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Mathematical Modeling of LDL-C and Blood Flow through an Inclined Channel with Heat in the Presence of Magnetic Field

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Abstract

This investigation revealed that LDL-C and blood flow through an inclined channel with heat in the presence of magnetic field. In this study, mathematical models were formulated for LDL-C and blood flow and energy transfer as a coupled system of partial differential equations (PDEs), the PDEs were scaled using the dimensionless quantities to dimensionless partial differential equations, they are further reduced to an ordinary differential equations (ODEs) using the perturbation method involving the oscillatory term, the governing equations are solved directly using the method of undetermined coefficient. The velocity and temperature functions are obtained with some governing parameters involved, and simulation codes were developed using Mathematica to study the effect of entering parameters on the profiles, and are found to be effective in controlling the flow profiles. It is observed that the governing parameters influenced the flow profiles; also the angle of inclination also influences the flow profile.

Keywords: Blood, LDL-C, Heat, Magnetic Field, Atherosclerosis, Treatment, Metabolism, Hypertension, Cardiovascular system

Introduction

Cardiovascular system is made up of blood cells, blood vessels and the heart. The main function of the heart is to pump blood into circulation, to the tissues and organs of the human body through the blood vessels. According to Bunonyo and Amos [1], blood as essential ingredient of the vitality of the body system, and the major constituents are red blood cells (erythrocytes), white blood cells (leukocytes), the platelet and the plasma fluid, any obstruction to the blood flow due to the accumulation of fatty substances, such as cholesterol and saturated fats, within the arteries creates a medical condition called atherosclerosis or stenosis.

The presence of stenosis inside an arterial blood vessel changes its flow pattern and haemodynamics conditions and its continuous growth inside the blood vessel increase the chances of heart failure, significantly [2,3]. As atherosclerosis is directly related to human health, many researchers have investigated its effects on the blood flow considering various shapes and sizes. Mekheimer and Kot [4] elucidated using mathematical model to study the geometry of stenosis by defining the shape and tapering parameters and concluded that impedance decreases with increasing the values of the stenosis length and stenosis shape parameter, while it increases as the size of the stenosis increases. Kamangar *et al.* [5] numerically studied the steady and transient blood flow for different geometrical shapes of stenosis (triangular, elliptical and trapezium) having 70 % (moderate), 80 % (intermediate) and 90 % (severe) stenosis. In view of a diseased artery, Tripathi and Sharma [6] used a mathematical model to study narrow arteries for both core and plasma regions separately. It was shown that the velocity profile of blood flow decreases with increasing height of the stenosis for both core and plasma regions.

Magneto-hydrodynamics (MHD) is very important in medical science as it helps to treat hyperthermia, cancerous tumor or magnetic wounds bleeding reduction during surgeries [7,8]. As blood shows the characteristics of bio magnetic fluids, MHD laws are used to study the flow of blood through under the influence of magnetic field. Blood contains haemoglobin; Haemoglobin is an iron (Fe) containing protein that transports oxygen around the body from the lungs to where it is needed, like the brain or muscles etc. In the 1930s, it was found that haemoglobin has magnetic properties that are different depending on whether it is carrying oxygen or not.

In the last 15 years, the difference in magnetic property has been used in magnetic resonance imaging (MRI) research. MRI uses a very strong magnetic field so this difference in the magnetic properties of oxygenated and deoxygenated haemoglobin in blood can be detected. This change is called the BOLD (blood oxygenation level dependent) signal.

Misra *et al.* [9] developed a mathematical model to study the behavior of blood circulation under the influence of external applied magnetic field by considering artery as a channel. It was observed that the velocity of the blood can be controlled (increase or decrease) by varying the strengths of an applied magnetic intensity in their study. Bhatti *et al.* [10] analyzed the effects of transverse magnetic field, heat and mass transfer on the peristaltic motion of the 2-phase flow. They found that the temperature profile of the flow increases as the influence of the magnetic intensity increases.

In a further study, Ponalagusamy and Selvi [11] elucidated the effects of the magnetic field for the2-phase blood circulation consisting of RBC containing core region and cell-free plasma region and depicted that wall shear stress and impedance increase as strengths of the external applied magnetic field intensity increases.

In recent time, the heat generated by Joule heating and viscous dissipation has caught the attention of many researchers and scientists alike.

Joule heating is the process in which heat is generated due to collision among the moving particles and in this procedure kinetic energy is transformed into heat which enhances the temperature of the human body [12,13]. On the other hand, viscous dissipation is calculated through the work done by the velocity against viscous stresses and also knownas viscous dissipation of energy [14]. The oxidation of metabolic fuels such as carbohydrate and fatty acids in the mitochondria of the muscle fibres produces adenosine triphosphate. Through the hydrolysis of adenosine triphosphate, energy is released to support muscle contraction which generates heat. However, the hydrolysis of adenosine triphosphate also releases heat. In further investigation, the ingredients in the balm, such as camphor, increase blood circulation to the surface of the skin, creating a warming sensation that can distract from pain and stiffness. Camphor and menthol may also improve blood circulation to the muscles, potentially speeding up healing time and reducing inflammation [20].

If there is too much LDL cholesterol in the body it can build up in various arteries, clogging them and reducing their flexibility. Hardening of the arteries is results to an atherosclerosis, and that restrict normal blood flow because of the loss of the vessel flexibility, so the heart has to work harder to push blood through to the downstream [15]. The extra work done by the heart to push blood through due to plaque formation on the blood channel can lead to hypertension, and it the channel is partially and totally impaired it could lead to stroke and heart attack.

Strokes and heart attacks, the leading causes of death in the industrialized world, are often linked to high blood viscosity. Thicker blood damages blood vessels, and in repairing the damage, the vessels build up fatty deposits, which make strokes and heart attacks more likely. Currently, the only way to reduce blood viscosity is with drugs like aspirin, which inhibit the tendency of blood to clot. But aspirin has side effects: In high doses, it can lead to stomach bleeding, ulcers, and even tinnitus, or ringing of the ears.

The study of the effect of chemical reaction in blood flow can't be overemphasized due to its importance in view of several physiological and physiochemical aspects of drug dynamics.

In most chemical reactions, the reaction rate is based on the concentration of the species itself, whether they areheterogeneous or homogeneous. Further, the chemical reaction depends upon whether it occurs at an interface or as asingle phase volume reaction. If the reaction rate of any chemical reaction is directly proportional to the concentrationit is called a 1^{st}-order homogeneous chemical reaction. The reaction rate is proportional to the *n*th power of theconcentration is considered *n*th order chemical reaction. Makinde [16] analyzed the mixed convection problem forvertical porous plate considering *n*thorder homogeneous chemical reaction between the fluid and the diffusing species.

In a further investigation, Tripathi and Sharma [17] explained the effects of the 1^{st}-order chemical reaction on the 2-phase modelof blood flow and resulted that the concentration profile decreases in both core and plasma regions as values of thechemical reaction parameter increase.

The main purpose of this present research is to investigate the flow of LDL-C and blood through an inclined atherosclerotic channel, motivated by the previous researches carried out by Bunonyo and Amos [18], Bunonyo *et al.*[15]and Midya*et al.* [19], where they all explicitly made it clear the various contributions of magnetic field intensity and heat sources. However, they didn't consider the angle of inclination situation. The angle of inclination is very important due to some circumstance where a patient is required to sit in a particular position for blood circulation to improve before a surgical procedure can be done. The flow of the mixed fluid, which is blood carrying blood proteins, is considered incompressible, viscous and electrically conducting. The governing equations of fluid flow are solved subjectto the specific and relevant boundary conditions. The effects of pertinent parameters entering intothe problem have been discussed in detail. This paper is organized as follows. The mathematical formulation in Section 2, solutions of governing equations is provided in Section 3 under the method of solution, simulated results are presented in Section 4 under presentation of results, discussion of results are presented in Section 5and Section 5 provides the conclusions.

Materials and methods

Mathematical formulation

In formulating the flow equations to investigate the mixed fluid flow in an irregular inclined parallel channel, we shall consider the fluid to be a mixture of LDL-C and blood, incompressible, viscous and conducting fluid, flowing in an inclined channel with the angle of inclination α , and the flow far field temperature and LDL-C concentration is T_∞ and C_∞ , also the temperature and concentration of the fluid at the atherosclerotic wall is T_w & C_w. The flow is also assumed to be oscillatory in a porous channel, and unidirectional with velocity $\vec{w} = \left(0, 0, w^*\right)$ towards the x^*-axis. The height δ^* of irregularity is caused by stenosis, the length of the irregularity is l_0^* , and the flow is also assumed to be influenced by magnetic field with intensity $\vec{B} = \left(0, 0, B_0\right)$. Following the aforementioned assumptions, we present the diagram showing the flow region and the irregularity. However, we assume that the growth rate of cholesterol induced atherosclerosis can be controlled by an inhibitor which we refer to as the treatment R_T and the flow could be accelerated by an angle of inclination (**Figure A**).

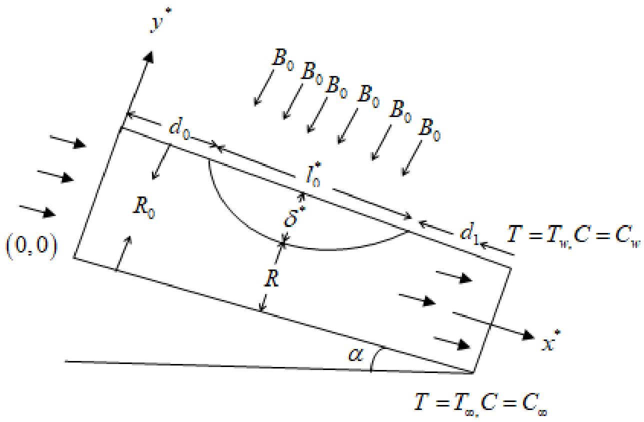

Figure A Schematic diagram showing the angle of inclination and irregular region on the flow profile.

From **Figure A**, the region of atherosclerosis modeled to be:

$$R = R_0 - \delta^*\left(cos2\frac{\pi x^*}{l_0^*}\right) \tag{1}$$

Continuity equation

$$\frac{\partial w^*}{\partial x^*} + \frac{\partial u^*}{\partial y^*} = 0 \tag{2}$$

Momentum equation

$$\rho_b \frac{\partial w^*}{\partial t^*} = -\frac{\partial P^*}{\partial x^*} + \mu_b \frac{\partial^2 w^*}{\partial y^{*2}} - \sigma_e B_0^2 w^* - \frac{\mu_b \varphi}{k^*} w^* + \rho_b g \beta_T \left(T^* - T_\infty\right) cos\alpha + \rho_b g \beta_C \left(C^* - C_\infty\right) cos\alpha \tag{3}$$

Energy equation

$$\rho_b c_p \frac{\partial T^*}{\partial t^*} = k_{bT} \frac{\partial^2 T^*}{\partial y^{*2}} + Q_0 \left(T^* - T_\infty\right) + Q_1 \left(C^* - C_\infty\right) \tag{4}$$

LDL-C concentration equation

$$\frac{\partial C^*}{\partial t^*} = D_m \frac{\partial^2 C^*}{\partial y^{*2}} - k_0 \left(C^* - C_\infty\right) \tag{5}$$

The corresponding boundary conditions are:

$$\left.\begin{array}{ll} w^* = 0, T^* = T_\infty, C^* = C_\infty & \text{at } y^* = 0 \\ w^* = 0, T^* = T_w, C^* = C_w & \text{at } y^* = R \end{array}\right\} \tag{6}$$

where k_{bT} is the thermal conductivity of blood, D_m is the molecular diffusivity , T^* is blood temperature, ρ_b is the density of blood, σ_e is electrical conductivity, c_{bp} is the specific heat capacity of blood, β_T is the volumetric expansion, β_C is the volumetric expansion due to concentration, μ_b is the dynamic viscosity of blood, φ is the porosity, k^* is the permeability of the porous medium, Q_0 is the is the dimensional heat source and Q_1 is dimensional heat due to body metabolism.

$$\left.\begin{array}{l} x = \dfrac{x^*}{l_0^*}, y = \dfrac{y^*}{R_0}, w = \dfrac{w^* R_0}{\upsilon}, t = \dfrac{\upsilon t^*}{R_0^2}, Rd_1 = \dfrac{Q_1 R_0^2}{k_{bT}} \dfrac{\left(C^* - C_\infty\right)}{\left(T_w - T_\infty\right)}, \\ \phi = \dfrac{C^* - C_\infty}{C_w - C_\infty}, \theta = \dfrac{T^* - T_\infty}{T_w - T_\infty}, P = \dfrac{P^* R_0^3}{l_0^* \upsilon \mu_b}, Rd_2 = \dfrac{Q_0 R_0^2}{k_{bT}}, \delta_0 = \dfrac{\delta^*}{R_0} \end{array}\right\} \tag{7}$$

Using the dimensionless quantities in Eq. (7), we reduce the Eqs. (1) to (6) as follows:

Considering the fact that the growth of cholesterol can be controlled using inhibitor in reducing the LDL-C, we have the region of atherosclerosis in Eq. (1) remodeled to:

$$\frac{R}{R_0} = 1 - \delta_0 e^{\frac{at}{R_T}} \left(cos 2\pi x\right) \tag{8}$$

The dimensionless equations governing the flow are as follows:

$$\frac{\partial w}{\partial t} = -\frac{\partial P}{\partial x} + \frac{\partial^2 w}{\partial y^2} + Gr\theta cos\alpha + Gc\phi cos\alpha - M^2 w - \frac{w}{Da} \tag{9}$$

$$Pr\frac{\partial\theta}{\partial t}=\frac{\partial^2\theta}{\partial y^2}+Rd_2\theta+Rd_1Pr\phi \tag{10}$$

$$\frac{\partial\phi}{\partial t}=\frac{1}{Sc}\frac{\partial^2\phi}{\partial y^2}-Rd_3\phi \tag{11}$$

The corresponding boundary conditions are:

$$\left.\begin{array}{l} w=0,\ \phi=0,\theta=0 \quad \text{at } y=0 \\ \\ w=0,\ \phi=1,\theta=1 \quad \text{at } y=h \end{array}\right\} \tag{12a}$$

where:

$$\left.\begin{array}{l} Ha=B_0R_0\sqrt{\dfrac{\sigma_e}{\mu_b}},Da=\dfrac{k^*}{R_0^2},Gc=\dfrac{g\beta_C\left(C_w-C_\infty\right)R_0^3}{\upsilon^2},Gr_1=Grhcos\alpha \\ Sc=\dfrac{\upsilon}{D_m},Gr=\dfrac{g\beta_T\left(T_w-T_\infty\right)R_0^3}{\upsilon^2},Pr=\dfrac{\mu_bc_p}{k_{bT}},Gc_1=Gchcos\alpha \end{array}\right\} \tag{12b}$$

Method of solution

Since the flow is purely oscillatory due to the pumping action of the ventricle, it is appropriate to seek oscillatory solutions of the form:

$$\left.\begin{array}{l} w\left(y,t\right)=w_0\left(y\right)e^{i\omega t} \\ \theta\left(y,t\right)=\theta_0\left(y\right)e^{i\omega t} \\ \phi\left(y,t\right)=\phi_0\left(y\right)e^{i\omega t} \\ -\dfrac{\partial P}{\partial x}=P_0e^{i\omega t},\chi=\dfrac{y}{h} \end{array}\right\} \tag{13}$$

Using the solution form in Eq. (13), we reduce the dimensionless coupled partial differential Eqs. (9)-(12a) to homogenous and non-homogenous ordinary differential equations in the form:

$$\frac{\partial^2 w_0}{\partial\chi^2}-h\left(i\omega+M^2+\frac{1}{Da}\right)w_0=P_0h-Gr_1\theta_0-Gc_1\phi_0 \tag{14}$$

$$\frac{\partial^2\theta_0}{\partial\chi^2}+h\left(Rd_2-i\omega\right)Pr\theta_0=-Rd_1h\phi_0 \tag{15}$$

$$\frac{\partial^2\phi_0}{\partial\chi^2}-h\left(Rd_3+i\omega\right)Sc\phi_0=0 \tag{16}$$

Subject to the following boundary conditions as:

$$\left.\begin{array}{ll} w_0 = 0,\ \phi_0 = 0, \theta_0 = 0 & \text{at } \chi = 0 \\ w_0 = 0,\ \phi_0 = e^{-i\omega t}, \theta_0 = e^{-i\omega t} & \text{at } \chi = 1 \end{array}\right\} \tag{17}$$

Solving for the LDL-C concentration Eq. (3.15), we have:

$$\phi_0(\chi) = A_1 sinh\left(\sqrt{\beta_3}\chi\right) + B_1 cosh\left(\sqrt{\beta_3}\chi\right) \tag{18}$$

where $\beta_3 = h\left(Rd_3 + i\omega\right)Sc$

Solving for the constant coefficients in Eq. (18) using the appropriate boundary condition in Eq. (17), we have:

$$\phi_0(\chi) = \frac{e^{-i\omega t}}{sinh\left(\sqrt{\beta_3}\right)} sinh\left(\sqrt{\beta_3}\chi\right) \tag{19}$$

Substitute Eq. (19) into perturbed energy Eq. (15), we have:

$$\frac{\partial^2\theta_0}{\partial\chi^2} + h\left(Rd_2 - i\omega\right)Pr\theta_0 = -\frac{Rd_1 h e^{-i\omega t}}{sinh\left(\sqrt{\beta_3}\right)} sinh\left(\sqrt{\beta_3}\chi\right) \tag{20}$$

Let $\beta_4 = -\frac{Rd_1 h e^{-i\omega t}}{sinh\left(\sqrt{\beta_3}\right)}$, so that Eq. (20) reduces to

$$\frac{\partial^2\theta_0}{\partial\chi^2} + \beta_2\theta_0 = \left(\beta_4 sinh\left(\sqrt{\beta_3}\chi\right)\right) \tag{21}$$

where $\beta_2 = h\left(Rd_2 - i\omega\right)Pr$

Solution of the homogenous part of Eq. (21) is:

$$\theta_{0h}(\chi) = A_2 sin\left(\sqrt{\beta_2}\chi\right) + B_2 cos\left(\sqrt{\beta_2}\chi\right) \tag{22}$$

and the solution of the particular part of Eq. (21) is:

$$\theta_{0p} = \frac{\beta_4}{\left(\beta_3 + \beta_2\right)} sinh\left(\sqrt{\beta_3}\chi\right) \tag{23}$$

So that, the general solution of Eq. (21) would be

$$\theta_0(\chi) = A_2 sin\left(\sqrt{\beta_2}\chi\right) + \frac{\beta_4}{\left(\beta_3 + \beta_2\right)} sinh\left(\sqrt{\beta_3}\chi\right) \tag{24}$$

where $B_2 = 0, A_2 = \frac{e^{-i\omega t}}{sin\left(\sqrt{\beta_2}\right)} - \frac{\beta_4}{(\beta_3+\beta_2)}\frac{sinh\left(\sqrt{\beta_3}\right)}{sin\left(\sqrt{\beta_2}\right)}$

Substitute the LDL-C concentration Eq. (19) and temperature profile Eq. (24) in the perturbed momentum Eq. (14), we have

$$\frac{\partial^2 w_0}{\partial \chi^2} - h\left(i\omega + M^2 + \frac{1}{k}\right)w_0 = P_0 h - \left(A_2 Gr_1 sin\left(\sqrt{\beta_2}\chi\right) + \frac{\beta_4 Gr_1}{(\beta_3+\beta_2)} sinh\left(\sqrt{\beta_3}\chi\right)\right) - \left(\frac{Gc_1 e^{-i\omega t}}{sinh\left(\sqrt{\beta_3}\right)} sinh\left(\sqrt{\beta_3}\chi\right)\right) \tag{25}$$

Further simplification of Eq. (25) reduces to

$$\frac{\partial^2 w_0}{\partial \chi^2} - \left(i\omega + M^2 + \frac{1}{k}\right)w_0 = P_0 h - A_2 Gr_1 sin\left(\sqrt{\beta_2}\chi\right) - \left(\frac{Gc_1 e^{-i\omega t}}{sinh\left(\sqrt{\beta_3}h\right)} + \frac{\beta_4 Gr_1}{(\beta_3+\beta_2)}\right) sinh\left(\sqrt{\beta_3}\chi\right) \tag{26}$$

Let $\beta_1 = h\left(i\omega + M^2 + \frac{1}{Da}\right)$, $\beta_5 = \left(\frac{Gc_1 e^{-i\omega t}}{sinh\left(\sqrt{\beta_3}h\right)} + \frac{\beta_4 Gr_1}{(\beta_3+\beta_2)}\right)$ so that Eq. (26) is reduce to:

$$\frac{\partial^2 w_0}{\partial \chi^2} - \beta_1 w_0 = P_0 h - A_2 Gr_1 sin\left(\sqrt{\beta_2}\chi\right) - \beta_5 sinh\left(\sqrt{\beta_3}\chi\right) \tag{27}$$

If the RHS of Eq. (27) is zero, the solution is

$$w_{0h}(\chi) = A_6 sinh\left(\sqrt{\beta_1}\chi\right) + B_6 cosh\left(\sqrt{\beta_1}\chi\right) \tag{28}$$

If the RHS ofEq. (27) is not zero, the particular solution will be of the form:

$$w_{0p} = A_{31} + A_4 sin\left(\sqrt{\beta_2}\chi\right) + A_5 sinh\left(\sqrt{\beta_3}\chi\right) \tag{29}$$

where $A_{31} = -\frac{P_0 h}{\beta_1}, A_4 = \frac{A_2 Gr_1}{(\beta_2+\beta_1)}, A_5 = \frac{\beta_5}{(\beta_1-\beta_3)}, B_4 = 0, B_5 = 0$

Then, the general solution to Eq. (27) is:

$$w_0(\chi) = A_6 sinh\left(\sqrt{\beta_1}\chi\right) + B_6 cosh\left(\sqrt{\beta_1}\chi\right) + A_{31} + A_4 sin\left(\sqrt{\beta_2}\chi\right) + A_5 sinh\left(\sqrt{\beta_3}\chi\right) \tag{30}$$

We use the boundary conditions in Eq. (17) to solve for the coefficients in Eq. (30) so that the solution becomes:

$$w_0(\chi) = A_6 sinh\left(\sqrt{\beta_1}\chi\right) - A_{31}\left(cosh\left(\sqrt{\beta_1}\chi\right) - 1\right) + A_4 sin\left(\sqrt{\beta_2}\chi\right) + A_5 sinh\left(\sqrt{\beta_3}\chi\right) \tag{31}$$

where:

$$A_6 = A_{31}\left(\frac{cosh\left(\sqrt{\beta_1}\right)}{sinh\left(\sqrt{\beta_1}\right)} - \frac{1}{sinh\left(\sqrt{\beta_1}\right)}\right) - A_4\frac{sin\left(\sqrt{\beta_2}\right)}{sinh\left(\sqrt{\beta_1}\right)} - A_5\frac{sinh\left(\sqrt{\beta_3}\right)}{sinh\left(\sqrt{\beta_1}\right)}$$

We can now substitute Eqs. (31), (24) and (19) into Eq. (13) in order to obtain our blood velocity profile, temperature profile and LDL-C profile respectively as:

$$w(\chi,t) = \left(A_6 sinh\left(\sqrt{\beta_1}\chi\right) - A_{31}\left(cosh\left(\sqrt{\beta_1}\chi\right) - 1\right) + A_4 sin\left(\sqrt{\beta_2}\chi\right) + A_5 sinh\left(\sqrt{\beta_3}\chi\right)\right)e^{i\omega t} \tag{32}$$

$$\theta(\chi,t) = \left(A_2 sin\left(\sqrt{\beta_2}\chi\right) + B_2 cos\left(\sqrt{\beta_2}\chi\right) + \frac{\beta_4}{(\beta_3+\beta_2)} sinh\left(\sqrt{\beta_3}\chi\right)\right)e^{i\omega t} \tag{33}$$

$$\phi(\chi,t) = \left(\frac{e^{-i\omega t}}{sinh\left(\sqrt{\beta_3}\right)} sinh\left(\sqrt{\beta_3}\chi\right)\right)e^{i\omega t} \tag{34}$$

The volumetric flow rate is solved analytically using Eq. (32), which is

$$Q = \int_0^1 \left(A_6 sinh\left(\sqrt{\beta_1}\chi\right) - A_{31}\left(cosh\left(\sqrt{\beta_1}\chi\right) - 1\right) + A_4 sin\left(\sqrt{\beta_2}\chi\right) + A_5 sinh\left(\sqrt{\beta_3}\chi\right)\right)e^{i\omega t} d\chi \tag{35}$$

$$Q = e^{i\omega t}\left(\frac{A_6}{\sqrt{\beta_1}} cosh\left(\sqrt{\beta_1}\chi\right) - A_{31}\left(\frac{sinh\left(\sqrt{\beta_1}y\right)}{\sqrt{\beta_1}} - \chi\right) - \frac{A_4}{\sqrt{\beta_2}} cos\left(\sqrt{\beta_2}\chi\right) + \frac{A_5}{\sqrt{\beta_3}} cosh\left(\sqrt{\beta_3}\chi\right)\right)\Bigg|_{y=0}^{y=1}$$

$$Q = e^{i\omega t}\left(\begin{array}{l}\frac{A_6}{\sqrt{\beta_1}} cosh\left(\sqrt{\beta_1}\right) - A_{31}\left(\frac{sinh\left(\sqrt{\beta_1}\right)}{\sqrt{\beta_1}} - 1\right) - \frac{A_4}{\sqrt{\beta_2}} cos\left(\sqrt{\beta_2}\right) + \frac{A_5}{\sqrt{\beta_3}} cosh\left(\sqrt{\beta_3}\right) \\ -\left(\frac{A_6}{\sqrt{\beta_1}} - \frac{A_4}{\sqrt{\beta_2}} + \frac{A_5}{\sqrt{\beta_3}}\right)\end{array}\right) \tag{36}$$

The rate of heat transfer at the wall of the vessel is calculated analytically as:

$$Nu = -\frac{\partial\theta}{\partial\chi}\bigg|_{\chi=1} = h\left(B_2\sqrt{\beta_2} sin\left(\sqrt{\beta_2}\right) - A_2\sqrt{\beta_2} cos\left(\sqrt{\beta_2}\right) - \frac{\beta_4\sqrt{\beta_3}}{(\beta_3+\beta_2)} cosh\left(\sqrt{\beta_3}\right)\right)e^{i\omega t} \tag{37}$$

The rate of LDL-C mass transfer at the wall of the vessel is calculated analytically as:

$$Sh = -\frac{\partial\phi}{\partial\chi}\bigg|_{\chi=1} = -h\left(\frac{\sqrt{\beta_3}e^{-i\omega t}}{sinh\left(\sqrt{\beta_3}\right)} cosh\left(\sqrt{\beta_3}\right)\right)e^{i\omega t} \tag{38}$$

Results and discussion

Results

This research is to investigate the mathematical modeling of mixed LDL-C and blood flow through an inclined channel with heat in the presence of magnetic field, and the object of study has been achieved with the simulated results showing the effect of the various parameters values such as radiation, metabolic heat, Prandtl number, Hartmann number, Schmidt number, angle of inclination, chemical reaction, oscillatory frequency parameter, height of stenosis, treatment parameter, growth rate of $a = 0.05$ at $x = 0.8$ on the axial blood velocity, LDL-C concentration and temperature profiles respectively with the aid of Mathematica, version 10.

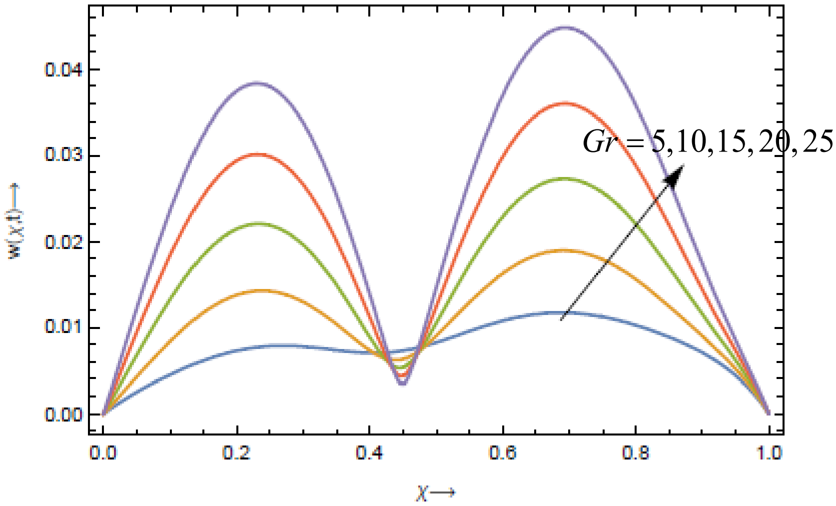

Figure 1 Effect of Grashof parameter Gr values on velocity $w(\chi,t)$.

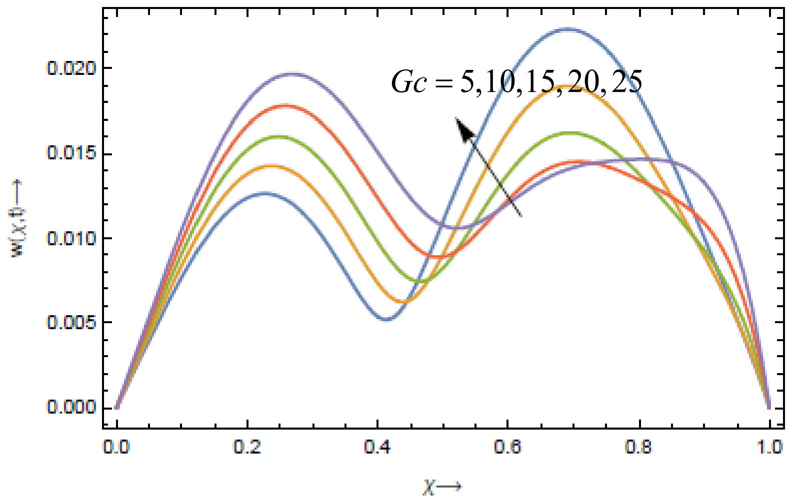

Figure 2 Effect of solutal Grashof Gc values on velocity $w(\chi,t)$.

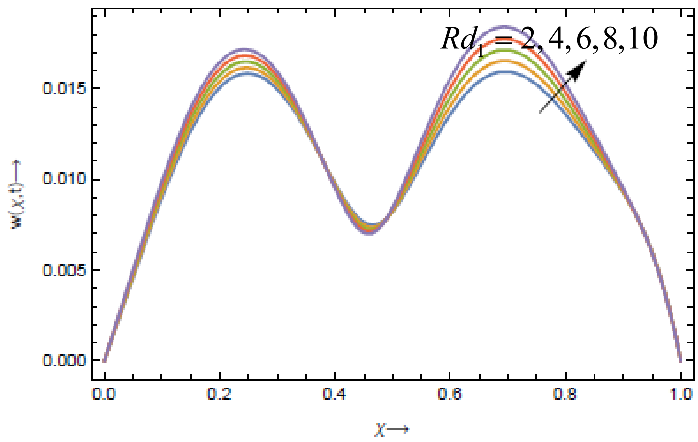

Figure 3 Effect of Radiation parameter Rd_1 values on velocity $w(\chi,t)$.

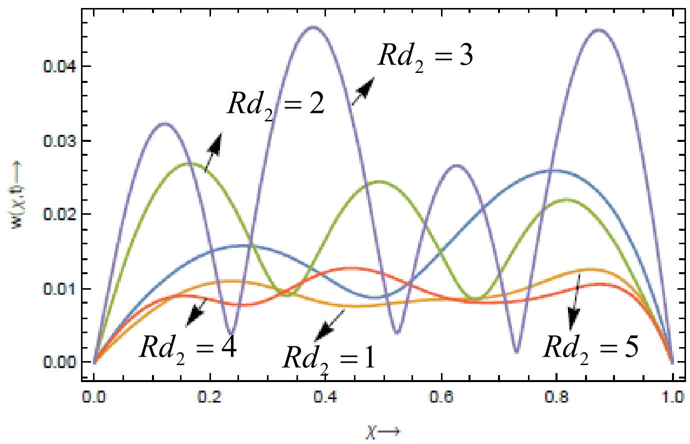

Figure 4 Effect of Metabolic parameter Rd_2 values on velocity $w(\chi,t)$.

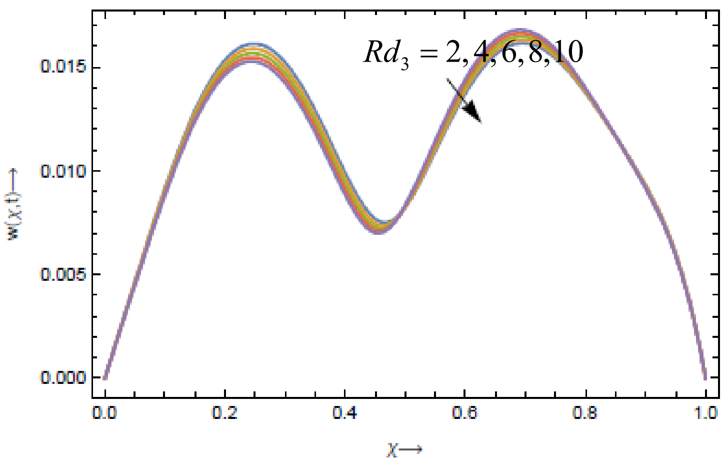

Figure 5 Effect of chemical parameter Rd_3 values on velocity $w(\chi,t)$.

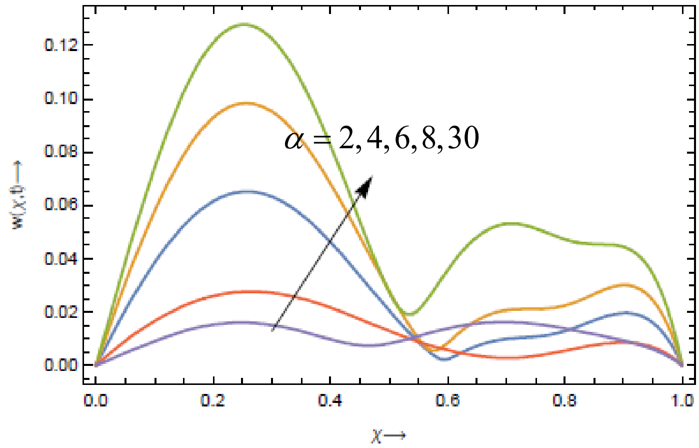

Figure 6 Effect of inclined angle α values on velocity $w(\chi,t)$.

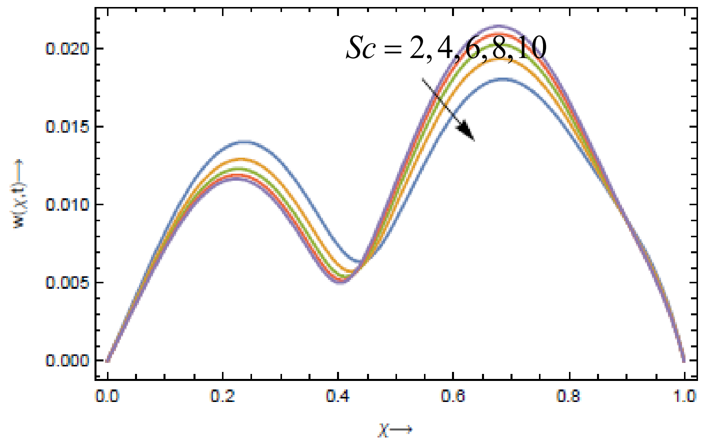

Figure 7 Effect of Schmidt number Sc values on velocity $w(\chi,t)$.

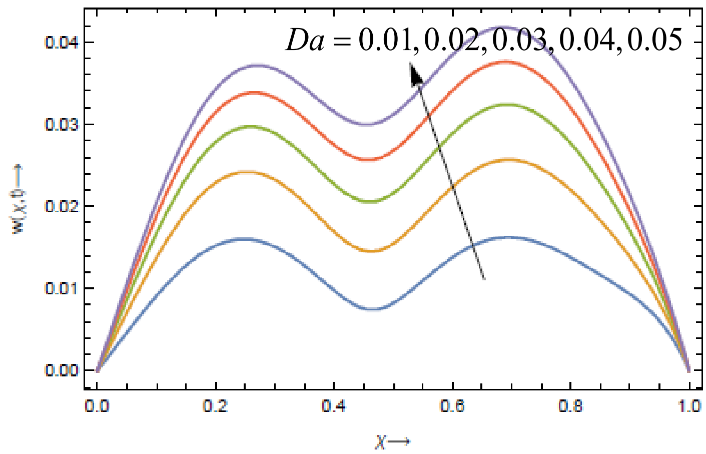

Figure 8 Effect of Darcy parameter Da values on velocity $w(\chi,t)$.

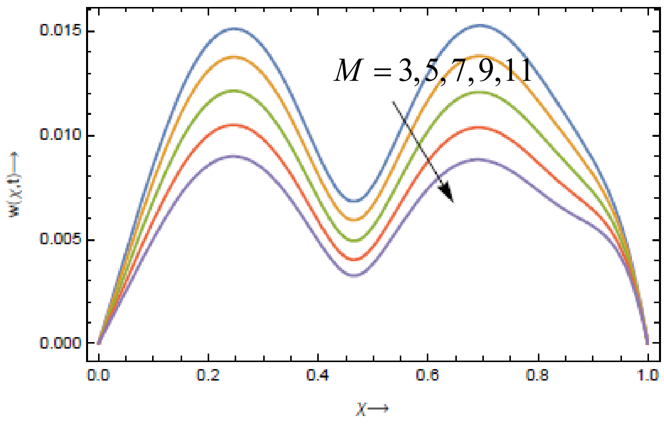

Figure 9 Effect of Magnetic parameter M values on velocity $w(\chi,t)$.

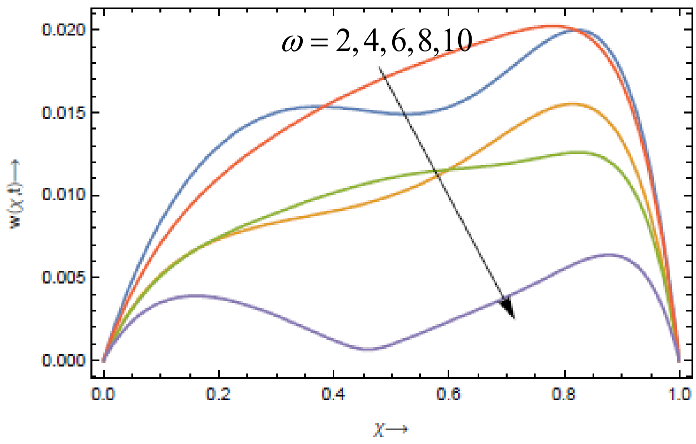

Figure 10 Effect of oscillatory parameter ω values on velocity $w(\chi,t)$.

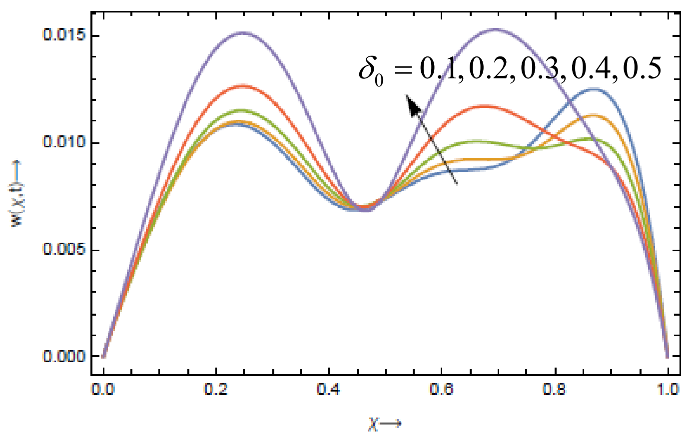

Figure 11 Effect of height of stenosis δ_0 values on velocity $w(\chi,t)$.

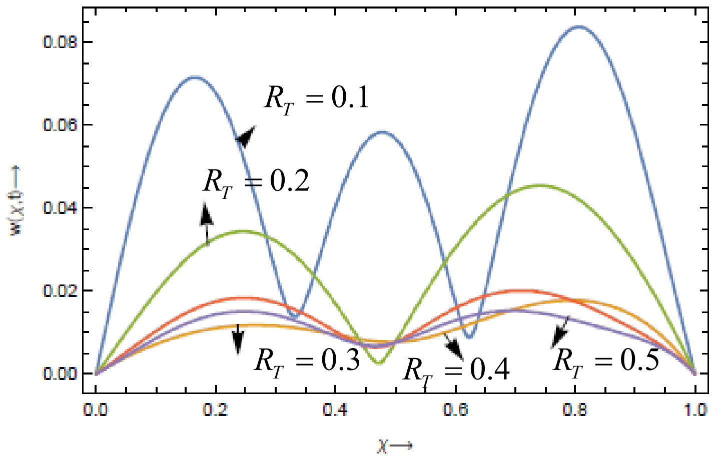

Figure 12 Effect of Treatment parameter R_T values on velocity $w(\chi, t)$.

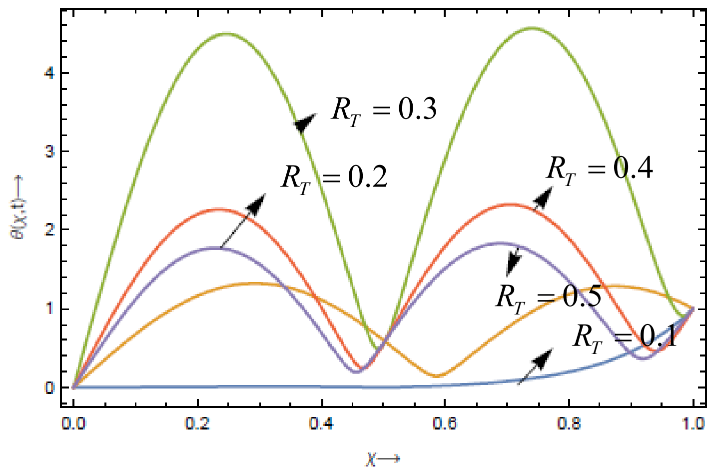

Figure 13 Effect of Treatment R_T values on temperature $\theta(\chi, t)$.

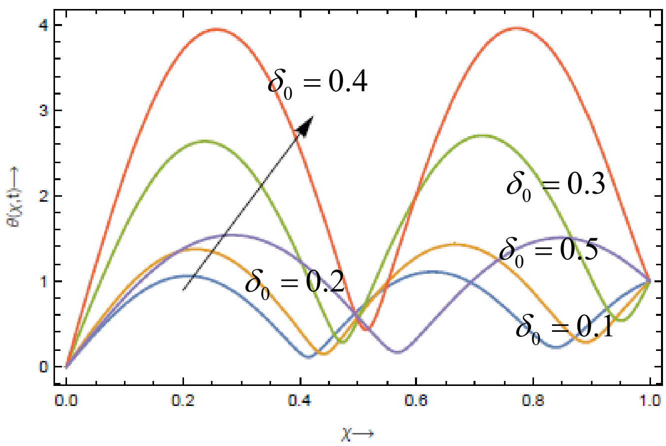

Figure 14 Effect of height of stenosis δ_0 values on temperature $\theta(\chi, t)$.

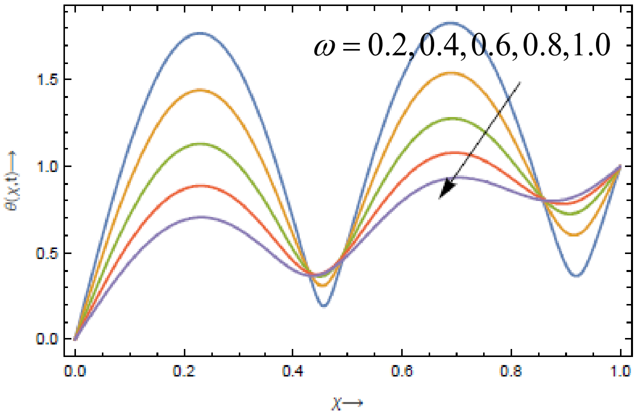

Figure 15 Effect of oscillatory frequency ω values on temperature $\theta(\chi,t)$.

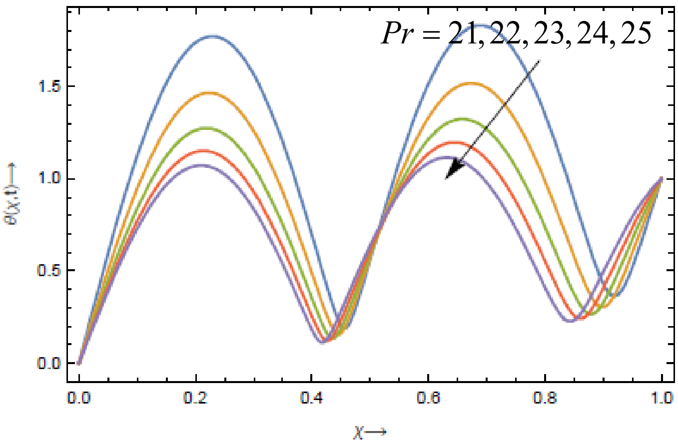

Figure 16 Effect of Prandtl number Pr values on temperature $\theta(\chi,t)$.

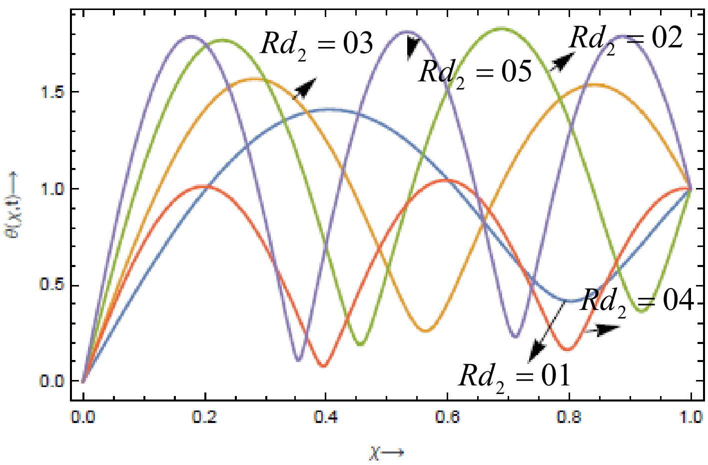

Figure 17 Effect of metabolic heat Rd_2 values on temperature $\theta(\chi,t)$.

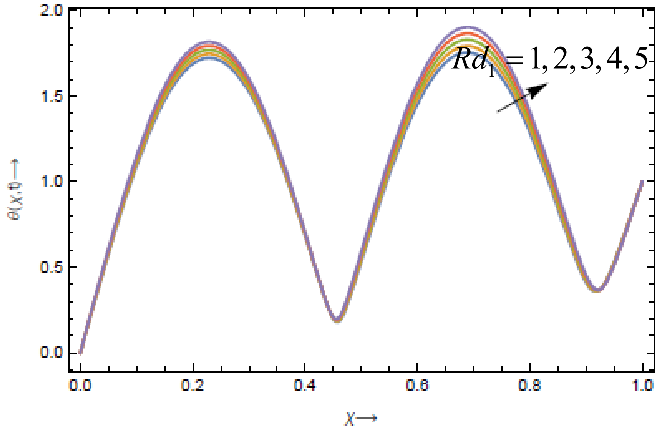

Figure 18 Effect of Radiation Rd_1 values on temperature profile $\theta(\chi, t)$.

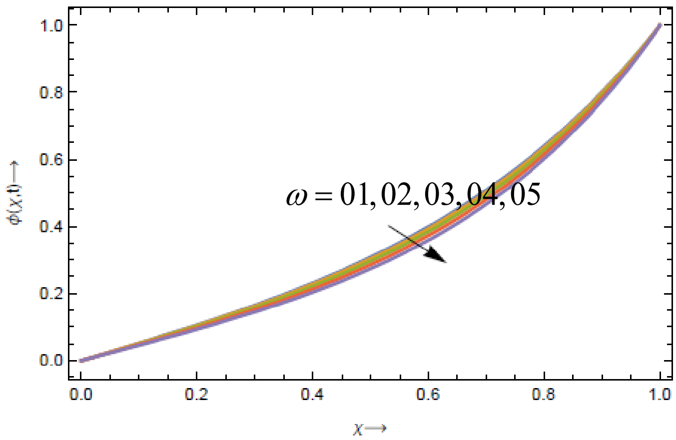

Figure 19 Effect of chemical Rd_3 values on LDL-C Concentration $\phi(\chi, t)$.

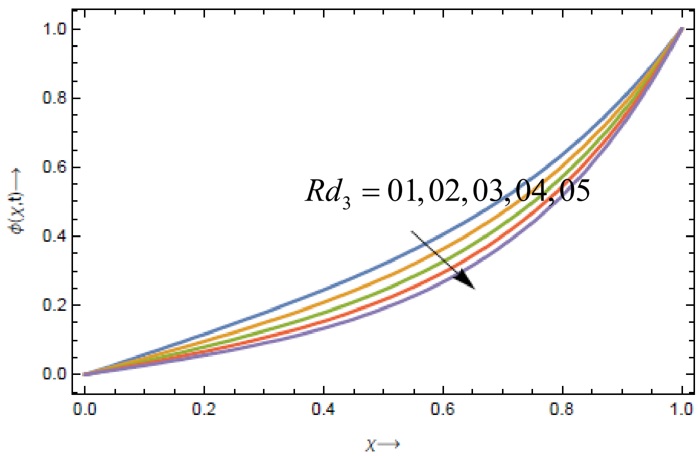

Figure 20 Effect of Schmidt number Sc values on LDL-C Concentration $\phi(\chi, t)$.

Discussion

The biological models led to an analytical solution of the velocity, LDL-C concentration and temperature profiles. A simulation was done to investigate the effect of the entering pertinent parameters where results were presented and the results are discussed below.

1) The increase in Grashof number Gr and solutal Grashof number Gc caused the velocity of the fluid to increase. This result shows that the velocity of the fluid increase as the buoyancy effect due to temperature and concentration lipid is increased as noticed in **Figures 2**-**3**.

This result depicts situation where by other pertinent parameters such;

$$Gr = 10, Rd_1 = 03, Rd_2 = 03, Rd_3 = 03, \alpha = 30, Pr = 21, Da = 0.01, M = 0.3, \omega = 0.2,$$

$$\delta_0 = 0.5, R_T = 0.5.$$

2) **Figure 4** shows that the velocity profile oscillates as it increases for different values of the metabolic heat parameter, while other parameters are kept constant. **Figure 5** illustrates a decrease in velocity profile for different values of the chemical reaction term. This result is of the view that the rate of chemical reaction between the drugs administered and the chemical composition of the body lead to a decrease in blood circulation.

3) The effect of an angle of inclination was also investigated and the simulation found that the velocity profile increases for different values of angle of inclination, and the result is shown in **Figure 6**. **Figure 7** illustrates a decrease in velocity profile for different values of the Schmidt number, this result is of the view that the increase in molecular diffusion could create an increase in lipid concentration in the bloodstream and that caused the velocity decrease. However, in **Figure 8**, the velocity profile of the fluid increases, for different values of Darcy number. The Darcy number has to do with increase in porosity of the fluid medium, which shows that the increase in the sizes of the pore matrix of the medium the faster the flow become.

4) The interaction between a moving electrically conducting fluid and a magnetic field would lead to a force called the Lorentz force which inhibits the velocity of the fluid as depicted by **Figure 9**. **Figure10** depicts the periodic increase in pulse rate and that led to decrease in velocity profile of the fluid, for an increase in values of the oscillatory frequency.

5) Ordinarily the increase in height of stenosis would have cause a decrease in velocity profile but **Figure 11** shows otherwise because the inhibitor in the form of statin drug has been able to block off the production of cholesterol into circulation and mob up process in the form of macrophages which helps in improving the velocity profile of the fluid.

6) **Figure 12** illustrate a typical oscillatory behavior of the fluid velocity for different values of the treatment parameters. This result is of the view that when administering drugs in treating any ailment be mindful of the chemistry of the individual's body in order to avoid drug reaction. As similar result being observed for the temperature profiles as shown in **Figure 13**.

7) The temperature profile is seen to increase for different values of the height of stenosis, this result is of the view that the quantity of cholesterol available at a particular time actually create a rise in temperature as illustrated in **Figure 14**. However, the temperature profile decrease for the increasing values of the oscillatory frequency and Prandtl number as depicted in **Figures 15** and **16** respectively.

8) **Figure 17** shows a pure oscillatory temperature profile for different values of the metabolic heat parameter, while the concentration increases for different values of the radiation parameter, this is of the view that increase in radiation expands the vessel to allow quicker passes and without sufficient blocker in the system denoted in **Figure 18**. Finally, it is seen in **Figure 19** that the concentration profile decreases for different values increase in the oscillatory frequency values

Conclusions

In conclusion, we have done justice to the matter in the sense that, we formulated the model and soled to and obtained an analytical solution for the flow expression, developed numerical simulation code using Mathematica, and simulated the various expression for different entering parameters in the velocity profile, temperature profile and LDL-C concentration profiles respectively. And found the significant flow parameters affecting the profiles and if well managed are of good information for health practitioners for studying cardiovascular ailments.

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