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Magnetic Dipole Moments, Occupation Numbers and Magnetic Form Factors of Some Neutron Rich Nuclei in *sdpf*-shell

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Abstract

In the *sdpf* shell model, static and dynamic characteristics of certain neutron-rich nuclei are measured by using the *SDPF* interactions: Magnetic dipole moments (μ), occupation numbers (*occ#*) and magnetic electron scattering form factors for the ground state, they are presented for some Vanadium ($_{23}$V) and Manganese ($_{25}$Mn) isotopes using radial wave functions for the single-particle matrix elements of harmonic-oscillator potential (HO). These isotopes are ^{48}V (4 1$^+$), ^{49}V (7/2$^-$ 3/2), ^{50}V (6$^+$ 2), ^{53}Mn (7/2$^-$ 3/2), ^{54}Mn (3$^+$ 2) and ^{56}Mn (3$^+$ 3). The calculations introduced with model space (MS) and core-polarization *CP* effects are included through effective *g* factors. In model space and with core-polarization effects, the magnetic transition probability *B(M1)* is also calculated. The calculations of one-body density matrix *OBDM* together with the interaction *SDPFK* are carried out in the *sdpf*-model space using the *OXBASH* code. The theoretical results of the μ moments agree with recent experimental data.

Keyword: Magnetic dipole moments, Occupation numbers magnetic, Form factors

Introduction

The study of the structure of exotic nuclei started at the beginning of this century, particularly nuclei far from the stable line, owing to the tremendous development in measuring instruments and nuclear accelerators. One important way of knowing the nuclear structure is to measure and calculate the magnetic dipole moment and the nuclear magnetic electron scattering form factors; it does cause some small shifts in the hyperfine structure [1,2].

The identified nuclear characteristics may help to understand the changes that govern the nuclear force under extreme conditions. The electric and magnetic properties such as magnetic dipole and electric quadrupole moments give a clear description of the nucleus as a system of particles moving independently in a central potential or as a system of particles moving collectively [3-6].

Many models have been proposed recently to describe the properties of exotic nuclei and to compare these results with recently measured values. By calculating the quadrupole, magnetic moments, and occupation numbers using FPBM intervention in the fp-shell model space, the static and dynamic characteristics of some exotic scandium isotopes are investigated. The values have been improved by using CP effects that include effective charge and effective nucleon g factors [7]. The magnetic dipole moments and the electric quadrupole moments in the shell model are investigated for isotopes of $_{24}$Cr neutron-rich nuclei using fp-model space and different interactions. The results are adequate to obtain a good agreement between the predicted and experimental values when including CP effects through effective charge and effective g factors [8].

Radhi *et al.* [9] studied the microscopic structure for the neutron-rich *sd-pf* cross-shell nuclei by calculating the magnetic and quadrupole moments by using the shell model. They found that the magnetic static and dynamic properties can be described by free *g* factors for the model space nucleons without introducing CP effect, on the contrary to the electric static and dynamic properties, which cannot be described properly by the model space nucleons without taking into account CP effects.

Several research studies some properties of exotic nuclei include energy levels, nuclear charge distribution, electric and magnetic moments, and nuclear form factors using different interaction and different models' space [10-14].

In this work, the shell model SM and the harmonic oscillator HO for single-particle wave functions are adopted to compute magnetic dipole moments μ, occupation numbers (*occ#*), magnetic transition probability $B(M1)$ and elastic magnetic form factors (EMFF) for *sdpf* -shell model nuclei by using *SDPFK* [15] interaction for the ground state of $^{48,49,50}V$ and $^{53,54,56}Mn$ isotopes. All calculations are provided for the model space (MS) only and include the discarded space through the CP effect

Theory

The decrease matrix item for the magnetic transition operator $\hat{O}_k(m1)$ was defined as the sum of the products of the items one-body intensity matrix *OBDM* times about single-particle matrix item, as-is determine by Eq. (1) [16];

$$\langle J_i \rangle = \sum_{a,b,T} \quad (-1)^{T_i - T_z} (T_i \; T \; T_i \; -T_z \; 0 \; T_z) OBDM(a,b,J=1,T) \langle b \rangle \tag{1}$$

where the bracket $\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$ denotes the 3j-symbol, $T_Z = \frac{Z-N}{2}$ and the single-particle states ($n\,l\,j$) are denoted by k.

For $J=1$, the magnetic dipole moment was identified in the terminology of ($m1$) operator as [16];

$$\mu(J=1) = (J_i \; 1 \; J_i \; -J_i \; 0 \; J_i) \sqrt{\frac{4\pi}{3}} \langle J \rangle \mu_N \tag{2}$$

where $\langle J \rangle$ the operator of the magnetic transition, and $\mu_N = \frac{e\hbar}{2m_p c} = 0.1051\, e.fm.$ which is the nuclear magnetrons, m_p are the proton mass.

For the proton, the orbit and spin for the free nucleon g factors were: $g_l\,(p) = 1$, $g_s\,(p) = 5.585$ and for the neutron, the $g_l\,(n) = 0$, the $g_s\,(n) = -3.826$ for [16].

The MESFFs include angular momentum λ and momentum transfer q, which is b/w the primary (i) and ultimate (f) for the nuclear shell model, as determine by Eq. (3) [16,17];

$$|F(M\lambda, q)|^2 = \frac{4\pi}{Z^2(2J+1)} \left| \langle J_i \rangle \, F_{c.m}(q) F_{f.s}(q) \right|^2 \tag{3}$$

$F_{f.s} = [1 + (\frac{q}{4.33})^2]^{-2}$ is the finite size nucleon form factor [18], and $F_{c.m} = e^{q^2 b^2/4A}$ is the correction for the center of mass form factors [19] . A defined as a mass number, and b is the length parameter for the harmonic oscillator (HO).

At the long-wavelength limit, the momentum transfer is $q = k = E_x / \hbar c$, where E_x is the excitation energy [20]. This limit is defined as a photon point. The reduced magnetic transition probabilities $B(MJ)$ can be expressed in terms of electron scattering form factors evaluated at the photon point as follows [20,21];

$$B(MJ) = \frac{Z^2}{4\pi} \frac{J}{J+1} \left[\frac{(2J+1)!!}{k^J} \right]^2 \left| F_J^M(k) \right|^2 \tag{4}$$

In each subshell j, the average occupation numbers (*occ#*) are given by [22];

$$occ\#(j, t_z) = OBDM\,(a, b, t_z, J=0) \sqrt{\frac{2j+1}{2J_i+1}} \tag{5}$$

Results and discussion

Calculations are presented using *SDPFK* effective interactions for the *sdpf*-shell model for the ground state of $_{23}V$ and $_{25}Mn$ isotopes to generate the *OBDM* elements. The *OBDM* is calculations in *OXBASH* code that is in the isospin formalism. *OXBASH* is set of codes for carrying out shell-model calculations with dimensions up to about 50,000 in the J-T scheme and about 2,000,000 in the M-scheme. *OXBASH* comes with a library of model spaces and interactions [23].

The $_{23}V$ and $_{25}Mn$ isotopes are composed of the core ^{16}O plus the valence active nucleons distributed over $1d_{5/2}$, $1d_{3/2}$, $2s_{1/2}$, $1f_{7/2}$, $1f_{5/2}$, $2p_{3/2}$, and $2p_{1/2}$ orbits. The results appear only with model space (MS) and core polarization (CP) effects. The occupation numbers (*occ#*) for the ground states of ${}_{23}V$ and ${}_{25}Mn$ isotopes are tabulated in **Table 1**. The calculated magnetic dipole moments (μ) being compared with the experimental values and the magnetic transition probability *B*(*M1*) are tabulated in **Table 2**.

Table 1 The occupation numbers (occ#) for the ground states of $1d_{5/2}$, $1d_{3/2}$, $2s_{1/2}$, $1f_{7/2}$, $1f_{5/2}$, $2p_{3/2}$ and $2p_{1/2}$ orbits outside the ^{16}O core for considered $^{48,49,50}V$ and $^{53,54,56}Mn$ isotopes.

Nucleus	Average no. of particles in each *j*-level						
	$1d_{5/2}$	$1d_{3/2}$	$2s_{1/2}$	$1f_{7/2}$	$1f_{5/2}$	$2p_{3/2}$	$2p_{1/2}$
^{48}V	12	8	4	6.106	0.2321	1.4512	0.2107
^{49}V	12	8	4	7	1	1	0
^{50}V	12	8	4	6	2	1	1
^{53}Mn	12	8	4	10	1	1	1
^{54}Mn	12	8	4	10	2	1	1
^{56}Mn	12	8	4	10	6	0	0

Table 2 The calculated transition probability and magnetic dipole moments (μ) of $^{48,49,50,51}V$ and $^{53,54,56}Mn$ isotopes are compared with experimental results.

Nucleus	$J^{\pi}T$	g(free) $\mu_{Calc.}$(*n m*)	g(eff.) g_l^n g_l^p / $\mu_{Calc.}$(*n m*)	$\mu_{exp.}$ (n m)[24]	*B(M1)* (n m)^2 MS	(MS+CP)
^{48}V	4^+ 1	1.7602	1.075 0.075 / 2.010	2.012	0.9246	1.203
^{49}V	$7/2^-$ 3/2	5.2345	0.072 0.072 / 4.4710	4.47(5)	8.410	2.185
^{50}V	6^+ 2	3.3331	1.005 0.0 / 3.3460	3.3456(14)	3.090	3.118
^{53}Mn	$7/2^-$ 3/2	2.8630	2.92 0.3 / 5.0318	5.035(1)	2.516	7.772
^{54}Mn	3^+ 2	1.5442	1.75 0.2 / 3.3088	3.306(1)	0.759	3.485
^{56}Mn	3^+ 3	0.6381	2.79 0.6 / 3.2294	3.2266(2)	0.1296	3.320

^{48}V nucleus ($J^{\pi}T = 4^+$ 1)

In the *sdpf*-shell model space, the ^{48}V nucleus presented consists of 32 nucleon perimeters for core ^{16}O. The occupation numbers of valence nucleons ^{16}O are shown in **Table 1**. The average number of nucleons in each j-level outside the core are $1d_{5/2} = 12$, $1d_{3/2} = 8$, $2s_{1/2} = 4$, $1f_7/2 = 6.106$, $1f_{5/2} = 0.2321$, $2p_{3/2} = 1.4512$ and $2p_{1/2} = 0.2107$. The major ratio of the occupying numbers of this neutron goes to $1d_{5/2}$ and a clear exotic behavior for the valence nucleons of ^{48}V. The calculated value μ of *g* (free) for this transition is $\mu = 1.7602\ nm$, which underestimates the empirical value [24]. Including the CP effects using orbital *g* factors for both proton and neutron as $g_l^p = 1.075$, $g_l^n = 0.075$ the result of μ becomes 2.010 nm which agrees with that of the experimental values shown in **Table 2**. **Figure 1** shows the calculation of the elastic magnetic electron scattering factors. **Figure 1(a)** shows the individual multipoles contributions M1, M3, M5, M7 and the solid black curves representing the total magnetic form factors in model space (MS) only, since the diffraction minimum for MS is located at momentum transfer q = 1.4 fm^{-1}. By adding the Core polarization effect (CP) using the effective g factor to the model space (MS), the total magnetic form factors with MS+CP (red solid curves) are close to total magnetic form factors in MS (black solid curves) at all momentum transfers except at $0.1 \leq q \leq 0.9$ fm^{-1} because the form factors depend on the size parameter b, as shown in **Figure 1(b)**. The magnetic transition probability $B(M1)$ for this transition in MS and with MS+CP is 0.9246 and 1.203 $(nm)^{2,}$ respectively. The value of $B(M1)$ increases when CP is included.

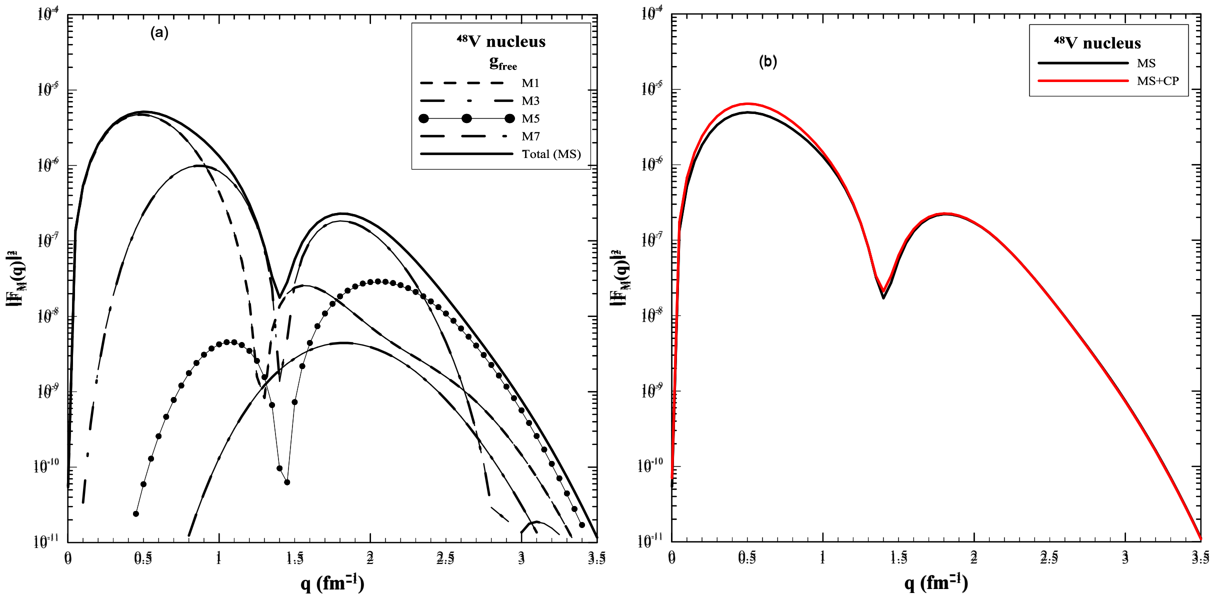

Figure 1 The magnetic form factors for ground state of ^{48}V calculated in sdpf model space. (a) The individual multipoles contributions of M1, M3, M5 and M7 are shown. (b) Comparison between the total form factors of ^{48}V with free gl (black solid curve), and with the effective gl (red solid curve).

^{49}V **nucleus ($J^{\pi}T$ = 7/2$^-$ 3/2)**

The occupation numbers of valence nucleons for ^{49}V nucleus are shown in Tab. (1). It is obvious that $1d_{5/2}$ orbit is operated by the major ratio of occupation numbers. The calculated μ moments for this transition with g(free) is $\mu = 5.2345\, nm$, which overestimates the experimental value $\mu_{exp.} = 4.47(5)\ nm$ [24]. Reducing the orbital g factors for both proton as well as neutron as g_l^p = 0.072, g_l^n = 0.072 modified the calculated value to be $\mu = 4.4710nm$ and in agreement with the experimental value. The effective g-factor has minor dependence on the form factors and plays significant role on the values of magnetic moment compared with that of g-free. The *EMESFF* calculations of the ground state shell model for ^{49}V nucleus with $J^{\pi}T$ = 7/2$^-$ 3/2, by using *sdpf*-shell model space with *SDPFK* interaction in MS only (black solid curves) with the contribution of M1, M3, M5, and M7 multipoles of value q between 0 and 3.5 $fm^{-1,}$ are shown in **Figure 2(a)**. The diffraction minimum is located at momentum transfer q ~ 0.9 fm^{-1}. Including CP (red solid curves) by using the effective g factor to the model space MS, the small suppress is observed for M1 for q < 0.9 fm^{-1} and the minimum is turned to small q, as is shown in **Figure 2(b)**. The $B(M1)$ for this transition in MS and with MS+CP are 8.410 and 2.185 $(nm)^{2,}$ respectively. The value of $B(M1)$ decreases when CP is adding.

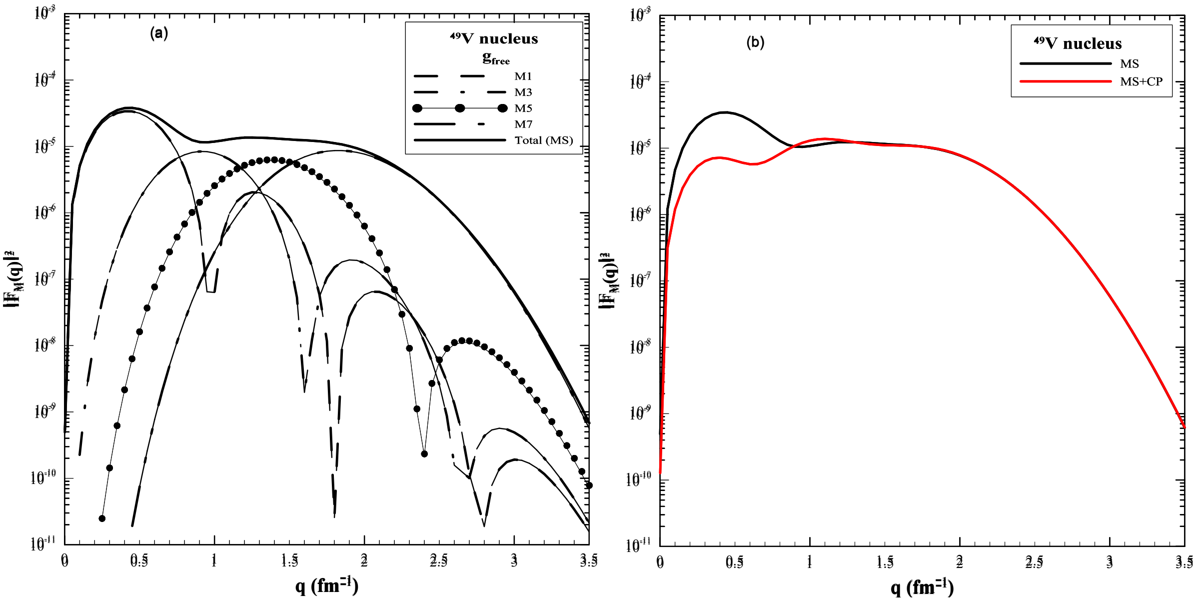

Figure 2 The *MFFs* of ground state of ^{49}V calculated in *sdpf* model space. (a) The individual multipoles contributions of M1, M3, M5 and M7 are shown. (b) compared between the total *FFs* of ^{49}V with the free g_l (black solid curve), and with the effective g_l (red solid curve).

^{50}V **nucleus ($J^{\pi}T$ = 6^+ 2)**

The ^{50}V nucleus is a *sdfp*-shell model space consisting of a core ^{16}O with a core ^{16}O perimeter of 34 nucleons. The calculations are done with the *SDPFK* reaction which gives occupation numbers outside the core $1d_{5/2}$ = 12, $1d_{3/2}$ = 8, $2s_{1/2}$ = 4, $1f_{7/2}$ = 6, $1f_{5/2}$ = 2, 2 $p_{3/2}$ = 1and $2p_{1/2}$ = 1. It is obvious that $1d_{5/2}$ orbit is occupied by the major ratio of occupation numbers. The calculated μ moment for this isotope with g(free) is μ = 3.3331 nm, this value of magnetic moment becomes close to the measured value, and after adding the error rate. By inclusion of the orbital g-factors for the proton only as g_l^p =1.005 the μ moment, it becomes 3.3456(14) nm [24] which is near to the experimental value. The EMFFs for this transition are also calculated. The contribution for each of the four multipoles of q between 0 and 3.5 fm^{-1}appears in **Figure 3(a)**. The total MFFs in the MS zone are almost entirely made up of the M1 portion, with minimum diffraction located at momentum transfer q ~ 1.1 fm^{-1}. Calculation of the form factors is important to know the internal structure of the exotic nucleus. The effect of the CP added to the MS wave functions (red solid curves) by using the efficient orbital g factors, makes the total MFFs (MS+CP) close to total magnetic form factors in MS (black solid curves) at all momentum transfers, as shown in **Figure 3(b)**. The $B(M1)$ values of this transition in MS and with MS+CP are 3.090 and 3.118 (nm)^2, respectively. The value of $B(M1)$ increases when CP is added.

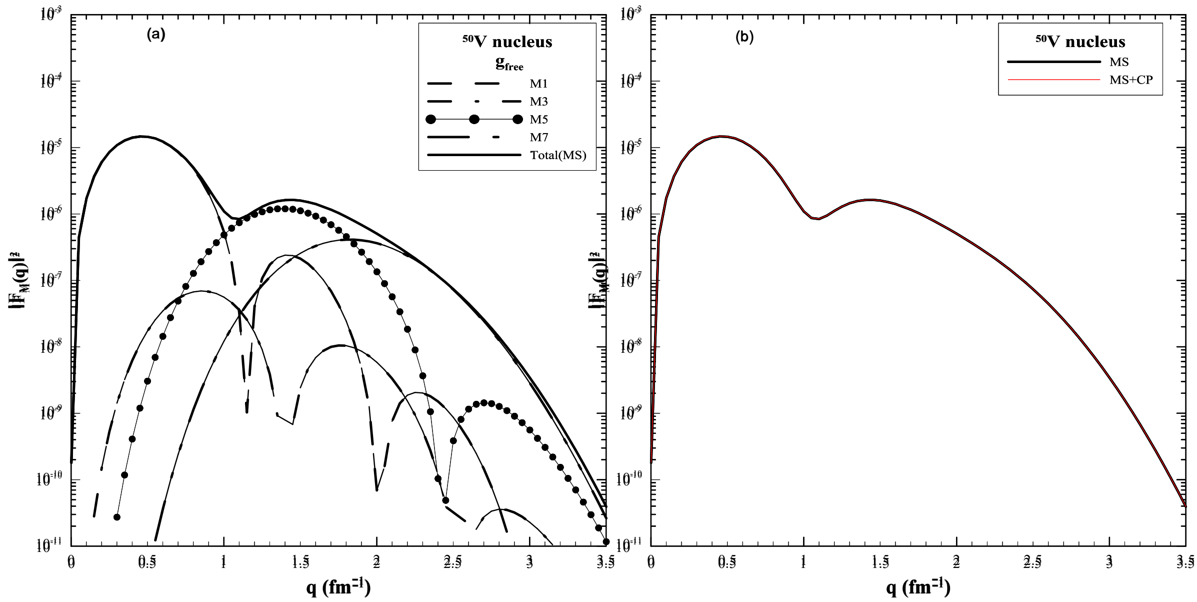

Figure 3 The magnetic form factors for ground state of ^{50}V calculated in *sdpf* model space. (a) The individual multipoles contributions of M1, M3, M5 and M7 are shown. (b) Comparison between the total *FFs* of ^{50}V with the free g_l (black solid curve), and with the effective g_l (red solid curve).

^{53}Mn nucleus ($J^{\pi}T = 7/2^{-}\ 3/2$)

^{53}Mn (exotic neutron-richly system) nucleus appearing in the *sdfp*- model space, is made up of the core ^{16}O with the core ^{16}O perimeter of 37 nucleons. The calculations are done with *SDPFK* reaction which gives occupation numbers outside the core $1d_{5/2} = 12$, $1d_{3/2} = 8$, $2s_{1/2} = 4$, $1f_{7/2} = 10$, $1f_{5/2} = 1$, $2p_{3/2} = 1$ and $2p_{1/2} = 1$. The $1d_{5/2}$ orbit outside the core is occupied by the major ratio of occupation numbers. Calculating a free g factor magnetic moment values of 2.863 nm, it is obvious that they do not agree with the experimental value of 5.035(1) nm [24]. This contradiction was resolved using the efficient nucleon *g* factors applied with the orbital *g* factors for both proton and neutron like $g_l^p = 2.92$, $g_l^n = 0.3$; μ moment becomes 5.0318 nm which agrees with the experimental value. Calculating the *EMESFFs* for this nucleus with $J^{\pi}T = 7/2^{-}\ 3/2$ is by using *sdpf*-shell model space with *SDPFK* interaction. The M1, M3, M5, and M7 components contribute to the total form factors in MS (black solid curves). The diffraction minimum for the M1 component is located at momentum transfer $q = 1.4$ fm^{-1} as shown in **Figure 4(a)**. Adding CP gives essentially the same enhancement to the form factors across the range of momentum transfer to the MS (red solid curves) by using the effective *g* factors gives a reasonable behavior compared with the MS only (black solid curves) and the minimum is shifted to small *q* as shown in **Figure 4(b)**. The *B*(*M1*) value is 2.516 (nm)^2 for this transition in MS and with MS+CP value is 7.772 (nm)^2. The value of *B(M1)* increases when CP is added.

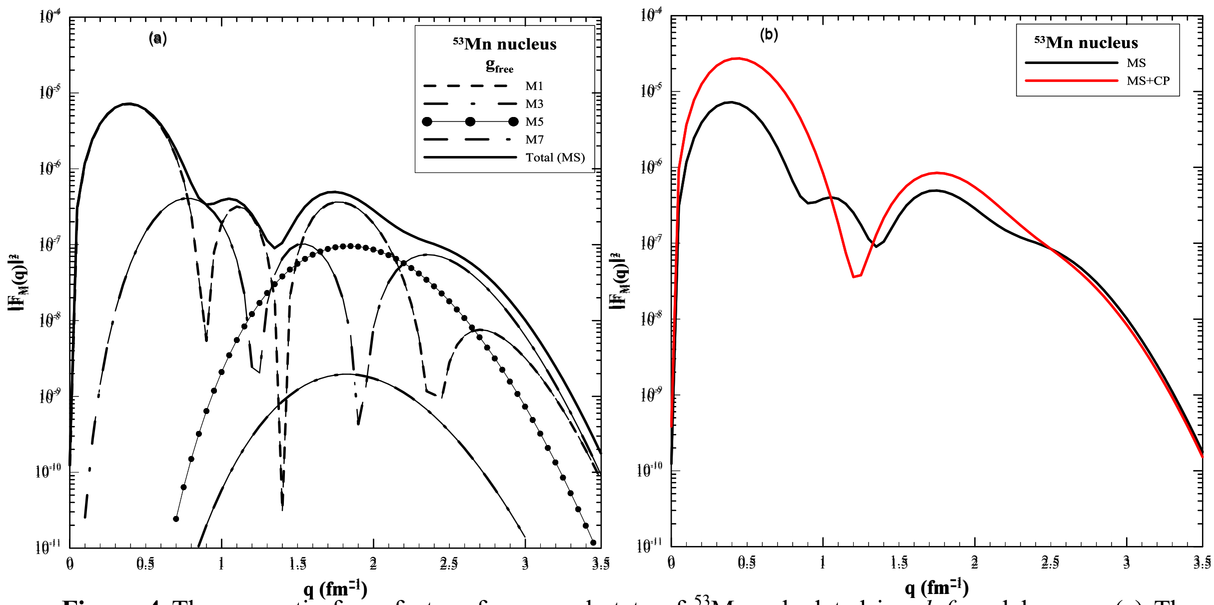

Figure 4 The magnetic form factors for ground state of ^{53}Mn calculated in *sdpf* model space. (a) The individual multipoles contributions of M1, M3, M5 and M7 are shown. (b) Comparison between the total form factors of ^{53}Mn with free g_1 (black solid curve), and with the effective g_l (red solid curve).

^{54}Mn nucleus ($J^{\pi}T = 3^{+}\ 2$)

The ground state of the neutron-rich nucleus ^{54}Mn is sdfp-model space, is made up of the core ^{16}O with the core ^{16}O perimeter of 38 nucleons. The occupation numbers of valence nucleons of this nucleus are shown in **Table 1**. The largest values of occupation numbers are found in the $1d_{5/2}$ orbit. The calculated μ moment with g (free) is 1.5442 nm which is less than the experimental value of 3.306(1) nm [24]. By adding the CP, this value is increased by using the effective nucleon g factors with orbital g factors for both proton and neutron, as = 1.75, 0.3 the μ moment becomes 3.3088 nm, which is the calculated value well reproduced. Effective g-factors have minor effect on the form factors along all range of q, and a major effect on the magnitude of magnetic dipole moments. **Figure 5(a)**, shows the elastic magnetic form factors for this nucleus with $J^{\pi}T = 3^{+}\ 2$ by using *sdpf*-shell model space with *SDPFK* interaction. The *M1*, *M3,* and *M5* components contribute to the total FFs of MS (black solid curves). The diffraction minimum for the M1 component is located at momentum transfer $q = 1.45$ fm^{-1}. **Figure 5(b)** shows the comparison between the total form factors in MS (black solid curves), and the total form factors in MS including CP (red solid curves). The form factors are closed very well by large momentum transfer at $q \geq 1.2$ fm^{-1}. The B(M1) value 0.759 (nm)^2 is increased by a factor of 3 when CP is applied and becomes 3.485 (nm)^2.

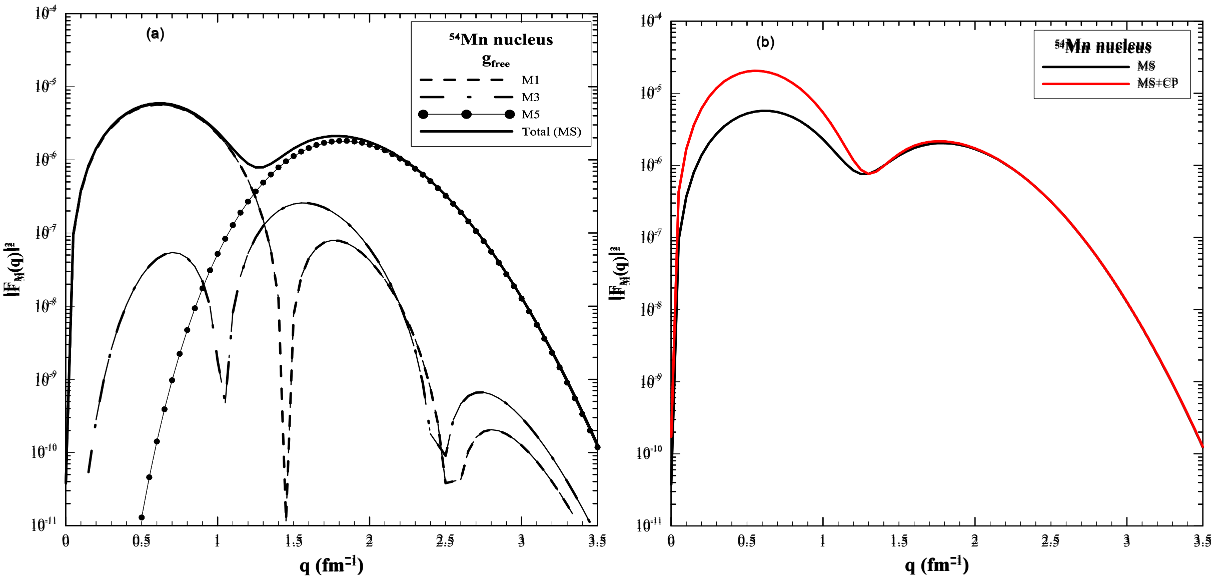

Figure 5 The magnetic form factors for ground state of ^{54}Mn calculated in *sdpf* model space. (a) The individual multipoles contributions of M1, M3 and M5 are shown. (b) Comparison between the total form factors of ^{54}Mn with free g_1 (black solid curve), and with the effective g_l (red solid curve).

^{56}Mn nucleus ($J^{\pi}T = 3^{+}\ 3$)

In the sdpf-shell model space, the ^{56}Mn nucleus consists of the core ^{16}O with 40 nucleons perimeter of the core^{16}O, and the active *sdpf*- MS Valence nucleon occupation numbers are shown in the Tab. (1). The average numbers of nucleons in each j-level outside the core are $1d_{5/2} = 12$, $1d_{3/2} = 8$, $2s_{1/2} = 4$, $1f_{7/2}$=10, $1f_{5/2}$=6, $2p_{3/2}$=0 and $2p_{1/2}$=0. It is obvious that $1d_{5/2}$ orbit outside the core is occupied by the major ratio of occupation numbers. The calculated μ of this transition for g(free) is $\mu = 0.6381\ nm$, which is an underestimate with the empirical value $\mu_{exp.} = 3.2266(2)\ \ nm$ [24]. Including the CP effects used in the orbital g factors for both proton and neutron alike: $g_l^p = 2.79$, $g_l^n = 0.6$ the result of μ becomes 3.2266(2) nm which agrees with that of experimental values shown in **Table 2**.

Figure 6(a), shows the elastic magnetic form factors for this nucleus with $J^{\pi}T = 3^{+}\ 3$ by using *sdpf*-shell model space with *SDPFK* interaction. The M1, M3, and M5 components contribute to the total *FFs* in MS (black solid curves). The diffraction minimum for the M1 component is located at momentum transfer q = 1.45 fm^{-1}. **Figure 6(b)**, shows the comparison between the total form factors in MS (black solid curves) and the total form factors in MS including CP (red solid curves). The form factors increase the discrepancy by small momentum transfer at $q \geq 1.3$ fm^{-1}. Minor effect for the core polarization contribution on the magnitude of the total form factors for all interactions and for all q-values in comparison with the model space results. The $B(M1)$ value 0.1296 (nm)^2 increases by about a factor of 3 by applying CP and becomes 3.320 (nm)^2.

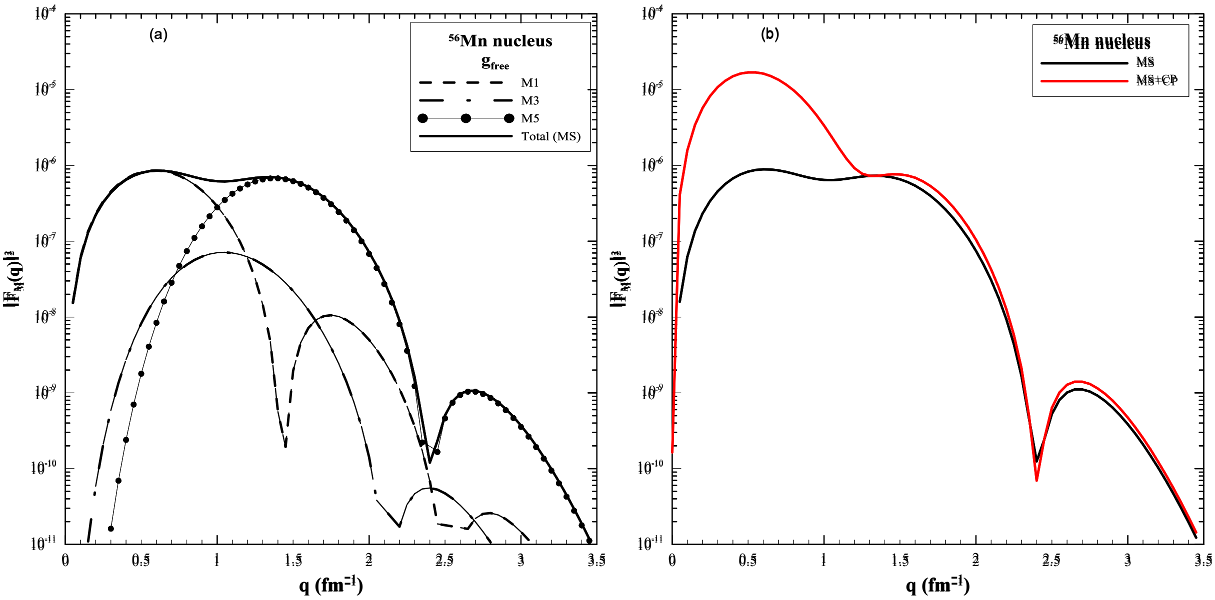

Figure 6 The magnetic form factors for the ground state of ^{56}Mn calculated in *sdpf* model space.

Conclusions

Experimental values for all V and Mn isotopes except ^{49}V isotope are underestimated by the theoretical results of magnetic dipole moment calculations based on *SDPFK* interaction for free nucleon g factors. The magnetic dipole moments are improved and give a good agreement between the predicted and experimental values when the CP effects are included using effective nucleon *g* factors. For all momentum transfers except at *q* 1.0 fm^{-1}, the elastic magnetic form factors calculated in model space alone and with CP effects are close to each other. Through analytical calculations of the occupation numbers of neutron-rich V and Mn isotopes in the *sdpf*-model space a significant contribution to the orbit of $1d_{5/2}$ and its obvious exotic conduct of the parity nucleons which is outside the core ^{16}O nucleus.

The inclusion of core polarization has sensitive effects on the magnitude of the probability of magnetic transition B(M1). In addition to the model space, the B(M1) values increase for all V and Mn isotopes except for the ^{49}V isotope after the CP effects Declaration of interested: Non

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