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Distribution of Charges on a Flat Disk as the Application of Far-Ranging Mathematics

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Abstract

The foremost grail of this academic indagation is to delineate a mathematical expression of the normalised charge density over a flat disk. Aiming to ensue, firstly, 2 different frameworks have been dealt with to formulate the potential distribution which allows stability of a non-uniform charge distribution. At first, a logical but mathematically toilsome integral method has been approached. Out of the unyielding territory, we reduced the expression into algebraical functions using the Bessel coefficient and Green's theorem, eventually inferring a new mathematical equivalence. Subsequently, this paper explores beta function as a solving tool of complete elliptic integral so that the normalization of charge apportion leads to 0 gradients of potential. Finally, the article deduces an integral equation whose implicit solution brings into the required charge distribution. The write-up also encounters finding a proximate graphical illustration of the assortment following the CAS system and direction fields. Beyond the conventional approach of real analysis, it facilitates proving the convergence of an acclaimed series. Consequently, it conceives a discussion on image charges for a flat disk. Even a short view of the article's impact on practical fields of biology and engineering sciences has been included as the denouement. So, it might be of interest to the wide-ranged audience of research scholars in both the fields of physical and mathematical sciences.

Keywords: Disk, Potential gradient, Charge density distribution, Transformation equation, Mathematical equivalence, Real number series

Introduction

Electrostatics is a vast field with an enormous sense of application in engineering as well. The mathematical modelling of charge distribution on a conductor is of great interest in numerous scientific disciplines. The concept of image charges brings a new aspect of finding a solution in search of a problem which opens up a huge practical problem-solving technique [1,2]. As a consequence, the mathematical estimation of the electrostatic potential at any arbitrary observation point in the plane of the charged disk has been the bull's eye for many subject areas. As far as physics is concerned, even in several 2D electronic models of quantum hall effect [3-7], the approximation of neutralizing background charges is an esteemed application of the disk models [3]. Understanding the interaction of a neutral conductor with a charge placed nearby requires an immense solution of the complex potential function. One of the most promising subject areas of electrostatics is the potential and field distribution caused by static charges. In the entire territory of physics, the electrostatic field and potential are the most wildly used concepts in almost every discipline. Apart from physics and mathematics, it has an immense sense of application in computer science engineering, electrical engineering, communication engineering, semiconductor sciences, and much more. To study the behaviour of numerous biological systems [8] like apoplastic and symplastic imbibition [9], EMP pathways, electron-transport systems [10,11], etc. requires knowledge of electric potentials. Genetics is an apodictically studied and enriched discipline of Modern Biology. Researches in genetic evolution and DNA-disk [10, 12, 13] as well as different phenomenon of nucleus also seek knowledge on electrostatics. Approximating the protein and polypeptide sol to charged disk models enables understanding their functioning [14-16]. Transportation of neurohumor essentially entails the idea of potential amongst interacting charges [17-20]. It is unnecessary to specify the applications of potential and charge distribution in the engineering domain. That's why introducing a detailed knowledge of charge distribution, their fields and potentials are eminently needed.

Whenever capacitors and it's regarding physics is talked over, the forces felt by the charges in the plates also entail the idea of the dissemination of charge density. The perception of charge dispensation over a plane disk is crucial for designing hyperbolically structured and more strengthened palates for regenerative technology, the heart of nowadays electric vehicles [21-22]. In the last decade it has been a piece of hot cake to discover an expression stating the potential distribution. Most recently an article [23] has successfully tried to formulate such a mathematical expansion by the year 2020, but my article demands a more scientific way to disregard the findings of [23].

The article comprises the general solution of the charge distribution on a flat disk along with 2 different pathways of working out the function of potential distribution in the plane of the disk. From mathematical sight, the solution of the problem is highly non-trivial as well as it takes into account some complex algebraical functions followed by a few special integral forms such as a solution of complete elliptic integration of different kinds. Keeping the calculation simple and relatively straightforward, we aimed to have an expression to be interpreted as a summation of contributions of all the infinitesimally constituent charged rings of the disk. In case, a direct integral approach has been opted. The formula is useful yet seems to be mathematically obdurate. In a separate appeal, we first reduced the expression using well-known coefficients of Bessel's function [24]. This expression is quite handy and instructive to derive new inferences as well as equivalences. Ultimately, setting an onerous equation to be satisfied by the charge density on a flat disk is of paramount importance.

Computing several methods, new insights of transformation equations have been achieved. A concrete and intriguing discussion on the charge density has also been done. In that light, a new vision has been attained for the concept of image charges on a finite flat disk. The solution of potential distribution simultaneously allows drawing an interesting concept regarding convergence theorems of real series. In brief, the article uniquely contributes in expressing the potential at any arbitrary 3D space point due to uniformly charged disk, natural unconstrained normalisation of charge density, implicating trigonometric solution in complete elliptic integral, and proving the convergence of a bivariate series along with finding transformation equation.

Equating potential gradient for uniform charge density

The charge distribution on a circular disk is a well-known but complex problem of higher-order mathematical physics. Before stepping into the distribution of charges, let us convince ourselves whether the charges can rearrange themselves. Consider that q amount of charge has been given to a disk of finite radius a. Assuming the charge distribution being restricted to only one surface (originally, if q is the net charge, then symmetrically $\frac{q}{2}$ amount charge should appear on both the surfaces of the disk), we are considering that the charge is uniformly spread with a density $\sigma = \frac{q}{\pi a^2}$. Let us produce the potential contribution of the charged disk at 2 different points, namely $r = 0\ and, a$. Refer to the **Figure 1**.

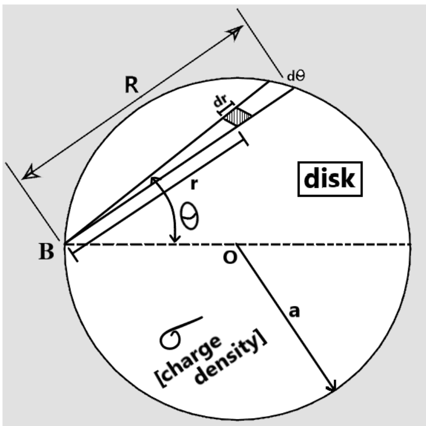


Figure 1 Derivation of potential at $r = a$.

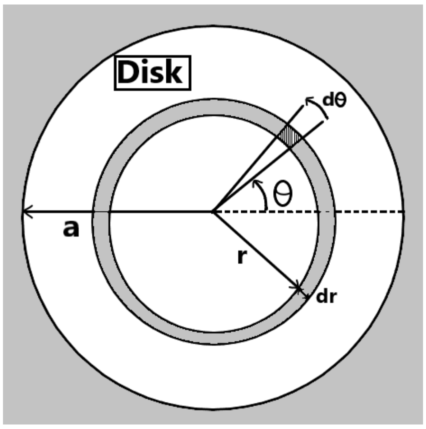


Figure 2 Derivation of potential at $r = 0$.

Potential at $r = 0$ and a

Take point B on the rim of the disk where $r = a$. The potential at point B is P_1(say). For evaluating the potential P_1, we may consider a thin wedge of total length R and angular width $d\theta$ as shown in **Figure 1**. An element of the wedge at a distance r from B has been illustrated as the shaded region in the figure. The elementary charge contained in the region is, $dq = \sigma(r.d\theta)dr$. Its contribution to the potential at point B is thus;

$$dV_w = \frac{dq}{4\pi\epsilon_0 r}$$

$$= \frac{\sigma(r.d\theta)dr}{4\pi\epsilon_0 r}$$

$$= \frac{\sigma.d\theta.dr}{4\pi\epsilon_0}$$

∴ The contribution of the entire wedge in the potential is, $V_w = \int_{r=0}^{R} dV_w$. Now, the potential at B due to the entire disk is;

$$P_1 = \int_{r=0}^{R} \int_{\theta=-\frac{\pi}{2}^+}^{\frac{\pi}{2}^-} dV_w$$

Since, at $\theta = \left\{-\frac{\pi}{2}, \frac{\pi}{2}\right\}$ the wedge element becomes tangential to the disk, it is the limiting contribution to the potential. But the potential due to wedge at limiting θ is simply zero because $r = 0$. Therefore the equation of P_1 can be modified as;

$$P_1 = \int_{r=0}^{R} \int_{\theta=-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sigma.d\theta.dr}{4\pi\epsilon_0}$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sigma.d\theta}{4\pi\epsilon_0} . \int_{r=0}^{R} dr$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sigma.d\theta}{4\pi\epsilon_0} .\ 2a\cos\theta$$

since $R = 2a\cos\theta$

$$= \frac{\sigma.2a}{4\pi\epsilon_0} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d(\sin\theta)$$
$$= \frac{\sigma.a}{\pi\epsilon_0} \tag{1}$$

Now, considering **Figure 2** we are considering a thin circular ring. The region denotes an elementary part of a thin circular ring with an area of $dA = (r.d\theta)dr$. Thus, the charge contained in it is, $dq = \sigma(r.d\theta)dr$. Hence, the contribution of the elementary charge in the potential P_0 is, [P_0 is the potential due to the entire disk at the center ($r = 0$)], $dV = \frac{\sigma.d\theta.dr}{4\pi\epsilon_0}$. Hence potential at the center of the flat disk is;

$$P_0 = \int_{r=0}^{a}\int_{\theta=-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sigma.d\theta.dr}{4\pi\epsilon_0}$$

$$= \int_{\theta=-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sigma.d\theta}{4\pi\epsilon_0}.\int_{r=0}^{a} dr$$

$$= \frac{\sigma.(2\pi).a}{4\pi\epsilon_0}$$

$$= \frac{\sigma.a}{2\epsilon_0} \tag{2}$$

Commencing with the potential of the disk at the points O and B for uniform charge distribution σ, we can infer that the potential $P_{0,}\, P_1$ are not the same rather, $P_0 = \frac{\sigma.a}{2\epsilon_0} > P_1 = \frac{\sigma.a}{\pi\epsilon_0}$ which is in accordance with [25]. Thus, it can be visualized that there is a continuous gradient of potential with the origin (centre of the disk) at a higher potential. Followingly, we can try shortly to produce the nature of potential variation on the plane of a finite and uniformly charged disk of radius ‘a’. It is a well-known source of scrupulous but smart mathematics.

Finding an expression for the potential distribution

Take a uniformly charged disk of radius ‘a’ carrying charge q spread homogenously over the flat disk with charge density $\sigma = \frac{q}{\pi a^2}$. Now, we are eager to estimate the electrostatic potential by the disk at a point with position vector $\vec{r}(x, y, z)$. It is to be noted that the z-direction denotes the direction of the disk's axis. Consider the points with position vector $\vec{r'}(r', \varphi', z)$ in the cylindrical polar coordinate of which the contribution in potential is to be evaluated.

$$\therefore V(r,z) = \frac{\sigma}{4\pi\epsilon_0}\int_0^R r'.dr' \int_0^{2\pi} \frac{d\varphi'}{\sqrt{r^2+r'^2-2rr'.\cos\varphi'+z^2}}$$

$$= 4\left(\frac{\sigma}{4\pi\epsilon_0}\right)\int_0^R \frac{r'.dr'}{\sqrt{(r+r')^2+z^2}} K\left[\frac{4rr'}{(r+r')^2+z^2}\right] \tag{3}$$

$$\therefore V(r,0) = 4\left(\frac{\sigma}{4\pi\epsilon_0}\right)\int_0^R \frac{r'.dr'}{(r+r')} K\left[\frac{4rr'}{(r+r')^2}\right] \tag{4}$$

Now, the function $K[\eta]$is given as follows, $K[\eta] = \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1-\eta\sin^2\theta}}$ and $\eta = \frac{4rr'}{\sqrt{(r+r')^2+z^2}}$, which is the complete elliptic integral of the first kind [26]. Substituting $\frac{r'}{R}$ with a new variable p, we can rewrite the equation as;

$$\frac{V(r,0)}{V(0,0)} = \frac{2}{\pi}\int_0^1 \frac{dp.p}{\frac{r}{R}+p} K\left[\frac{4\frac{r}{R}p}{\left(\frac{r}{R}+p\right)^2}\right] \tag{5}$$

where $V(0,0) = \frac{2\pi\sigma R}{4\pi\epsilon_0}$.

Now, let us approach the integral using the Bessel function as following [27];

$$\frac{1}{|\vec{r_1}-\vec{r_2}|} = \sum_{m=-\infty}^{\infty} \int_0^\infty dk.\, e^{im(\varphi_1-\varphi_2)} J_m(kr_1).J_m(kr_2) e^{-k.|z_1-z_2|} \tag{6}$$

Now the potential is given as;

$$V(r,z) = K_e \sigma \int_0^R \int_0^{2\pi} \frac{r'.dr'.d\varphi'}{\sqrt{(r+r')^2+z^2}}$$

$$\therefore V(r,0) = K_e \sigma \int_0^R r'.dr' \int_0^{2\pi} \frac{d\varphi'}{|\vec{r}-\vec{r'}|} \tag{7}$$

where $K_e = \frac{1}{4\pi \epsilon_0}$.

Computing the angular integration over 0 to 2π, the equation can be reduced to the following form, using Bessel Function;

$$\therefore V(r,z) = 2\pi K_e \sigma R \int_0^\infty \frac{dk}{k} J_0(kr).J_1(kR) e^{-k.|Z|} \tag{8}$$

where it is to be noted that, $2\pi K_e \sigma A = \frac{K_e Q}{2R}$.

Now let us again inculcate a dummy variable $q = kR$. Therefore;

$$\frac{V(r,0)}{V(0,0)} = \int_0^\infty \frac{dq}{q} J_0\left(\frac{q}{R} r\right).J_1(q) \tag{9}$$

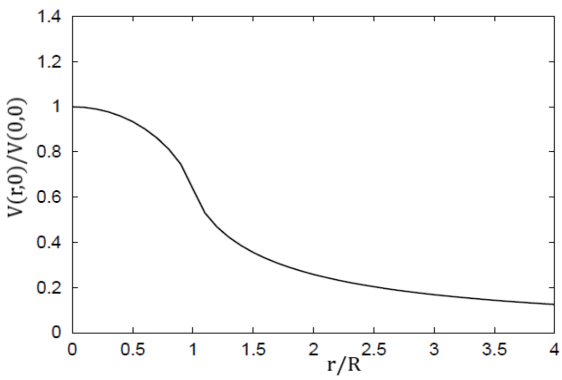


Figure 3 Plot of electrostatic potential versus r/R.

This mathematical expansion is quite easy to tackle. As depicted in **Figure 3**, the electrostatic potential on the plane of the disk has been shown as a function of $\frac{r}{R}$ for the ranges $0 \le r \le R$ and $R < r < \infty$. Imparting integral forms for Bessel's functions we have;

$$\therefore \frac{V(r,0)}{V(0,0)} = \frac{2}{\pi} E\left[\left(\frac{r}{R}\right)^2\right] \qquad \forall\, 0 \le \left(\frac{r}{R}\right)^2 \le 1$$

$$\frac{2}{\pi}\left[\frac{r}{R} E\left[\left(\frac{r}{R}\right)^{-2}\right] + \frac{1-\left(\frac{r}{R}\right)^2}{\left(\frac{r}{R}\right)}.K\left[\left(\frac{r}{R}\right)^{-2}\right]\right] \qquad \forall\, 1 \le \left(\frac{r}{R}\right)^2 < \infty \tag{10}$$

where the function $E(m) = \int_0^{\frac{\pi}{2}} \sqrt{1 - m\,(\sin\theta)^2}.\,d\theta$ is the complete elliptic integral of the second kind [28,31].

Deriving a transformation formula

Quite evidently this allows identifying some prominent mathematical transformation in terms of a new variable, $\beta = \frac{r}{R}$. The potential distribution achieved following the first and second kind of complete elliptic integral can be expressed as Eq. (10). The Eq. (5) can be rewritten using the new variable as;

$$\frac{V(r,0)}{V(0,0)} = \frac{2}{\pi}\int_0^1 \frac{dp.p}{\beta+p}\, K\left[\frac{4\beta p}{(\beta+p)^2}\right] \tag{11}$$

Again, we can delineate Eq. (10) using the variable β such that;

$$\therefore \frac{V(r,0)}{V(0,0)} = \frac{2}{\pi}E[\beta^2] \qquad \forall\, 0 \le \beta^2 \le 1$$

$$\frac{2}{\pi}\left[\beta E[\beta^{-2}] + \frac{1-\beta^2}{\beta}.K[\beta^{-2}]\right] \qquad \forall\, 1 \le \beta^2 < \infty \tag{12}$$

Hence Equating, (11) and (12), the final mathematical equivalence that can be drawn is;

$$\int_0^1 \frac{dp.p}{\beta+p}\, K\left[\frac{4\beta p}{(\beta+p)^2}\right] = \frac{2}{\pi}E[\beta^2] \qquad \forall\, 0 \le \beta^2 \le 1$$

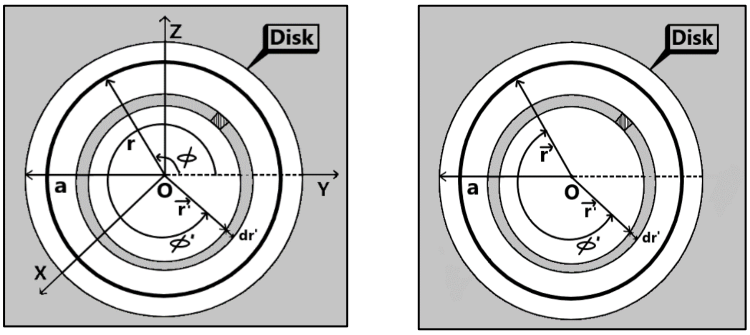
$$\frac{2}{\pi}\left[\beta E[\beta^{-2}] + \frac{1-\beta^2}{\beta}.K[\beta^{-2}]\right] \qquad \forall\, 1 \le \beta^2 < \infty \tag{13}$$

where $K(\eta)$ and $E(\eta)$ denotes the first and second kinds of complete elliptic integrals, respectively.

The article is not intended to draw any additional mathematical inferences. It adjoins the preceding sections for vehemently convincing that the potential distribution is quite eccentric with a maximum at the center followed by a gradual depression towards the rim. It is impressive to note that naturally induced charges on any flat and finite disk can never be spread uniformly. Since there is a difference in potential with a positive slope from the center towards the edge, the charges are naturally subjected to a force to flow outward. As a result, the density of charges consistently decreases at the center with an increment on the rim.

Stabilization of charge density and its novel mathematics

Thus, we are keen to understand the natural charge distribution and, for that sake, it is likely to proceed with the details of the elliptical integral. As much as physics is concerned, it is really easy to formulate the equations but complicated enough to solve them. The prime base of physics we are going to use in these formulations is the fact that the potential throughout the disk for the region $0 < r \le a$ becomes uniform as the charge distribution is normalized. So, the charge density will be uplifted at the edge rather than a uniform assortment and the only possible way is the radial flow of charges. Hence quite evidently, we can infer that the final arrangement of charge density is radially symmetric. As a fact of consequence, the charge density is independent of φ $(azimuthal\ angle)$ and z (axile coordinate).



(a) (b)

Figure 4 (a) Elementary charged ring as the differential contributor to potential, and (b) Resolving the system in a relative azimuthal frame.

Expressing the potential of the entire disk as the sum of differential ring element

Consider **Figure (4a)**, the axis OY has been taken as the initial line to which the azimuthal angle can be assigned. Now, $\overrightarrow{r}$ denotes the position vector of the observation point and $\overrightarrow{r'}$ is the position vector of the point whose contribution in potential is to be analyzed. $\overrightarrow{r} \equiv (r, \varphi, z)$ denotes a certain point, whereas, $\overrightarrow{r'} \equiv (r', \varphi', 0)$ denotes an element of the shaded ring illustrated in the above-mentioned figures. Let us assume a thin ring of thickness dr' with a radius r' be termed as the source charges. So, the potential of an elementary source charge at the point $\overrightarrow{r}$ can be given by;

$$V\ (source\ point) = \frac{dq'}{4\pi\epsilon_0 \left|\overrightarrow{r} - \overrightarrow{r'}\right|}$$

$$= \frac{dA'.\sigma(r')}{4\pi\epsilon_0 |\overrightarrow{r} - \overrightarrow{r'}|}$$

$$= \frac{\sigma(r').r'.d\varphi'.dr'}{4\pi\epsilon_0.\sqrt{r^2 + r'^2 - 2rr'.\cos(\varphi - \varphi') + z^2}}$$

∴ The potential of the entire ring at $\vec{r}$ is;

$$V\ (ring) = \int_0^{2\pi} \frac{\sigma(r').r'.d\varphi'.dr'}{4\pi\epsilon_0.\sqrt{r^2 + r'^2 - 2rr'.\cos(\varphi - \varphi') + z^2}} \tag{13a}$$

We have produced these equations on the basis of the fact that, for a given r', $\sigma(r')$ is constant through out the ring. Therefore, the potential of charged disk at $\vec{r}$ is mathematically as follows;

$$\therefore V(disk) = \int_0^a \int_0^{2\pi} \frac{\sigma(r').r'.d\varphi'.dr'}{4\pi\epsilon_0.\sqrt{r^2 + r'^2 - 2rr'.\cos(\varphi - \varphi') + z^2}}$$

Since we are analysing just on the plane of the disk, we can assume, $\lim(z) = 0$.

$$\therefore V(disk) = \int_0^a \int_0^{2\pi} \frac{\sigma(r').r'.d\varphi'.dr'}{4\pi\epsilon_0.\sqrt{r^2 + r'^2 - 2rr'.\cos(\varphi - \varphi')}}$$

$$= \int_0^a \frac{\sigma(r').r'.dr'}{4\pi\epsilon_0} \left[\int_0^{2\pi} \frac{d\varphi'}{\sqrt{r^2 + r'^2 - 2rr'.\cos(\varphi - \varphi')}} \right] \tag{13b}$$

Assessing the ambiguity of implanting 2 different azimuthal angles

Let us first evaluate the integral, $I = \int_0^{2\pi} \frac{d\varphi'}{\sqrt{r^2 + r'^2 - 2rr'.\cos(\varphi - \varphi')}}$ (14)

Differentiating both sides with respect to φ,

$$\frac{\partial I}{\partial \varphi} = \frac{\partial}{\partial \varphi} \left[\int_0^{2\pi} \frac{d\varphi'}{\sqrt{r^2 + r'^2 - 2rr'.\cos(\varphi - \varphi')}} \right]$$

$$= \int_0^{2\pi} \frac{\partial}{\partial \varphi} \left[r^2 + r'^2 - 2rr' + 2rr'.\{1 - \cos(\varphi - \varphi')\} \right]^{-\frac{1}{2}}.d\varphi'$$

$$= \int_0^{2\pi} \frac{\partial}{\partial\varphi}\left[(r-r')^2 + 4rr'.\left(\sin\frac{\varphi-\varphi'}{2}\right)^2\right]^{-\frac{1}{2}}.d\varphi'$$

$$= \int_0^{2\pi} -\frac{1}{2}\left[(r-r')^2 + 4rr'.\left(\sin\frac{\varphi-\varphi'}{2}\right)^2\right]^{-\frac{3}{2}}.\frac{1}{2}\left(4rr'.2\sin\frac{\varphi-\varphi'}{2}.\cos\frac{\varphi-\varphi'}{2}\right).d\varphi'$$

$$= -rr'\int_0^{2\pi}\left[(r-r')^2 + 4rr'.\left(\sin\frac{\varphi-\varphi'}{2}\right)^2\right]^{-\frac{3}{2}}.\sin 2\left(\frac{\varphi-\varphi'}{2}\right).d\varphi'$$

$$= -\frac{1}{4}\int_{\frac{\varphi}{2}}^{\frac{\varphi}{2}-\pi}\frac{(-2)B\sin 2\psi}{[A+B(\sin\psi)^2]^{\frac{3}{2}}}d\psi$$

where, A=$(r-r')^2$, B=$4rr'$ and, $\psi = \frac{\varphi-\varphi'}{2}$ such that, $d\varphi' = -2d\psi$.

φ'	0	2π
ψ	$\frac{\varphi}{2}$	$\frac{\varphi}{2}-\pi$

$$= -\frac{1}{2}\int_{A+B(\sin\frac{\varphi}{2})^2}^{A+B(\sin\frac{\varphi}{2})^2}\frac{dt}{t^{\frac{3}{2}}} \quad = 0$$

where $A + B(\sin\psi)^2 = t$ $implies, dt = B\sin 2\psi$.

Since the function $A + B(\sin\psi)^2$ gives similar output for both the inputs $\frac{\varphi}{2}$ and $\frac{\varphi}{2}-\pi$ the integral becomes zero. Thus, it implies that;

$$\frac{\partial I}{\partial\varphi} = 0 \Rightarrow I = f(r.r',\varphi') \qquad i.e.\, independent\ of\ \varphi. \tag{14a}$$

'I being independent of φ'' stands for the irrelevance of considering an initial axis referred to which the azimuthal angles are to be defined rather, the equation can be solved in a relative azimuthal frame i.e., we can simply consider a single variable φ' between the 2 position vectors. Refer to **Figure 4(b)**.

Expressing the potential as a function of r

$$\therefore V(disk) = \int_0^a \frac{\sigma(r').r'.dr'}{4\pi\epsilon_0}\left[\int_0^{2\pi}\frac{d\varphi'}{\sqrt{r^2+r'^2-2rr'.\cos\varphi'}}\right]$$

Following this, let us first evaluate the angular integral part of it, and further, we assume that it is to be denoted by, S.

$$\therefore s = \int_0^{2\pi}\frac{d\varphi'}{\sqrt{r^2+r'^2-2rr'.\cos\varphi'}}$$

Now, $(r-r')^2 \geq 0$

$\Rightarrow r^2 + r'^2 - 2rr' \geq 0$

$\Rightarrow r^2 + r'^2 \geq 2rr'$

$\Rightarrow r^2 + r'^2 \geq 2rr'.\cos\varphi'$

$\Rightarrow r^2 + r'^2 - 2rr'.\cos\varphi' \geq 0$

with this, we are just confirming whether S is subjected to assess any complex integration.

$$\therefore s = \int_0^{2\pi} \frac{d\varphi'}{\sqrt{(r-r')^2+4rr'.\left(\sin\frac{\varphi'}{2}\right)^2}}$$

$$= 2\int_0^{\pi} \frac{d\psi}{\sqrt{A+B\,(\sin\psi)^2}}$$

where, A = $(r-r')^2$, B = $4rr'$ and, $\psi = \frac{\varphi'}{2}$

$$\therefore d\varphi' = 2d\psi$$

φ'	0	2π
ψ	0	π

For a function f, $\int_0^{2a} f(t).dt = 2.\int_0^{a} f(t).dt$ $\quad when\, f(2a-t) = f(t)$
$= 0$ $\quad when\, f(2a-t) = -f(t)$

Now, $[\sin(\pi-\psi)]^2 = (\sin\psi)^2$ implies $\int_0^{\pi} \frac{d\psi}{\sqrt{A+B\,(\sin\psi)^2}} = 2\int_0^{\frac{\pi}{2}} \frac{d\psi}{\sqrt{A+B\,(\sin\psi)^2}}$.

$$\therefore S = 4\int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{A}} \frac{d\psi}{\sqrt{1+\frac{B}{A}(\sin\psi)^2}}$$

$$= \frac{4}{\sqrt{A}}\int_0^{\frac{\pi}{2}} \frac{d\psi}{\sqrt{1+\xi\,(\sin\psi)^2}}$$

where let us assume $\xi = \frac{B}{A}$

$$= \frac{4}{\sqrt{A}}\int_0^{\frac{\pi}{2}} \frac{d\psi}{\sqrt{1-\epsilon}}$$

where the dummy variable be $\epsilon = -\xi\,(\sin\psi)^2$.

Now further suppose we reckon that, $f(\epsilon) = (1-\epsilon)^{-\frac{1}{2}}$

$$f'(\epsilon) = \frac{1}{2}(1-\epsilon)^{-\frac{3}{2}}$$

$$f''(\epsilon) = \frac{1}{2}.\frac{3}{2}(1-\epsilon)^{-\frac{5}{2}}$$

$$f'''(\epsilon) = \frac{1}{2}.\frac{3}{2}.\frac{5}{2}(1-\epsilon)^{-\frac{7}{2}}$$

… …

$$f^k(\epsilon) = \frac{1.3.5\ldots(2k-1)}{2^k}(1-\epsilon)^{-\frac{2k+1}{2}}$$

$$= \frac{(2k-1)!!}{2^k}(1-\epsilon)^{-\frac{2k+1}{2}}$$

$$\therefore f^k(0) = \frac{(2k-1)!!}{2^k}$$

where the double factorial notation is the extended form of writing $\binom{-\frac{1}{2}}{n}$ without using ‘Pochhammer symbol’. It indicates the product of even and odd positive integers as follows [29];

$1.3.5\ldots\,(2n-1)=(2n-1)!!=\frac{(2n)!}{2^n.n!}$

$and, 2.4.6\ldots 2n=(2n)!!=n!\,2^n$

Using Maclaurin power series expansion, the function $f(\epsilon)$ can be expanded as follows;

$$f(\epsilon)=\sum_{k\geq 0} f^k(0).\frac{\in^k}{k!}$$

$$\Rightarrow f(\epsilon)=\sum_{k\geq 0}\frac{(2k-1)!!}{2^k}.\frac{\in^k}{k!}$$

It readily implies that, $f_0(\epsilon)=\frac{(-1)!!}{0!2^0}\in^0=1$ where $f_0(\epsilon)$ denotes $f(\epsilon)$ at k=0 and thus, the equality sign holds good for all the situations.

Putting $\in=-\xi(\sin\psi)^2$ we have, $f(\epsilon)=\sum_{k\geq 0}\frac{(2k-1)!!}{2^k}.\frac{(-\xi(\sin\psi)^2)^k}{k!}$ (14b)

which eventually reduces the integral to;

$$S=\frac{4}{\sqrt{A}}\int_0^{\frac{\pi}{2}}\sum_{k\geq 0}\frac{(2k-1)!!}{2^k.k!}.(-1)^k\xi^k(\sin\psi)^{2k}d\psi$$

$$=\frac{4}{\sqrt{A}}\sum_{k\geq 0}\frac{(2k-1)!!}{2^k.k!}\xi^k.(-1)^k\int_0^{\frac{\pi}{2}}(\sin\psi)^{2k}d\psi$$

$$=\frac{4}{\sqrt{A}}\sum_{k\geq 0}\frac{(2k-1)!!}{2^k.k!}\xi^k.(-1)^k\int_0^{\frac{\pi}{2}}(\sin\psi)^{2k}.(\cos\psi)^0 d\psi$$

$$=\frac{4}{\sqrt{A}}\sum_{k\geq 0}\frac{(2k-1)!!}{2^k.k!}\xi^k.(-1)^k\int_0^{\frac{\pi}{2}}(\sin\psi)^{2\left(\frac{2k+1}{2}\right)-1}.(\cos\psi)^{2\left(\frac{1}{2}\right)-1}d\psi$$

Now, note that the expression of S is only valid if the summation stated by Eq. (14b) is uniformly convergent i.e., it can result in a finite quantity. By real analysis of bivariate series, it is impossible to find out the nature of convergence since both the correlation of the variable turns out to be 0 (being independent variable). If we complete the angular integration to eliminate ψ, still then there is no defined nature of ξ. So, by the test of absolute convergence, the series with holds to be absolutely divergent. Running Simulink programs, it has been found that, for certain discrete ranges the equations result in a finite quantity otherwise, it reaches infinity since computer can assess a series for huge numbers (1 lakh, in the program used here) but exactly it is unable to run an infinity. This issue can be solved by equating the integral in a different form. Let us first consider that Eq. (14b) is valid and proceeding further, it results in Eq. (14c). We will later prove its convergence in the section.

β function can be defined as;

$$\beta(x,y)=\frac{\Gamma(x).\Gamma(y)}{\Gamma(x+y)}$$

$$=2.\int_0^{\frac{\pi}{2}}(\sin t)^{2x-1}.(\cos t)^{2y-1}dt$$

Considering $x=\frac{2k+1}{2}$ and $y=\frac{1}{2}$, the integration is further befitted to;

$$s=\frac{4}{\sqrt{A}}\sum_{k\geq 0}\frac{(2k-1)!!}{2^k.k!}\xi^k.(-1)^k\beta\left(\frac{2k+1}{2},\frac{1}{2}\right)$$
$$=\frac{4}{\sqrt{A}}\sum_{k\geq 0}\frac{(2k-1)!!}{2^k.k!}\xi^k.(-1)^k\frac{\Gamma\left(\frac{2k+1}{2}\right).\Gamma\left(\frac{1}{2}\right)}{\Gamma(k+1)}$$

One of the most prominent value of gamma function is, $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. Let us explore further values of the gamma function as required.

Since by definition, $\Gamma(x) = \int_0^\infty e^{-t}.t^{x-1}.dt$ it readily implies that, $\Gamma(1) = 1$. Thus, for an input of $(n+1)\ \forall n \in N$ the gamma function can be resolved as;

$$\Gamma(n+1) = \int_0^\infty e^{-t}.t^n.dt$$

$$= [-e^{-t}t^n]_0^\infty + \int_0^\infty e^{-t}nt^{n-1}\,dt$$

$$= 0 + n\int_0^\infty e^{-t}t^{n-1}\,dt$$

$$= n\Gamma(n)$$

Quite similarly, $\Gamma(n) = \int_0^\infty e^{-t}t^{n-1}\,dt = (n-1)\Gamma(n-1)$

Hence it can be deduced that, $\Gamma(n+1) = n(n-1)(n-2)\ldots 2\cdot 1\cdot \Gamma(1)$ [by reduction]

$$= n! \quad [\because \Gamma(1) = 1]$$

As a corollary we can infer $\Gamma(x) = (x-1)\Gamma(x-1)$

$$\therefore \Gamma\left(\frac{2k+1}{2}\right) = \left(\frac{2k-1}{2}\right)\Gamma\left(\frac{2k-1}{2}\right)$$

$$= \left(\frac{2k-1}{2}\right)\left(\frac{2k-3}{2}\right)\Gamma\left(\frac{2k-3}{2}\right)$$

$$= \left(\frac{2k-1}{2}\right)\left(\frac{2k-3}{2}\right)\left(\frac{2k-5}{2}\right)\cdots\frac{3}{2}\cdot\frac{1}{2}\cdot\Gamma\left(\frac{1}{2}\right)$$

$$= \frac{(2k-1)!!}{2^k}\sqrt{\pi} \qquad \left[\because \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}\right]$$

Therefore putting $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}, \Gamma(n+1) = n!, and, \Gamma\left(\frac{2k+1}{2}\right) = \frac{(2k-1)!!}{2^k}\sqrt{\pi}$ in equation …we have;

$$S = \frac{4}{\sqrt{A}}\sum_{k\geq 0}\frac{(2k-1)!!}{(k!)2^k}(-1)^k\xi^k\frac{(2k-1)!!}{2^k}\sqrt{\pi}\cdot\frac{\sqrt{\pi}}{2\cdot k!}$$

$$= \frac{2\pi}{\sqrt{A}}\sum_{k\geq 0}\left[\frac{(2k-1)!!}{(k!)2^k}\right]^2(-i)^k\xi^k$$

Let us have a look on the denominator;

$$[k!\,2^k]_{k=0} = 1 = 0!!$$

$$[2^k\cdot k!]_{k=1} = 2\cdot 1 = 2!! \qquad [0!! = 1]$$

$$[2^k\cdot k!]_{k=2} = 2\cdot 2\cdot 1\cdot 2 = 4!!$$

$$[2^k\cdot k!]_{k=3} = 2\cdot 2\cdot 2\cdot 3\cdot 2\cdot 1 = 6!!$$

… …

Thus, by principle of mathematical induction, $k!\,2^k = (2k)!!$

Therefore, the equation … becomes;

$$S = \frac{2\pi}{\sqrt{A}} \sum_{k\geq 0} \left(\frac{(2k-1)!!}{(2k)!!}\right)^2 (-1)^k \xi^k$$

$$= \frac{2\pi}{\sqrt{A}} \left[1 - \left(\frac{1}{2}\right)^2 \xi^2 + \left(\frac{1\cdot 3}{2\cdot 4}\right)^2 \xi^4 - \left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^2 \xi^6 + \cdots \infty\right] \tag{14c}$$

So, the potential of the charged disk at $\overrightarrow{r}$ is

$$V(r) = \int_{r'=0}^{a} \frac{\sigma(r')\cdot r' \cdot dr'}{4\pi\epsilon_0} . \frac{2\pi}{\sqrt{A}} \left[1 - \left(\frac{1}{2}\right)^2 \xi^2 + \left(\frac{1\cdot 3}{2\cdot 4}\right)^2 \xi^4 - \left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^2 \xi^6 + \cdots \infty\right] \tag{15}$$

Formulating an integral equation introducing the charge density as the dependent variable

After redistribution of charges, the potential being constant for any observation point on the disk, its partial derivative with respect to r is 0. So, the following differential equation must be satisfied by $\sigma(r')$.

$$\therefore \frac{\partial}{\partial r}\left[\int_0^a \frac{\sigma(r')\cdot r'\cdot dr'}{4\pi\epsilon_0} . \frac{2\pi}{|r-r'|}\left[1 - \left(\frac{1}{2}\right)^2 \left(\frac{4r.r'}{(r-r')^2}\right)^2 + \left(\frac{1\cdot 3}{2\cdot 4}\right)^2 \left(\frac{4r.r'}{(r-r')^2}\right)^4 - \left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^2 \left(\frac{4r.r'}{(r-r')^2}\right)^6 + \cdots \infty\right]\right] = 0$$

$\Rightarrow \frac{\partial}{\partial r}\left[\int_0^a G(r,r').dr'\right] = 0$ where $G(r,r') = \frac{\sigma(r')\cdot r'}{4\pi\epsilon_0} . \frac{2\pi}{|r-r'|}\left[1 - \left(\frac{1}{2}\right)^2 \left(\frac{4r.r'}{(r-r')^2}\right)^2 + \left(\frac{1\cdot 3}{2\cdot 4}\right)^2 \left(\frac{4r.r'}{(r-r')^2}\right)^4 - \left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^2 \left(\frac{4r.r'}{(r-r')^2}\right)^6 + \cdots \infty\right]$

$$\Rightarrow \int_0^a \frac{\partial G(r,r')}{\partial r} \cdot dr' + G(a,r) \cdot \frac{da}{dr} - G(0,r) \cdot \frac{d(0)}{dr} = 0$$

$$\Rightarrow \int_0^a \frac{\sigma(r')\cdot r'}{4\pi\epsilon_0}\left[\left(-\frac{1}{|r-r'|(r-r')}\right)\left[1 - \left(\frac{1}{2}\right)^2 \xi^2 + \left(\frac{1\cdot 3}{2\cdot 4}\right)^2 \xi^4 - \left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^2 \xi^6 + \cdots \infty\right] + \frac{2\pi}{|r-r'|}\left(-\left(\frac{1}{2}\right)^2 2\xi \cdot \xi' + \left(\frac{1\cdot 3}{2\cdot 4}\right)^2 \cdot 4\xi^3\xi' - \left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^2 6\xi^5\xi' + \cdots\right)\right] dr' = 0$$

$$\Rightarrow \int_0^a \frac{\sigma(r')\cdot r'}{2\epsilon_0}\left[\frac{1}{|r-r'|(r'-r)}\left\{1 - \left(\frac{1}{2}\right)^2 \left(\frac{4r.r'}{(r-r')^2}\right)^2 + \left(\frac{1\cdot 3}{2\cdot 4}\right)^2 \left(\frac{4r.r'}{(r-r')^2}\right)^4 - \left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^2 \left(\frac{4r.r'}{(r-r')^2}\right)^6 + \cdots\right\} - \frac{1}{2|r-r'|}\left\{\frac{4r.r'}{(r-r')^2} - \left(\frac{1\cdot 3}{2}\right)^2 \frac{1}{2}\left(\frac{4r.r'}{(r-r')^2}\right)^3 + \left(\frac{1\cdot 3\cdot 5}{2\cdot 4}\right)^2 \frac{1}{3}\left(\frac{4r.r'}{(r-r')^2}\right)^5 - \cdots\right\}\left(\frac{4r}{(r-r')^2} - \frac{8r.r'}{(r-r')^3}\right)\right] \cdot dr' = 0 \tag{16}$$

The integral equation is of implicit form which cannot be resolved into explicit solutions of the required charge density. So, using the concepts of lineal elements and isoclines, the closest direction field can be plot with the help of Computer algebra systems and Simulink. In a similar fashion with a bit of physical sense, the nature of the curve has been produced as shown in **Figure 5**. By the concept of image charges, a charge q kept at a small distance d from the plane, disk produces an image of density $\frac{q}{4\pi}\left|\frac{-2d\hat{k}}{[x^2+y^2+d^2]^{3/2}}\right|$ which is a planar distribution without any dependence on the z-axis. To maintain the neutrality of the disk another positive charge q is to be distributed over the surfaces. But the distribution in neutral condition can not anymore be predicted by the aforesaid equation since, another equal amount of charge is to be distributed according to Eq. (16), thus, the image charge density in practice can be shown in **Figure 6**. In regenerative technologies, the kinetic energy stored by the rotor disk has to maintain such metallic striping as a good endure of shock. Following **Figure 6**, it is quite easily understandable that the rotor disk must be built in such a design that the plate should be thicker centrally and by the rim to absorb a greater electrostatic pressure. So, for modern-day technologies, understanding, and mapping the charge distribution is quite an impressive aspect and most importantly the Eq. (16) theoretically facilitates making out all these.

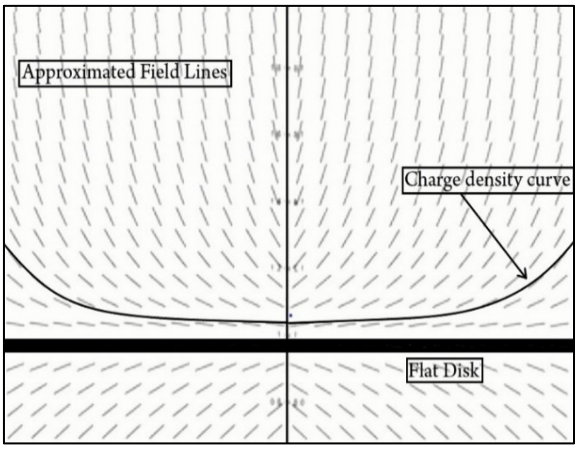


Figure 5 Closest approximation of direction field plot.

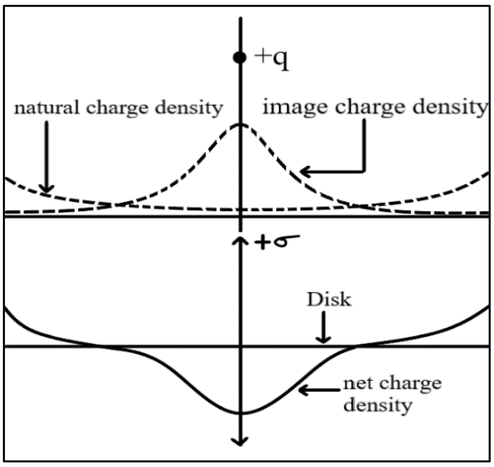


Figure 6 Graphing the charge density distribution of image charges.

If one attentively notices the curve, it can observe that the slope of the curve in the central region is too small to be considered and, there is a steep rise at the edge. So, for an infinite flat disk, the charge density seems to be constant. Based on the observer, a disk may behave as infinitesimal. In case the charge-density is often assumed to be q/A provided, A is the surface area of the disk. Although in reality, the density, σ, should be sufficiently lesser than the predicted density, q/A. Briefly, the abstract concept of uniform charge-density is a specific case of the Eq. (16).

The article [24] predicts a solution, $\sigma(r) = \frac{Q}{2\pi R} . \frac{1}{\sqrt{R^2 - r^2}}$ $\forall\ 0 \leq r \leq R$ which is not lugged by any derivation or logic. Treating it with further operations, it is seen that the solution does not satisfy the integral equation. Hence, the article as well demands the solution of [24] as an incorrect prediction.

Proving the convergence of a so-called absolutely divergent sequence

The following section deals with an establishment of a novel mathematical transformation as a natural consequence. Let us reformulate the expression of 'I' as in Eq. (14).

$$I = \int_0^{2\pi} \frac{d\varphi'}{\sqrt{r^2 + r'^2 - 2rr'.\cos(\varphi')}}$$

Inculcating another dummy variable $\theta = \frac{\varphi' - \pi}{2}$ in the previous equation we can rewrite the expression as;

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{2d\theta}{\sqrt{r^2 + r'^2 + 2rr'.\cos 2\theta}}$$

$$\Rightarrow I = 4\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{r^2 + r'^2 + 2rr'.\cos 2\theta}}$$

since, the function is an even function

$$\Rightarrow I = 4\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{r^2 + r'^2 + 2rr' - 2rr'.(1 - \cos 2\theta)}}$$

$$\Rightarrow I = 4\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{(r + r')^2 - 2rr'.2\sin^2\theta}}$$

$$\Rightarrow I = \frac{4}{|r - r'|}\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1 - \frac{4rr'}{(r - r')^2}\sin^2\theta}} \quad (17)$$

Reducing the equation to the form, $\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1 - m\sin^2\theta}}$ we can infer that the outcome of the integration is a finite quantity since, it is a complete elliptic integral of the first kind with, $m \leq 1$, where $m = \frac{4rr'}{(r - r')^2}$ following the report of [31]. Now the Eq. (17) is equivalent to the expression (14b). By comparing the 2 mathematical equivalences, we can infer a wonderful transformation equation. Up to the knowledge of well-known mathematics of series, the series in Eq. (14b) is not convergent but, if the variable ξ can be related to 2 parameters of circular frames, then the series must be convergent because the elliptic integral of the first kind results in a finite quantity provided the above-mentioned conditions are satisfied. That's why the article also claims a neoteric transformation formula that the following bivariate series;

$\left[1 - \left(\frac{1}{2}\right)^2 \left(\frac{4mn}{(m-n)^2}\right)^2 + \left(\frac{1\cdot3}{2\cdot4}\right)^2 \left(\frac{4mn}{(m-n)^2}\right)^4 - \left(\frac{1\cdot3\cdot5}{2\cdot4\cdot6}\right)^2 \left(\frac{4mn}{(m-n)^2}\right)^6 + \cdots \infty\right]$ is always convergent for, $m, n \in \mathbb{N}$.

Inference

Most commonly, whenever a charge is given to a conductor, the distribution is assumed to be uniform since there is neither an additional condition nor any constraint applied to it. The nature of charge density is no exception. In our work, to find the real distribution of charges we first pointed to deriving analytical mathematics for the potential of a flat disk bearing uniform charge density. Using aforesaid methods, the expression of potential came to be reflected in Eq. (15) which we believe to be of great engrossment for both the mathematical and physical science pursuer. More opportunely, the results helped to concretely claim the instability of uniform charge density.

The article successfully deduced 2 different sets of equations for expressing the potential at an arbitrary point in 3D space. Eventually, it resulted in a novel transformation Eq. (13). This set of equality, in general, relates the first and second kinds of elliptic integral. As a consequence, it proves that uniformity in charge density distribution is not naturally feasible. Proceeding further, we uniquely managed to derive an Eq. (16) satisfiable by the charge density. Eq. (16) further potentially denies the 'unobtained solution' predicted by the article [24]. It involves the solution as the dependent variable in an implicit form so, it is intractable to find an explicit expression. As a possible way out, the article eccentrically plots the closest nature of the distribution in **Figure 5**. It compels us to infer that the charge density on a finite disk gradually increases from the center towards the edge with positive upward concavity. Although when the disk is large enough in comparison, it almost seems to be uniform. Thus, for a tiny observer, a finite disk may seem to be infinitely large. In case only a snippet is observed bearing almost constant density. So, a 'literally infinite disk' upholds a uniform charge density concerning an observer. At the last, the article establishes an indigenous and thought pinching mathematical conclusions as follows, the corroboration of manifesting the convergence of the series in Eq. (14c) beyond conventional methods of real analysis.

The potential distribution for the flat disk model is also very important to other disciplines. In the case of nanostructured materials [31-32], the understanding of confinement of electrons undertakes the concept of electrostatic charge distribution which is accompanied by equivalent structured potential distribution with similar patterns and symmetries. The dynamics of various biological systems consisting of charged particles like DNA, colloidal sol of proteins, amino acids, and such heavier particles are often described by approximating them with disk models [33-36]. The distribution of charge density here plays a crucial role to well interpret the interactions and process of transferring biological information. As mentioned earlier, the idea of charge density over a finite disk is also indispensable in structuring the rotor wheel of a regenerative shock absorber. Thus, the physical concept is simultaneously an element of wide interest for numerous engineering branches including biological sciences as well as the novel equivalences and transformation formulas are also of some importance for mathematics lovers.

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References

[1] CL Ladera and G Donoso. A rigorous and simpler method of image charges. *Eur. J. Phys.* 2016; **37**, 045208.

[2] P Räcke, D Spemann, JW Gerlach, B Rauschenbach and J Meijer, Detection of small bunches of ions using image charges. *Sci. Rep.* 2018; **8**, 9781.

[3] R Morf and BI Halperin. Monte carlo evaluation of trial wave functions for the fractional quantized hall effect: Disk geometry. *Phys. Rev. B Condens. Matter.* 1986; **33**, 2221-46.

[4] FDM Haldane and EH Rezayi. Finite-size studies of the incompressible state of the fractionally quantized hall effect and its excitations. *Phys. Rev. Lett.* 1985; **54**, 237.

[5] O Ciftja. Exact results for systems of electrons in the fractional quantum Hall regime II. *Phys. B. Condens. Matter.* 2009; **404**, 2244-6.

[6] O Ciftja. Monte carlo study of bose laughlin wave function for filling factors 1/2, 1/4 and 1/6. *Europhys. Lett.* 2006; **74**, 486.

[7] J Xia. An estimate of the ground state energy of the fractional quantum hall effect. *J. Math. Phys.* 1999; **40**, 150.

[8] AY Grosberg, TT Nguyen and BI Shklovskii. *Colloquium*: The physics of charge inversion in chemical and biological systems. *Rev. Mod. Phys.* 2002; **74**, 329.

[9] M Binazadeh, M Xu, A Zolfaghari and H Dehghanpour. Effect of electrostatic interactions on water uptake of gas shales: The interplay of solution ionic strength and electrostatic double layer. *Energ. Fuel.* 2016; **30**, 992-1001.

[10] WM Gelbart, RF Bruinsma, PA Pincus and VA Parsegian. DNA-Inspired electrostatics. *Phys. Today* 2000; **53**, 38.

[11] A Fernández-Nieves, A Fernández-Barbero, FJDL Nieves and B Vincent. Ionic correlations in highly charge-asymmetric colloidal liquids. *J. Chem. Phys.* 2005; **123**, 054905.

[12] A El-Assaad, Z Dawy and G Nemer. Electrostatic study of Alanine mutational effects on transcription: Application to GATA-3: DNA interaction complex. *In*: Proceedings of the 37th Annual International Conference of the IEEE Engineering in Medicine and Biology Society, Milan, Italy. 2015, p. 4005-8.

[13] KL Kounovsky-Shafer, JP Hernandez-Ortiz, K Potamousis, G Tsvid, M Place, P Ravindran, K Jo, S Zhou, T Odijk, JJD Pablo and DC Schwartz. Electrostatic confinement and manipulation of DNA molecules for genome analysis. *Proc. Natl. Acad. Sci. Unit. States. Am.* 2017; **114**, 13400-5.

[14] M Quesada-Pérez, E González-Tovar, A Martín-Molina, M Lozada-Cassou and R Hidalgo-Álvarez. Overcharging in colloids: Beyond the poisson-boltzmann approach. *ChemPhysChem* 2003; **4**, 234-48.
[15] FJ Solis and MODL Cruz. Flexible polymers also counterattract. *Phys. Today* 2001; **54**, 71.
[16] AP dos Santos, A Diehl and Y Levin. Electrostatic correlations in colloidal suspensions: Density profiles and effective charges beyond the poisson-boltzmann theory. *J. Chem. Phys*. 2009; **130**, 124110.
[17] JR Poganik and Y Aye. Electrophile signaling and emerging immuno- and neuro-modulatory electrophilic pharmaceuticals. *Front. Aging Neurosci*. 2020; **12**, 1.
[18] DS Faber and AE Pereda. Two forms of electrical transmission between neurons. *Front. Mol. Neurosci.* 2018; **11**, 427.
[19] AE Pereda. Electrical synapses and their functional interactions with chemical synapses. *Nat. Rev. Neurosci.* 2014; **15**, 250-63.
[20] SG Hormuzdi, MA Filippov, G Mitropoulou, H Monyer and R Bruzzone. Electrical synapses: A dynamic signaling system that shapes the activity of neuronal networks. *Biochim. Biophys. Acta Biomembr.* 2004; **1662**, 113-37.
[21] P Zheng, J Gao, R Wang, J Dong and J Diao. Review on the research of regenerative shock absorber. *In*: Proceedings of the 24th International Conference on Automation and Computing, Newcastle Upon Tyne. 2018.
[22] R Archisman and B Tushar. A mystery car without fuel and battery. *Res. Rev. Int. J. Multidisciplinary* 2019; **4**, 463-8.
[23] O Ciftja. Results for charged disks with different forms of surface charge density. *Results Phys.* 2020; **16**, 102962.
[24] KS Gehlot. Differential equation of K-Bessel's function and its properties. *Nonlinear Anal. Differ. Equat.* 2014; **2**, 61-7.
[25] EM Purcell. *Electricity and magnetism.* McGraw Hill Education, New York, 1963, p. 53-5.
[26] GB Arfken, HJ Weber and F Harris. *Mathematical methods for physicists.* Academic Press, Massachusetts, 2001, p. 355-6.
[27] JD Jackson. *Classical Electrodynamics*. John Wiley & Sons, New York, 1998, p. 140.
[28] PF Byrd and MD Friedman. *Handbook of elliptic integrals for engineers and scientists*. *In*: GDM Wissenschaften (Ed.). 2nd ed. Springer, Heidelberg, 1971, p. 360.
[29] G Arfken, H Weber and FE Harris. *Mathematical methods for physicists*. 7th ed. Academic Press, Massachusetts, 2012, p. 1220.
[30] BC Carlson. Numerical computation of real or complex elliptic integrals. *Numer. Algorithm.* 1995; **10**, 13-26.
[31] EF Pecora, A Irrera, S Boninelli, L Romano, C Spinella and F Priolo. Nanoscale amorphization, bending and recrystallization in silicon nanowires. *Appl. Phys. Mater. Sci. Process.* 2011; **102**, 13-9.
[32] B Tian, X Zheng, TJ Kempa, Y Fang, N Yu, G Yu, J Huang and CM Lieber. Coaxial silicon nanowires as solar cells and nanoelectronic power sources. *Nature* 2007; **449**, 885-9.
[33] Y Levin. Electrostatic correlations: From plasma to biology. *Rep. Progr. Phys.* 2002; **65**, 1577.
[34] AP dos Santos, A Diehl and Y Levin. Colloidal charge renormalization in suspensions containing multivalent electrolyte. *J. Chem. Phys*. 2010; **132**, 104105.
[35] R Agra, E Trizac and L Bocquet. The interplay between screening properties and colloid anisotropy: Towards a reliable pair potential for disc-like charged particles. *Eur. Phys. J. E* 2004; **15**, 345-57.
[36] R Archisman and B Tushar. Short analysis of quantum entanglement. *J. Emerg. Technol. Innovat. Res.* 2019; **6**, 1832-6.