm{to}\,6;
\end{array}\right\}, \tag{38}$$

Set 2:In this case, the elements of $\Pi_{1j}, \Pi_{2j}, \Pi_{3j}, \Pi_{4j}$; $j = 1, ---, 6$, remains same as given inEq. (38). The remaining elements of Π_{5j}, Π_{6j} ; $j = 1, ---, 6$ in Eq. (37) for stress-free isothermal boundary condition are;

$$\left.\Pi_{5j} = J_0\left(k_i\right);\ i, j = 1,2,3, \quad \Pi_{5j} = Y_0\left(k_i\right);\ i = 1,2,3;\ j = 4,5,6;\right\}, \tag{39}$$

The elements of Π_{2j}, Π_{4j}, Π_{6j} ; $j = 1,\ 2, ---, 6$ are obtained by inserting η along with k_i , in the elements of Π_{1j}, Π_{3j}, Π_{5j} ; $j = 1,\ 2, ---, 6$.

Deduction of analytical results

Generalized transversely magneto-thermoelastic voids hollow cylinder

If the nonlocal constant is assumed to be absent, i.e. $\zeta_0 = 0$, then the analysis has been reduced to transversely isotropic magneto-thermoelastic voids hollow cylinder with the TPL model of generalized thermoelasticity.

Generalized and classical magneto-thermoelastic cylinder

If we establish thermal equilibrium and the nonlocal parameter and voids constants are ignored, i.e. $\zeta_0 = 0$, $\alpha = b = M = \xi_1 = \xi_2 = 0$, then the governing equations and analysis has been reduced to the 3-phase-lag model of generalized transversely isotropic electro-magneto-thermoelastic hollow cylinder, which completely agree with the analysis and governing equations of Das *et al*. [39]. Again, if $t_q = t_v = t_T = 0$, then the analysis reduced to classical magneto-thermoelastic cylinder.

Generalized thermoelastic LS model transversely isotropic cylinder

Again, if the nonlocal parameter, magnetic field constants and voids constants are ignored i.e. $\zeta_0 = 0$, $\mu_e = H_0 = 0$ and $\alpha = b = M = \xi_1 = \xi_2 = 0$, and also $K^* = t_q = t_T = 0$, $t_v = t_0$, therefore, the analysis reduced to transversely isotropic thermoelastic hollow cylinder whose governing equations and free vibration analysis agree with Sharma *et al*. [48], in the absence of functionally graded materials.

Elastic cylinder

If the constants i.e. the nonlocal, voids, magneto, relaxation times and thermo-mechanical parameters are removed i.e. $\zeta_0 = 0$, $\alpha = b = M = \xi_1 = \xi_2 = 0$, $\mu_e = H_0 = 0$, $K^* = t_q = t_v = t_T = 0$ and $T = \beta_R = \varepsilon_T = 0$, then the governing equations and the free vibration analysis have been reduced to transversely isotropic elastic cylinder which agree with Kele and Tutuncu [45] in the absence of functionally graded materials.

Numerical results

The numerical computational results have been proposed to validate the analytical results for TPL model of nonlocal magneto-thermoelastic hollow cylinder with voids. The simulated results have been performed for generalized thermoelastic models, i.e. coupled thermoelasticity (CTE), Lord-Shulman (LS), dual-phase-lag (DPL) and 3-phase-lag (TPL) in absence/presence of magnetic fields for nonlocal and local elastic materials with voids in thermoelastic hollow cylinder by taking the ratio of outer to inner radius $\eta = 1.5,\ 2.0$. For computation purpose the transversely isotropic material of single crystal of zinc thermoelastic solid with voids material has been assumed and its constant values are given in SI units (Chadwick and Seet [52]);

$$K = 1.24\times10^{2} Wm^{-1}\deg^{-1}, C_e = 3.9\times10^{2} JKg^{-1}\deg^{-1},\ \rho = 7.14\times10^{3} Kg\,m^{-3},\ \chi = 1.753\times10^{-15} m^{2},$$

$$c_{11} = 1.628\times10^{11}\ Nm^{-2}, c_{12} = 1.562\times10^{11}\ Nm^{-2}, \beta_r = 5.75\times10^{6}\ Nm^{-2}\ \deg^{-1}, T_0 = 296K,\ \omega = 10,\quad .$$

$$\alpha = 3.688\times10^{-5} N, M = 2.0\times10^{6} Nm^{-2}\deg^{-2}, \xi_1 = \xi_2 = 1.475\times10^{10} Nm^{-2}, b = 1.13849\times10^{10} Nm^{-2}.$$

The 3-phase-lag (TPL) parameters have been considered from Mondal and Kanoria [42] as $t_v = 0.05$, $t_T = 0.07$, $t_q = 0.09$, $K^* = 7.0$. The magnetic field parameters have been assumed as $\mu_e = 4\pi\times10^{7} H/m$, $H_0 = 10^{8} A/m$ from Othman and Hilal [41]. The nonlocal parameter value has been considered as $\xi_0 = 2.3102$ from Bachher and Sarkar [27]. The frequency dispersion relations have been attained from considered boundary conditions, which are transcendental equations, whose solution is in the form of complex numbers, is because of rate of dissipative term in heat conduction Eq(4). The numerically analyzed computations and simulations have been applied to Eq. (37) for thermally insulated cases till 4 places of decimals.

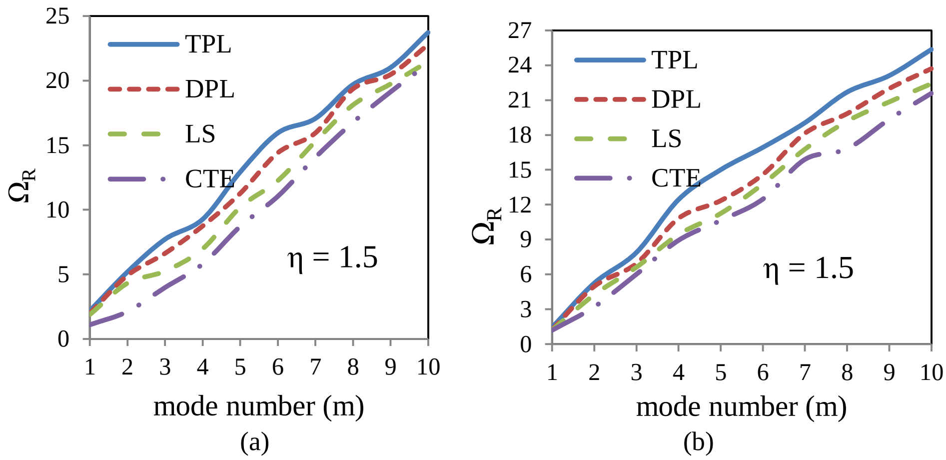


Figure 1 Natural frequencies (Ω_R) against mode number (m) for TPL, DPL, LS and CTE models at $\eta = 1.5$ in **nonlocal** thermoelastic cylinder with voids (a) **with** magnetic field (b) **without** magnetic field.

The numerical Iteration method has been applied to evaluate the roots of the Eq.(37), which is of the type $g(\Omega) = 0$. The required substitution for the method i.e. $\Omega = \Phi(\Omega)$, so that the sequence (Ω_n) of iterations has been generated for desired accuracy level. If the condition $|\Phi'(\Omega)| \ll 1$, holds for all $\Omega \in I$, then the root of approximations will converge to the actual value $\Omega = \Omega_a$ of the root, provided $\Omega_0 \in I$, here I is the expected interval. The Iteration method's condition for numerical convergence is $|\Omega_{n+1} - \Omega_n| < \varepsilon$. Here ε has been considered small arbitrary number to achieve the accuracy level selected randomly, which may be satisfied. Therefore, this procedure is repeated continuously for the values of Ω until desired level of accuracy achieved. The numerically analyzed complex values (frequencies) of Ω might be written as $\Omega^m = \Omega_R^m + i\Omega_I^m$. The real and imaginary parts have been considered as natural frequencies $\Omega_R^m = \Omega_R$ and dissipation factor $\Omega_I^m = \Omega_I$ respectively. The value m has been considered as mode number, which corresponds to root of the equation. The numerically analyzed natural frequencies have been presented graphically for TPL, DPL, LS and CTE models of thermoelasticity for nonlocal/local thermoelastic hollow cylinder in presence and absence of magnetic field.

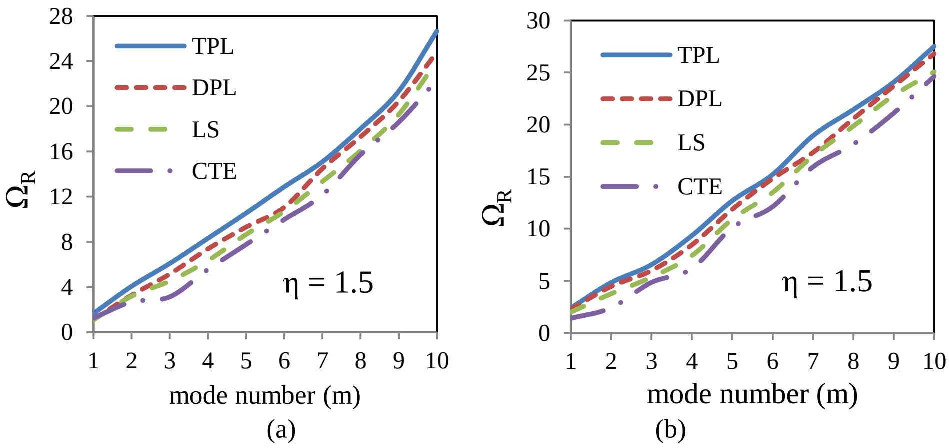


Figure 2 Natural frequencies (Ω_R) against mode number (m) for TPL, DPL, LS and CTE models at $\eta = 1.5$ in **local** thermoelastic cylinder with voids (a) **with** magnetic field (b) **without** magnetic field.

The real parts have been assumed as natural frequencies (Ω_R) against mode number (m) for nonlocal as well as local elastic cylinder with and without magnetic field at $\eta = 1.5$ have been shown graphically in **Figures 1** and **2**. These **Figures 1** and **2** (nonlocal and local case) depict that initially the vibrations are low and with increasing values of m, the variation of vibrations goes on increasing with increasing mode number. The behaviors of vibrations are lower in the presence of a magnetic field in contrast to the absence of magnetic field for nonlocal and local elastic materials. This is noticed from **Figures 1** and **2** that the behavior of variation of natural frequencies is larger in case of TPL model of generalized thermoelasticity in comparison with other models of thermoelasticity.

The frequency shift (Ω_{shift}) and the thermo-elastic damping related to inverse quality factor (Q^{-1}) for transversely isotropic electro-magneto generalized thermoelastic hollow cylinder have been calculated by Moosapour *et al*. [50] as $\Omega_{shift} = \left|\dfrac{\Omega_R^{\Upsilon^*} - \Omega_R^{CTE}}{\Omega_R^{CTE}}\right|$, $Q^{-1} = 2\left|\dfrac{\Omega_I}{\Omega_R}\right|$, Here CTE stands for coupled thermoelasticity and Υ^* denotes for LS, DPL, TPL models of generalized thermoelasticity. Here, in the figures thermoelastic damping has been denoted as $Q^{-1} = D_F$.

Figures 3 and **4** have been represented for frequency shift (Ω_{shift}) against mode number (m) for different models of generalized thermoelasticity i.e. TPL, DPL and LS at $\eta = 1.5$ for nonlocal/local elastic cylinders with voids in presence/absence of magnetic field. It is observed from **Figures 3(a)** - **3(b)**(nonlocal case with/without magnetic field) that initially the variation of frequency shift vibrations is meager, the peak values have been noticed at $m = 2.0$, and keep on decreasing linearly with increasing value of mode number. This has been noticed from **Figure 4(a)** (local with magnetic field) that the frequency shift vibrations are lower initially, accomplish maximum amplitude at $2.0 \le m \le 4.0$ and with increasing values of m, the behavior of vibrations goes on decreasing and become linear after $m = 7.0$. **Figure 4(b)** (local without magnetic field) shows that initially vibrations have lower behavior, accomplish maximum amplitude at $m = 2.0$, then decreases to attain small peaks at $m = 4.0$ and go on decreasing to become linear at $m = 6.0$.

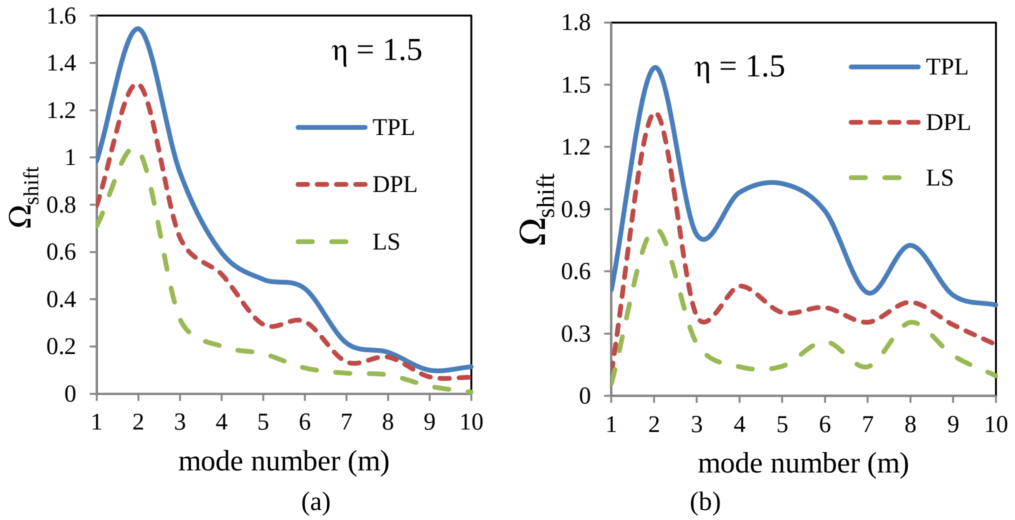


Figure 3 Frequency shift (Ω_{Shift}) against mode number (m) for TPL, DPL and LS models at $\eta = 1.5$ in **nonlocal**thermoelastic cylinder with voids (a) **with**magnetic field (b) **without** magnetic field.

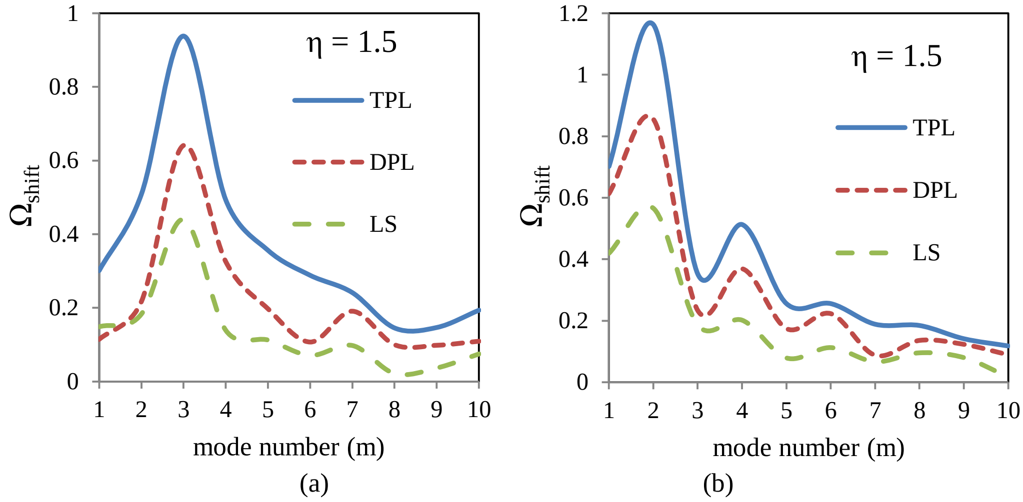


Figure 4 Frequency shift (Ω_{Shift}) against mode number (m) for TPL, DPL and LS models at $\eta = 1.5$ in **local**thermoelastic cylinder with voids (a) **with**magnetic field (b) **without** magnetic field.

The frequency shift is represented in **Figures 5** and **6** for TPL, DPL and LS models of generalized thermoelasticity at $\eta = 2.0$ for nonlocal/local elastic voids hollow cylinder with/without magnetic field. It has been observed from **Figures 5(a)** - **5(b)** that initially the frequency shift vibrations are low, after attaining its maximum amplitude at $m = 2.0$, it decreases slightly at $m = 4.0$ and keep on decreasing linearly with increasing mode number. It is revealed from **Figure 6(a)** that the behavior of frequency shift vibrations is low initially, attain its peak value at $m = 2.0$, decreases up to $m = 3.0$ and become linear with an increase in value of m. **Figure 6(b)** tells that initially the frequency shift vibrations are meager, attains its maximum amplitude between $2.2 \le m \le 4.3$, and keep on decreasing with increase in mode number. This is to be noticed that the variation of vibrations are larger in TPL case than DPL and LS cases.

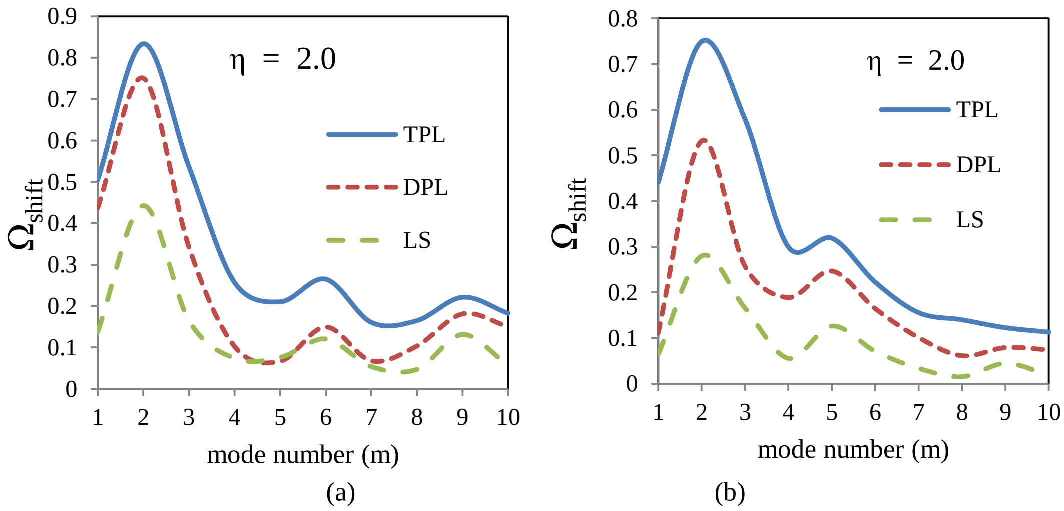
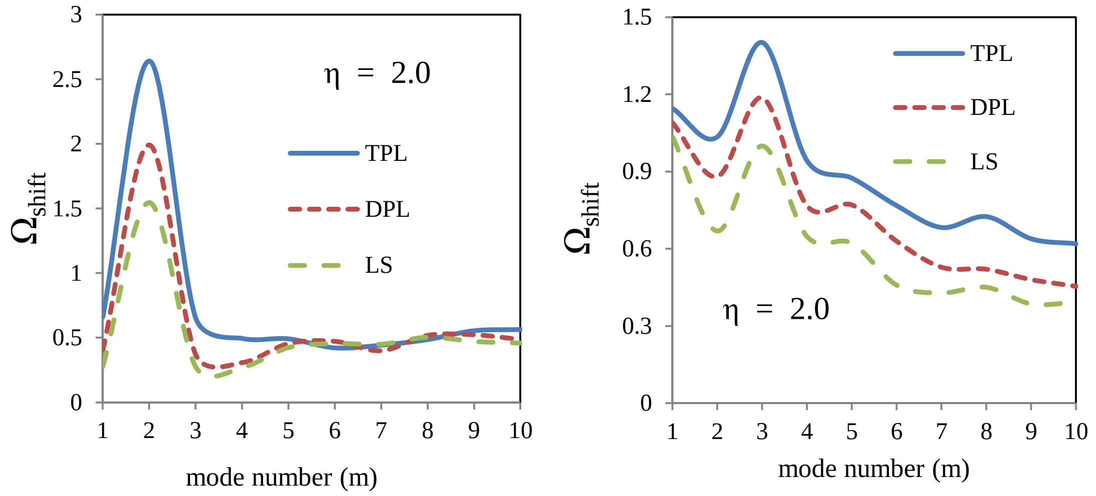


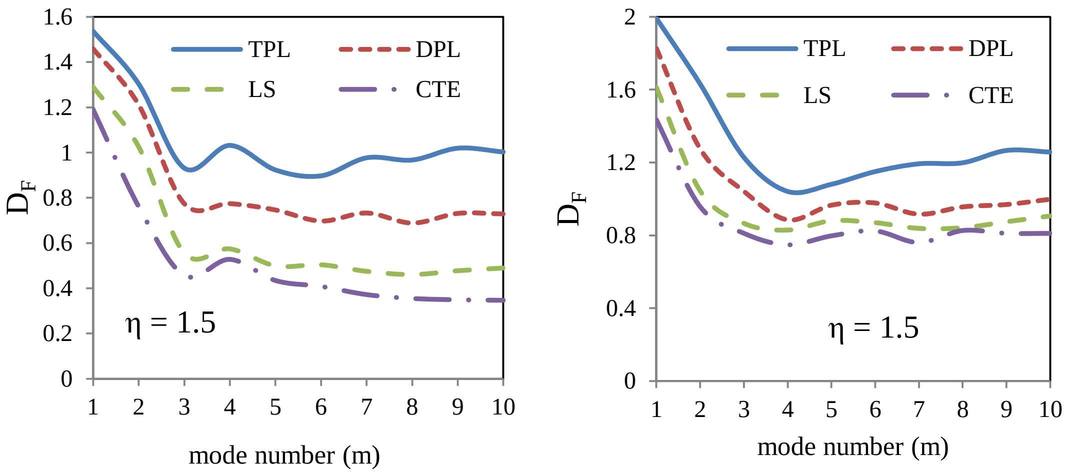
Figure 5 Frequency shift (Ω_{Shift}) against mode number (m) for TPL, DPL and LS models at $\eta = 2.0$ in **nonlocal**thermoelastic hollow cylinder with voids (a) **with** magnetic field (b) **without** magnetic field.



(a) (b)

Figure 6 Frequency shift (Ω_{Shift}) against mode number (m) for TPL, DPL and LS models at $\eta = 2.0$ in **local**thermoelastic hollow cylinder with voids (a) **with** magnetic field (b) **without** magnetic field.

The thermoelastic damping (D_F) against mode number (m) have been drawn in **Figures 7** and **8** for the generalized thermoelastic models, i.e. TPL, DPL, LS and CTE at $\eta = 1.5$ for nonlocal/local elastic cylinder with voids in presence/absence of magnetic field. This is noticed from **Figures 7(a) - 7(b)** (nonlocal case) that initially the thermoelastic damping vibrations are larger, go on decreasing up to $m = 3.0$ and from left to right, the variation of vibrations becomes linear. It has been revealed form **Figure8(a)** (local case with magnetic field) depict that initially the thermoelastic damping vibrations are larger, achieve its minimum amplitude between $2.0 \le m \le 3.0$, increases up to $m = 4.0$ and with increasing mode number values, the vibrations become linear. **Figure 8(b)** (local case without magnetic field) tells that initially the vibrations are larger, decreases up to $m = 3.0$, and with increasing values of mode number, the vibrations keep on increasing linearly.



(a) (b)

Figure 7 Thermoelastic damping (D_F) against mode number (m) for TPL, DPL, LS and CTE models at $\eta = 1.5$ in **nonlocal** thermo-elastic hollow cylinder with voids (a) **with** magnetic field (b) **without** magnetic field.

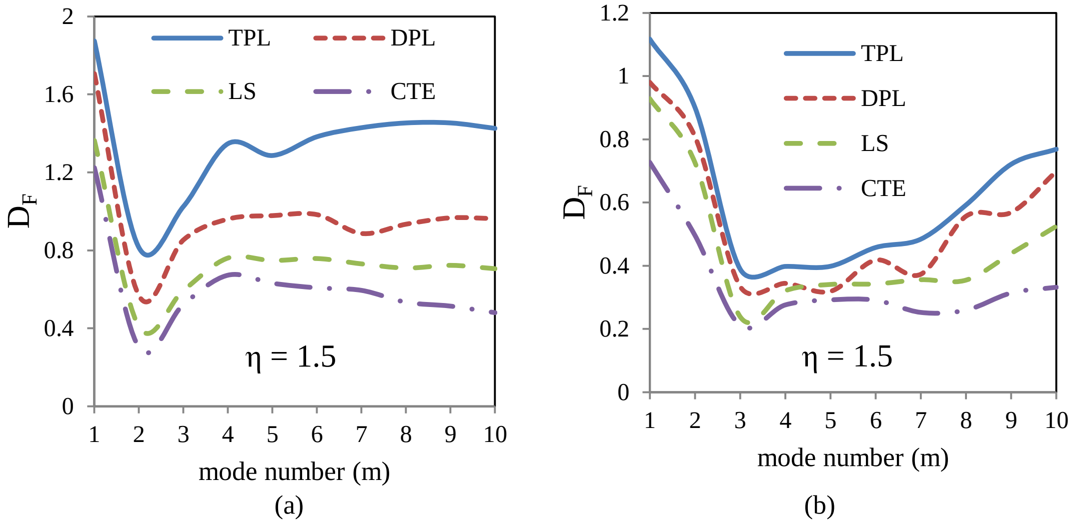


Figure 8 Thermoelastic damping (D_F) against mode number (m) for TPL, DPL, LS and CTE models at $\eta = 1.5$ in **local** thermo-elastic hollow cylinder with voids (a) **with** magnetic field (b) **without** magnetic field.

The thermoelastic damping (D_F) versus mode number (m) has been represented in **Figures9** and **10** for TPL, DPL, LS and CTE models of thermoelasticity at $\eta = 2.0$ for nonlocal/local elastic hollow cylinders with voids in presence/absence of magnetic field. It has been concluded from **Figures9(a)** - **9(b)**(nonlocal case) that initially the variation of thermoelastic damping vibrations is larger, decreases up to $m = 3.0$ and with increasing values of mode number the vibrations become linear. It has been noticed from **Figure 10(a)** (local case) that initially the thermoelastic damping vibrations are larger, decreases up to $m = 4.5$, and with increasing values of m, the vibrations become linear. This is to be observed from all the figures that the vibrations are larger in case of TPL model in comparison with DPL, LS and CTE models of thermoelasticity. Also, due to the effect of magnetic field the behavior of vibrations is noted to be larger without magnetic field in contrast to with magnetic field. It has been observed that thermoelastic damping vibrations noted to be decreasing between $m = 3.0$ to $m = 5.0$ and then become linear.

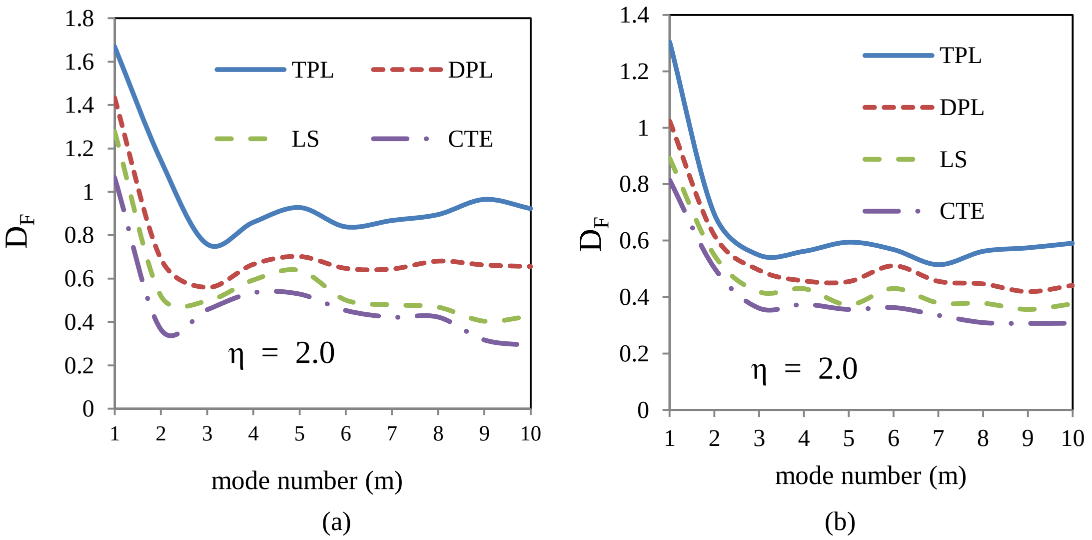


Figure 9 Thermoelastic damping (D_F) against mode number (m) for TPL, DPL, LS and CTE models at $\eta = 2.0$ in **nonlocal** thermoelastic hollow cylinder with voids (a) **with** magnetic field (b) **without** magnetic field.

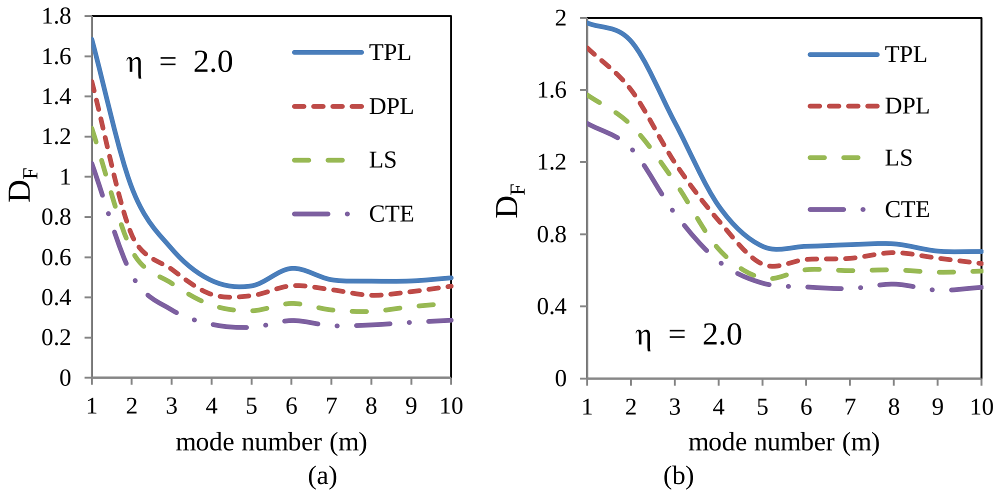


Figure10 Thermoelastic damping (D_F) against mode number (m) for TPL, DPL, LS and CTE models at $\eta = 2.0$ in **local** thermo-elastic hollow cylinder with voids (a) **with** magnetic field (b) **without** magnetic field.

Conclusions

Vibration analysis of electro-magneto transversely isotropic generalized nonlocal thermoelastic hollow cylinder with voids material has been presented in the reference of TPL model**.** The outer and inner surfaces of hollow cylinder have been assumed stress free and thermally insulated/isothermal**.** From the discussion of analytical and numerical results, following conclusions have been observed;

1) It is clearly indicated from the effect of magnetic field that the variations are larger in absence of magnetic field in contrast to the presence of magnetic field.

2) The effect of TPL model of magneto thermoelastic hollow cylinder is presented numerically for field functions i.e. thermoelastic damping and frequency shift in presence/absence of magnetic field. All the figures depict that the variation of vibrations has larger behaviour in the TPL model of generalized thermoelasticity in contrast to DPL. LS and CTE cases because of effect of phase-lags of relaxation time parameters.

3) It is observed from the analysis of graphs that the natural frequencies clearly indicate that as mode number increases, the vibrations go on increasing. This has been noticed that after attaining maximum and minimum amplitudes of variations, the behavior of thermoelastic damping becomes linear because of the coupling between elastic, voids equilibrated volume fraction and thermal fields.

4) The free vibration functions i.e. thermoelastic damping and frequency shift are influenced by non-locality effect and represented for nonlocal and local cases with/without magnetic fields. From present work, researchers may receive the motivation to inspect the analysis of free vibrations of thermoelastic and magneto-thermoelastic materials with voids as novel applications in continuum mechanics such as material science, designing of new materials and useful in practical situations such as geomagnetic, optics, geophysics and acoustics, oil prospecting etc.

5) From literature study, It has been found that the TPL models provide better approach to allow voids and relaxation time parameters, which have many applications in the field of science, technology and engineering. This research manuscript gives useful applications in the area of seismology for mining and drilling in the earth's crust.

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