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A New Extension of Generalized Extreme Value Distribution: Extreme Value Analysis and Return Level Estimation of the Rainfall Data

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Abstract

This paper presents an extension of the generalized extreme value (GEV) distribution, based on the T-X family of distributions: Gompertz-generated family of distributions that make the existing distribution more flexible called the Gompertz-general extreme value (Go-GEV) distribution. Some properties of the proposed distribution are introduced, and a new distribution is applied to actual data, namely rainfall in Lopburi Province, by comparing the proposed model with the traditional GEV distribution and estimating the return levels of the rainfall in Lopburi Province. Results showed that the Go-GEV was an alternative flexible distribution for extreme values that fitted with actual data and described the maximum rainfall better than the traditional GEV distribution. The probability density functions of the Go-GEV distribution had various shapes including left-skewed, right-skewed and close to symmetric. Estimation of the return levels of rainfall values in Lopburi Province by the Go-GEV distribution indicated that Buachum Station should be monitored because it had higher precipitation and return levels than Lopburi Station at 2, 5, 10 and 15 years.

Keywords: Extreme value theory, T-X family of distributions, GEV distribution, Gompertz-GEV distribution, rainfall, return level

Introduction

Extreme values are a fundamental statistical topic that almost all analysts are aware of but few know how to deal with appropriately. The analysis of data with extreme values is complex but ignoring an observation just because it is unacceptable is not best practice, especially when dealing with the probability of extreme events. One of the widely used tools in such situations is "Extreme Value Theory". This branch of statistics deals with extreme deviations from the median of probability distributions. The theory and practice of extreme value distributions is applied probability and statistics as a tool for predicting floods, extreme value distributions are applied analysis extreme value data [1,2]. The theory uses the block maxima approach to derive extreme value Fréchet, Weibull and Gumbel distributions. The generalized extreme value (GEV) distribution is then developed within the extreme value theory to combine these 3 distributions [3].

The extreme value model is widely applied to extreme statistical model events in environmental, structural and financial research on topics such as air pollution [4], climate change [5,6], temperature analysis [7], wave modeling [8], earthquake severity analysis [9,10], rainfall [11-13], drought [14,15] and finance [16,17]. However, developing a fitted distribution to the actual data would allow greater model accuracy, with more flexible distributions displaying wide-ranging data types. Many researchers use new generalizations to increase distribution cover and flexibility. Applying new generalizations for continuous distributions has now become more attractive because this methodology can improve the goodness of fit and determine tail properties. Guloksuz and Celik presented an extension of GEV distribution by generating the T-X family of distributions, called the uniform-GEV distribution, which applied the developed distribution to the magnitude of earthquakes in Turkey from 1970 (January) to 2018 (October), while Esfeh *et al.* [18] proposed a new class of extreme value distribution, called compound generalized extreme value (CGEV) distribution, to investigate the effects of monthly and seasonal variations in extreme travel road network delays. The CGEV distribution was obtained by linking 2 multiplicative error models and deriving a new compound distribution that simultaneously represented monthly and seasonal variation in extreme travel delays. In other words, the developed CGEV distribution was the product of 2 GEV distributions used to describe extreme travel delay variability across monthly and seasonal levels. El-Bassiouny *et al.*

[19] introduced a new distribution called new generalized extreme value, derived from an exponentiated Weibull distribution, while Provost *et al*. [20] proposed q-analogs of the generalized extreme value and Gumbel distributions, with the additional parameter q allowing for increased modeling flexibility. Aryal and Tsokos [21] first introduced the transmuted GEV (TGEV) distribution. This is a more flexible model than the GEV distribution to model extreme or rare events because the right tail of the TGEV is heavier than the GEV [13].

Statistical distributions for continuous data play an essential role in many applications such as engineering, medicine, biological science, management and public health. Statistical distributions also provide helpful information that results in conclusions and decisions. Many researchers use new generalizations for more cover and flexible distributions. Applying new generalizations for continuous distributions has now become more attractive because this methodology can improve the goodness of fit and determine tail properties. Generalizing distributions mainly depends on adding more flexibility to known distributions by implanting a primary distribution into a more capable structure [22,23]. These features have been established by the results of many generalizations such as the beta family [24] and transformed transformer [25]. In 2017, Alizadeh et al. proposed a Gompertz-Generated (Go-G) family of distributions using the Gompertz distribution as the parent distribution. The primary motivations for using the Go-G family in practice are (i) making the kurtosis more flexible than the parent distribution, (ii) producing skewness for symmetrical distributions, (iii) constructing heavy-tailed distributions that are no longer tailed for modeling real data, (iv) generating distributions with symmetric, left-skewed, right-skewed and reversed-J-shaped, (v) defining special models with all types of hazard rate functions, and (vi) consistently providing better fits than other generated models under the same baseline distribution [26].

Coverage and flexibility are both areas of great importance for developing quality improvements in statistical models that are very important and necessary tools in data analysis. The GEV distribution often proves somewhat inadequate in practice, requiring more flexible generalizations for modeling purposes. A new extension of the GEV distribution is proposed in this study and some properties of the proposed distribution are introduced. The method for estimating the parameters of the proposed distribution is presented for extreme value analysis and return level estimation of rainfall data in Lopburi Province, Thailand. Finally, some conclusions are drawn.

Materials and methods

This section introduces the probability function of the GEV distribution, T-X family of distributions and Go-G family of distributions to derive a new extension of the GEV distribution for extreme value analysis.

The GEV distribution

The GEV distribution was developed within the extreme value theory to combine the Gumbel, Fréchet and Weibull distributions [3]. The GEV distribution had cumulative density function (c.d.f.) and probability density function (p.d.f.) as:

$$G_{\mathbf{GEV}}(x) = \exp\left(-\tau(\mathrm{x})\right), \text{ and } \; g_{\mathbf{GEV}}(x) = \frac{1}{\sigma}\tau(x)^{\xi+1}\exp\left(-\tau(\mathrm{x})\right), \tag{1}$$

where,

$$\tau(x) = \begin{cases} \left(1+\left(\xi/\sigma\right)\left(x-\mu\right)\right)^{-1/\xi} & ;\xi \neq 0, \\ \exp\left[-\left(1/\sigma\right)\left(x-\mu\right)\right] & ;\xi = 0, \end{cases} \tag{2}$$

and the parameters μ, σ and ξ represent location, scale and shape parameters, i.e., $-\infty < \mu < \infty$, $\sigma > 0$ and $-\infty < \xi < \infty$. The mean and variance of the GEV distribution are given by:

$$\mathbf{E}(X) = \mu - \frac{\sigma}{\xi} + \frac{\sigma}{\xi}\mathbf{\Gamma}\left(1-\xi\right) \text{ and } \; \mathbf{Var}(X) = \frac{\sigma^2}{\xi^2}\left[\mathbf{\Gamma}\left(1-2\xi\right) - \left(\mathbf{\Gamma}\left(1-\xi\right)\right)^2\right],$$

where $\Gamma(\cdot)$ is a complete gamma function. The class of the GEV distribution has 3 sub-models as the Gumbel $(\xi = 0)$, Fr**é**chet $(\xi > 0)$, and Weibull $(\xi < 0)$ distributions.

The T-X family of distributions

In 2013, Alzaatreh et al. proposed the T-X family of distributions with the c.d.f. as:

$$F_{\textbf{T-X}}(x) = \int_{a}^{W[G(x)]} r(t)dt \tag{3}$$

where $r(t)$ is the p.d.f. of T for $T \in [a,b]$, $-\infty < a < b < \infty$, and $G(x)$ as the c.d.f. of a parent distribution. $W\left[G(x)\right]$ is the function of $G(x)$, which satisfies the following conditions: (i) $W\left[G(x)\right] \in [a,b]$, (ii) $W\left[G(x)\right]$ is differential and monotonically non-decreasing, and (iii) $W\left[G(x)\right] \to a$ as $x \to -\infty$ and $W\left[G(x)\right] \to b$ as $x \to \infty$ [25].

The Go-G family of distributions

In 2017, Alizadeh *et al.* [26] proposed the Go-G family of distributions, using the Gompertz distribution as the parent distribution and $W\left[G(x)\right] = -\log\left[1 - G(x)\right]$ with the c.d.f. as follows:

$$F_{\textbf{Go-G}}(x) = \int_{0}^{-\log[1-G(x)]} \lambda \exp(\gamma t) \exp\left\{-\frac{\lambda}{\gamma}\left[\exp(\gamma t) - 1\right]\right\} dt\ . \tag{4}$$

Results and discussion

This section proposes a new extension of the GEV distribution for extreme value analysis and return level estimation of extreme data. Some properties of the proposed distribution are presented, together with a simulation of the method for estimating the parameters of the proposed distribution. Moreover, the extreme value analysis and return level estimation of rainfall data in Lopburi Province in Thailand are also provided.

A new extension of the GEV distribution

Let T and X be random variables distributed as the Gompertz (Go) distribution with the p.d.f. $r(t)$ and the GEV distribution with the c.d.f. $G(x)$, respectively, i.e., T ~Go (λ, γ) and X ~GEV (μ, σ, ξ). Using the definition of the T-X family of distributions [25] and letting $W\left[G(x)\right] = -\log\left[1 - G(x)\right]$ [25] gives a new distribution called the Gompertz-generalized extreme value (Go-GEV) distribution with parameters μ, σ, ξ, λ and γ denoted by X ~Go-GEV $(\mu, \sigma, \xi, \lambda, \gamma)$. Its c.d.f. is:

$$\begin{aligned} F_{\textbf{Go-GEV}}(x) &= \int_{0}^{-\log[1-\exp(-\tau(x))]} \lambda \exp(\gamma t) \exp\left\{-\frac{\lambda}{\gamma}\left[\exp(\gamma t) - 1\right]\right\} dt \\ &= 1 - \exp\left\{\frac{\lambda}{\gamma}\left[1 - \left(1 - \exp\left(-\tau(x)\right)\right)^{-\gamma}\right]\right\}, \end{aligned} \tag{5}$$

where $\tau(x)$ as in Eq. (2), supports the distribution that becomes: $x \in (-\infty, \mu - \sigma/\xi]$ for $\xi < 0$; $x \in [\mu - \sigma/\xi, \infty)$ for $\xi > 0$; $x \in (-\infty, \infty)$ for $\xi \to 0$. The corresponding p.d.f. is:

$$f_{\textbf{Go-GEV}}(x) = \frac{\lambda}{\sigma} \tau(x)^{\xi+1} \left[1 - \exp\left(-\tau(\mathrm{x})\right)\right]^{-\gamma-1} \exp\left\{-\tau(\mathrm{x}) + \frac{\lambda}{\gamma}\left[1 - \left(1 - \exp\left(-\tau(\mathrm{x})\right)\right)^{-\gamma}\right]\right\}, \tag{6}$$

for a location parameter $\mu,$ scale parameters σ and $\lambda,$ and shape parameters ξ and γ, i.e., $-\infty < \mu < \infty,$ $\sigma > 0,$ $\lambda > 0,$ $\gamma > 0,$ and $-\infty < \xi < \infty.$

Some properties of the Go-GEV distribution

The class of the Go-GEV distribution can be represented by the parameter ξ which controls the tail behavior and has 3 sub-models as follows: (i) If $\xi = 0,$ the Go-GEV distribution has thin tail behavior (see **Figure 1**), which reduces to the Go-Gumbel distribution. The shapes of the distribution include the left-skewed (**Figures 1(b), 1(d)** and **1(f)**), right-skewed (**Figures 1(a), 1(c)** and **1(e)**), and symmetrical shape (**Figures 1(b), 1(d)** and **1(f)** for $\lambda = 1.5, 3$). The parameter λ changes the overall shape of the graph. The distribution shape is not change when the parameters μ and γ is changed. That is, the parameter μ simply shifts the graph left or right on the horizontal axis. The large the parameter γ, the less spread out the distribution. (ii) If $\xi > 0,$ the Go-GEV distribution has fat tail behavior (see **Figure 2**), which reduces to the Go-Fr**é**chet distribution. All shapes of the distribution have a right-skewed except the case of $\lambda = 2$ and $\gamma = 1.5$ are closed to symmetrical shape. And (iii) if $\xi < 0,$ the Go-GEV distribution has short tail behavior (see **Figure 3**), which reduces to the Go-Weibull distribution. All shapes of the distribution have a left-skewed.

From **Figures 1 - 3** indicates that λ is a shape parameter because of the parameter λ changes the overall shape of the graph. The large the parameter γ, the less spread out the distribution but its shape is not changed, which indicates γ is a scale parameter. The parameter μ is a location parameter because of the distribution shifts the graph left or right on the horizontal axis without changing shape.

Quantile function

Let X ~Go-GEV $(\mu,\sigma,\xi,\lambda,\gamma)$ and $F_{Go\text{-}GEV}(x) = U$, where U is a random variable distributed as a uniform distribution on (0,1), then the quantile function of X is:

$$Q_{\textbf{Go-GEV}}(u) = F^{-1}_{Go\text{-}GEV}(x) = G^{-1}_{GEV}\left\{1-\left[1-\frac{\gamma}{\lambda}\log(1-u)\right]^{-1/\gamma}\right\}, 0 < u < 1. \tag{7}$$

From the quantile function, the GEV distribution is as follows:

$$Q_{\textbf{GEV}}(p) = \begin{cases} \mu + \dfrac{\sigma}{\xi}\left[\left(-\log(p)\right)^{-\xi} - 1\right], \xi \neq 0 \\ \mu - \sigma\log\left(-\log(p)\right) \qquad , \xi = 0 \end{cases} \tag{8}$$

where $0 < p < 1,$ and the quantile function of the Go-GEV distribution is:

$$Q_{\textbf{Go-GEV}}(u) = \begin{cases} \mu + \dfrac{\sigma}{\xi}\left[\left(-\log\left\{1-\left[1-\dfrac{\gamma}{\lambda}\log(1-u)\right]^{-1/\gamma}\right\}\right)^{-\xi} - 1\right] & , \xi \neq 0 \\ \mu - \sigma\log\left(-\log\left\{1-\left[1-\dfrac{\gamma}{\lambda}\log(1-u)\right]^{-1/\gamma}\right\}\right) & , \xi = 0 \end{cases} . \tag{9}$$

Return level

Let $F_{\text{Go-GEV}}(x_p) = 1 - p,$ and x_p is called return level with return period $1/p$. This means that x_p is exceeded by the annual maximum in any particular year with probability p [2]. For $p = 1/M$, M is called a return period. The return level for the Go-GEV distribution is:

$$x_p = \begin{cases} \mu + \dfrac{\sigma}{\xi}\left[\left(-\log\left\{1-\left[1-\dfrac{\gamma}{\lambda}\log(p)\right]^{-1/\gamma}\right\}\right)^{-\xi} - 1\right] & ,\xi \neq 0 \\ \mu - \sigma\log\left(-\log\left\{1-\left[1-\dfrac{\gamma}{\lambda}\log(p)\right]^{-1/\gamma}\right\}\right) & ,\xi = 0 \end{cases} \quad . \tag{10}$$

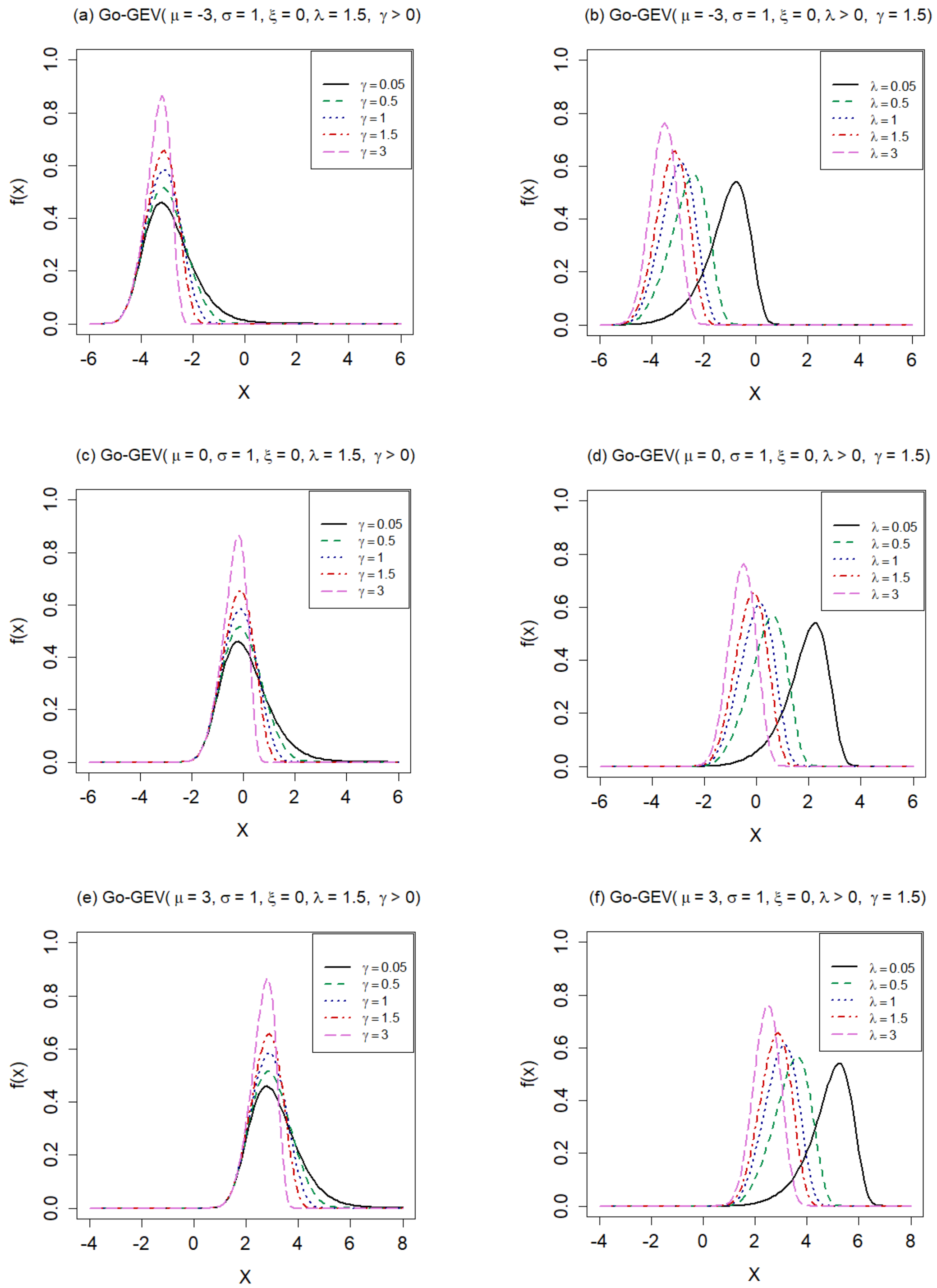

Figure 1 The p.d.f. plots of X for X ~Go-GEV $(\mu,\sigma,\xi,\lambda,\gamma)$ with fixed values of $\sigma = 1$ and $\xi = 0$.

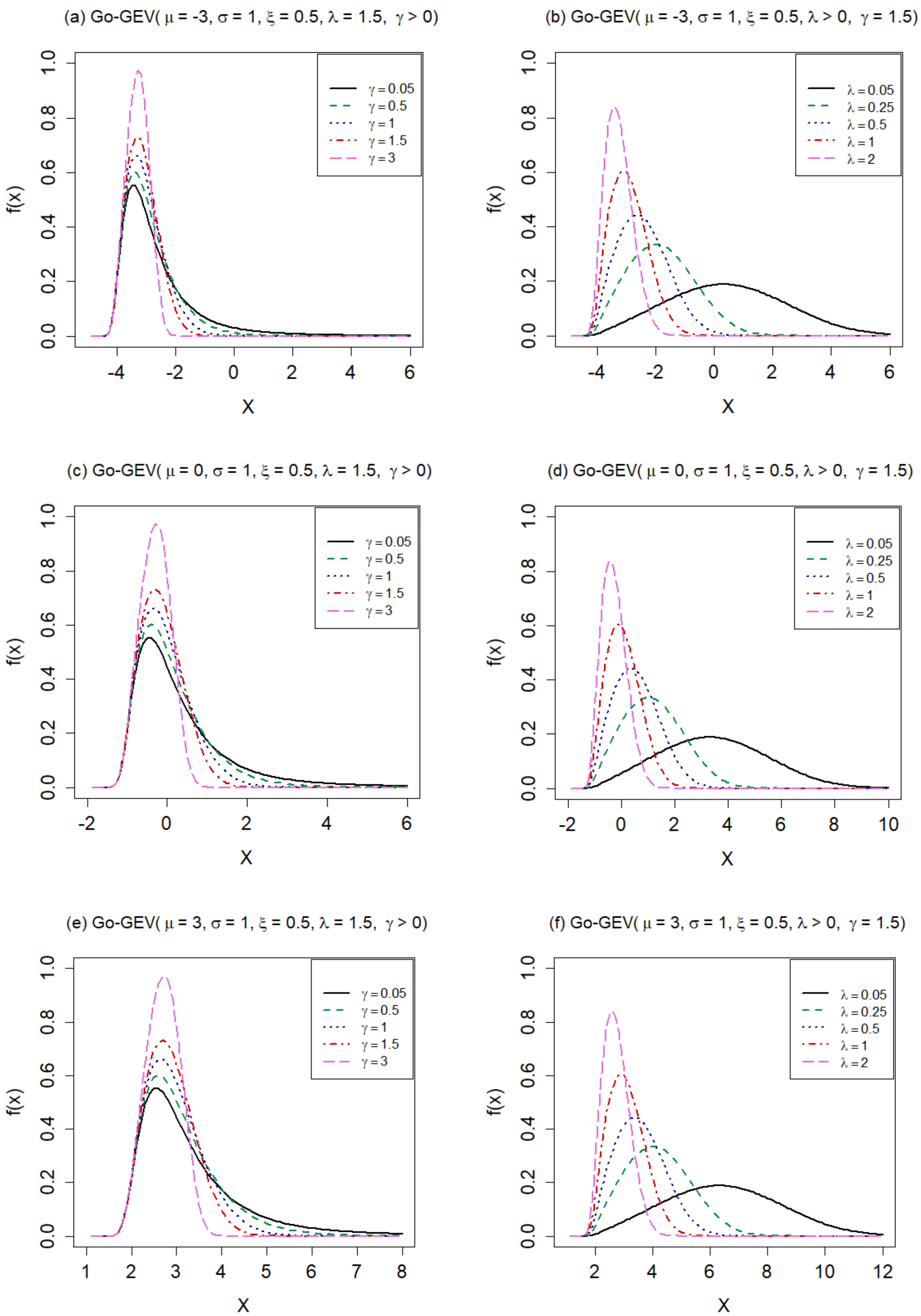

Figure 2 The p.d.f. plots of X for X ~Go-GEV$(\mu,\sigma,\xi,\lambda,\gamma)$ with fixed values of $\sigma=1$ and $\xi>0$.

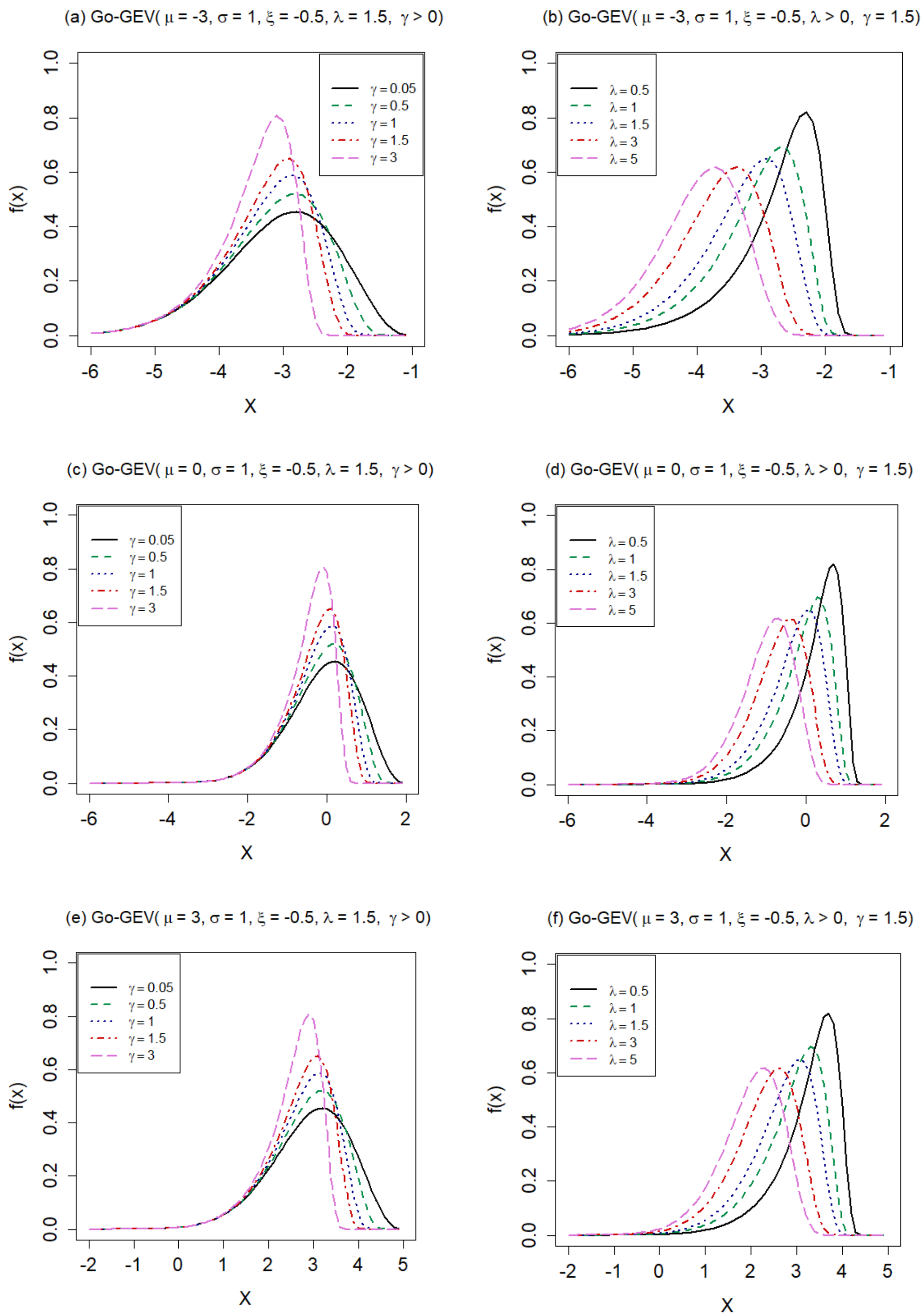

Figure 3 The p.d.f. plots of X for X ~Go-GEV $(\mu,\sigma,\xi,\lambda,\gamma)$ with fixed values of $\sigma = 1$ and $\xi < 0$.

Moments

Let X ~Go-GEV $(\mu,\sigma,\xi,\lambda,\gamma)$, then the moments of X becomes:

$$\mathbf{E}(X^k)=\sum_{r=0}^{\infty}(r+1)b_{r+1}\pi_{k,r} \tag{11}$$

where, $b_r=-\sum_{i=0}^{\infty}\sum_{j=0}^{i}\frac{(-1)^{i+r}}{i!}\binom{i}{j}\binom{-j\gamma}{\gamma}\left(\frac{\lambda}{\gamma}\right)^i$, $\pi_{k,r}=\int_0^1 u^r\left[Q_{GEV}(u)\right]^k du,$ and $Q_{GEV}(u)$ as in Eq. (8).

From the moments of X, the mean and variance of the Go-GEV distribution are:

$\mathbf{E}(X)=b_1\pi_{1,0}+b_2\pi_{1,1}$ and $\mathbf{V}(X)=b_1\left(\pi_{2,0}-b_1\pi_{1,0}^2\right)+b_2\left(\pi_{2,1}-b_2\pi_{1,1}^2\right)+b_3\pi_{2,2}-2b_1b_2\pi_{1,0}\pi_{1,1}$.

Parameter estimation

This study uses the maximum likelihood (ML) method to estimate the parameters of the Go-GEV distribution, with the assumption that $\tilde{x}=(x_1,\ldots,x_n)$ is a random sample $(X_1,\ldots,X_n)$ such that X_i are independent and identically distributed random variables of size n when X_i ~Go-GEV $(\mu,\sigma,\xi,\lambda,\gamma)$ with the p.d.f. in Eq. (6). Its log-likelihood function is:

$$\ell(\mathbf{\Theta})=n\log(\lambda)-n\log(\sigma)+(\xi+1)\sum_{i=1}^{n}\log(\tau(x_i))-(\gamma+1)\sum_{i=1}^{n}\left[1-\exp\left(-\tau(x_i)\right)\right]$$
$$-\sum_{i=1}^{n}\left(-\tau(x_i)\right)+\frac{\lambda}{\gamma}\sum_{i=1}^{n}\left[1-\left(1-\exp\left(-\tau(x_i)\right)\right)^{-\gamma}\right]$$

The unknown parameters $\mathbf{\Theta}=(\mu,\sigma,\xi,\lambda,\gamma)^{\mathrm{T}}$ are estimated by taking the partial derivatives with respect to each parameter and equating them to zero as follows:

$$\frac{\partial\ell(\mathbf{\Theta})}{\partial\mu}=0,\frac{\partial\ell(\mathbf{\Theta})}{\partial\sigma}=0,\frac{\partial\ell(\mathbf{\Theta})}{\partial\xi}=0,\frac{\partial\ell(\mathbf{\Theta})}{\partial\lambda}=0,\frac{\partial\ell(\mathbf{\Theta})}{\partial\gamma}=0.$$

The above equation cannot be derived in a closed form. Therefore, the numerical method of the 5-dimensional Newton-Raphson type procedure was used. The solutions of the ML estimators of $\mathbf{\Theta}$ were obtained by using the *optim* function in the R stats package [27].

Simulation study

The behavior of the parameters of the Go-GEV distribution was investigated by conducting simulation studies with the ML method, using the *optim* function in R software, which the R code are provided as follow:

```
#==============ML estimation for the Go-GEV distribution================#
logGoGEV<-function(x,par){
  mu<-par[1]; sigma<-par[2]; xi<-par[3]; lambda<-par[4]; gamma<-par[5];
  loglike<--sum(log(dGoGEV(x,mu,sigma,xi,lambda,gamma)))
  return(loglike)
}
out_mle<-optim(par=c(mu0,sigma0,xi0,lambda0,gamma0), fn=logGoGEV, x=x)
```

Data sets were generated from the Go-GEV distribution with a replication number of 1000. Simulated data X_i ~Go-GEV $(\mu,\sigma,\xi,\lambda,\gamma)$ were generated from Eq. (9) of sizes 30, 60, 100 and 200. The simulation was conducted for 3 different cases using varying true parameter values. The selected true parameter values were $\mu=3$, $\sigma=1$, $\lambda=1.5$, $\gamma=1.5$ where $\xi=-0.5,0,0.5$. The ML estimates of the true parameters were obtained by including the estimated ML estimators and the root mean square error (RMSE) as $\hat{\theta}_i = \frac{1}{1,000}\sum_{j=1}^{1,000}\hat{\theta}_{ij}$ and $\text{RMSE}\left(\hat{\theta}_i\right) = \sqrt{\frac{1}{1,000}\sum_{j=1}^{1,000}\left(\hat{\theta}_{ij} - \theta_i\right)^2}$.

Results of the simulation studies are shown in **Table 1**. Larger samples showed greater promise to give estimators closer to their parameter and the RMSE decreased. Thus, the ML method offered greater efficiency to determine the parameters as the sample size increased. The estimated value of each parameter are close to the true value when the sample is large (see **Figure 4**).

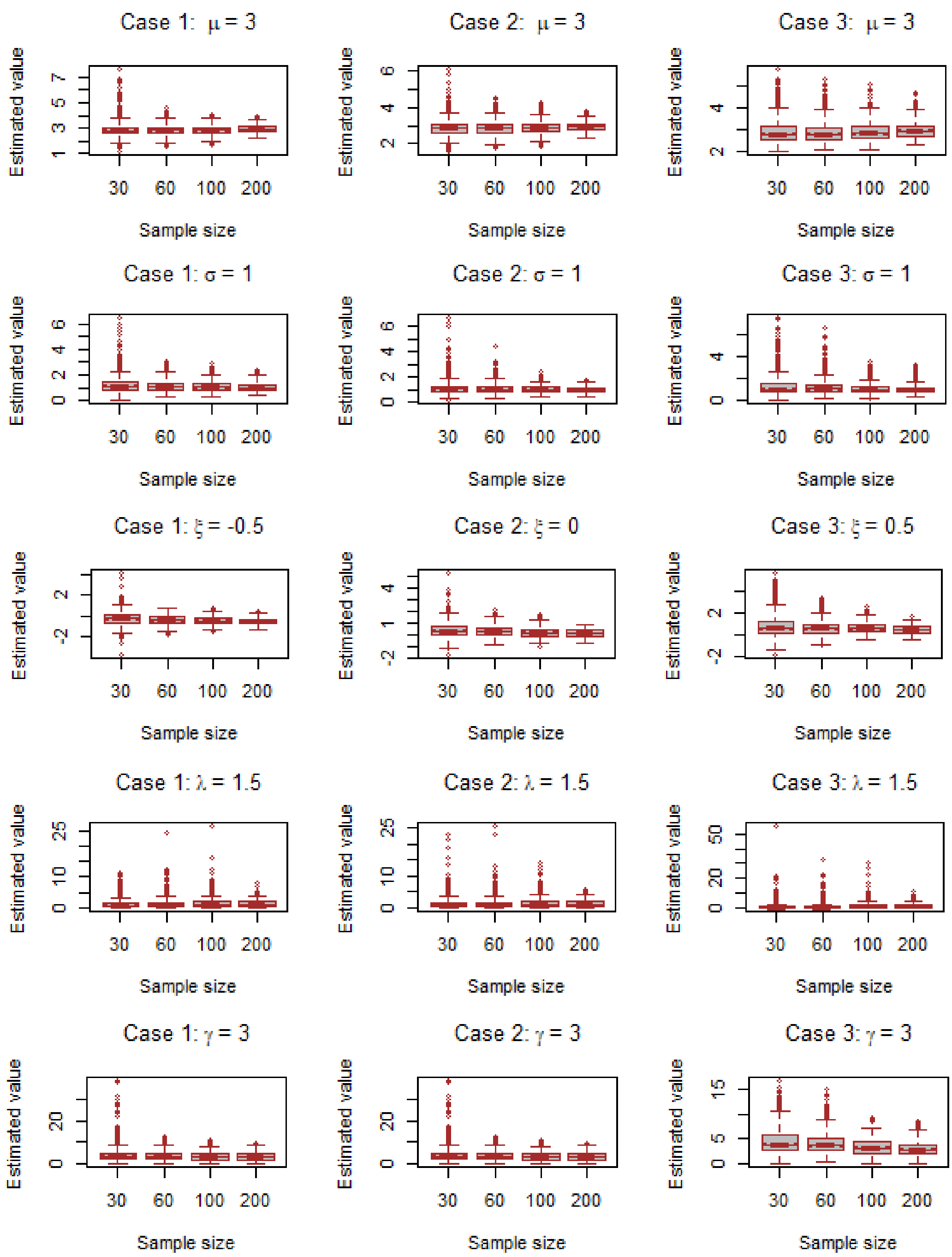

Figure 4 Box plots of the estimated values (1000 times) of the Go-GEV parameters.

Table 1 ML estimates and RMSE values of each estimator for parameters of the Go-GEV distribution.

n	Parameter	$\xi=-0.5$		$\xi=0$		$\xi=0.5$	
		Estimate	**RMSE**	**Estimate**	**RMSE**	**Estimate**	**RMSE**
30	$\mu=3$	2.8627	0.5896	2.9004	0.4515	2.8846	0.5480
	$\sigma=1$	1.2493	0.7494	1.1753	0.6221	1.3571	1.0568
	ξ	−0.2192	0.7071	0.3330	0.7159	0.8587	1.1334
	$\lambda=1.5$	1.4026	1.9652	1.5038	2.1978	1.1146	2.5641
	$\gamma=3$	4.2689	4.0078	3.9958	2.3959	4.4866	3.1223
60	$\mu=3$	2.8439	0.4331	2.8556	0.3995	2.8837	0.5073
	$\sigma=1$	1.1296	0.4616	1.0976	0.4130	1.2803	0.8307
	ξ	−0.3825	0.4296	0.2380	0.5436	0.7015	0.6924
	$\lambda=1.5$	1.5692	1.8841	1.4379	2.0052	1.1295	2.2364
	$\gamma=3$	3.5690	2.3434	3.6933	1.9069	4.0961	2.5481
100	$\mu=3$	2.8464	0.3960	2.8812	0.3441	2.8891	0.4454
	$\sigma=1$	1.0857	0.3984	1.0534	0.3033	1.1179	0.4847
	ξ	−0.4257	0.3496	0.1445	0.4110	0.6602	0.4886
	$\lambda=1.5$	1.5554	1.8142	1.4837	1.5567	1.3680	2.1772
	$\gamma=3$	3.2942	2.1043	3.3843	1.6252	3.3202	1.8073
200	$\mu=3$	2.9124	0.3129	2.9248	0.2567	2.9618	0.3954
	$\sigma=1$	1.0606	0.3579	1.0229	0.2506	1.0779	0.4426
	ξ	−0.4754	0.2752	0.0678	0.3235	0.5305	0.3615
	$\lambda=1.5$	1.5063	1.1461	1.5454	1.1413	1.4534	1.5003
	$\gamma=3$	3.1644	2.0357	3.1636	1.4473	3.0573	1.6408

Extreme value analysis and return level rainfall estimation

This section proposes the properties of a new extended GEV distribution. Parameters of the proposed distribution were estimated by maximum likelihood estimation for extreme value analysis and return level estimation of the rainfall at Lopburi Station, Thailand. The data used in this study were collected by the Thailand Meteorological Department from 1982 to 2021 at Lopburi Station and Buachum Station.

Let $X_t = \max\{X_{t,1}, X_{t,2}, X_{t,3}, \ldots, X_{t,12}\}$, as the maximum monthly mean rainfall (millimeters) in the tth year, where $t = 1, 2, \ldots, 40$. The data sets are shown in **Figures 5** and **6**.

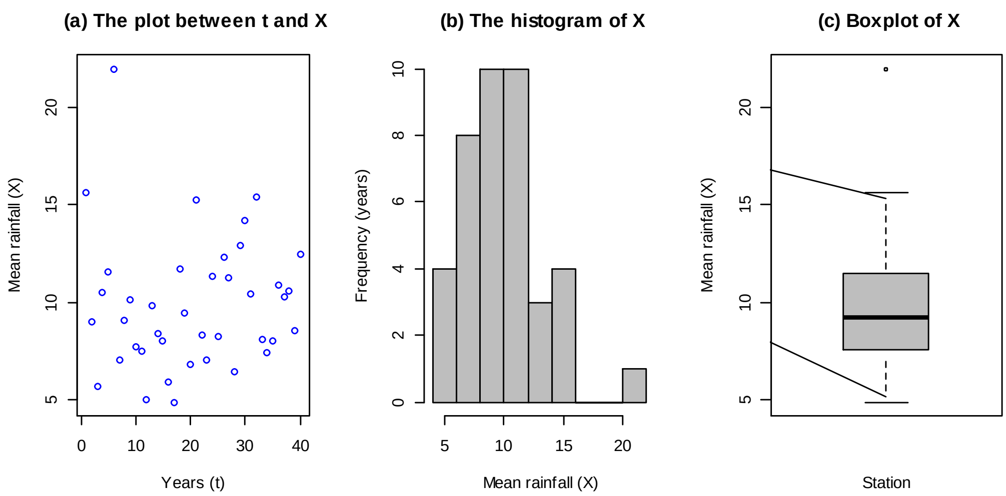

Figure 5 Plots of maximum mean rainfall values from 1982 to 2021 at Buachum Station.

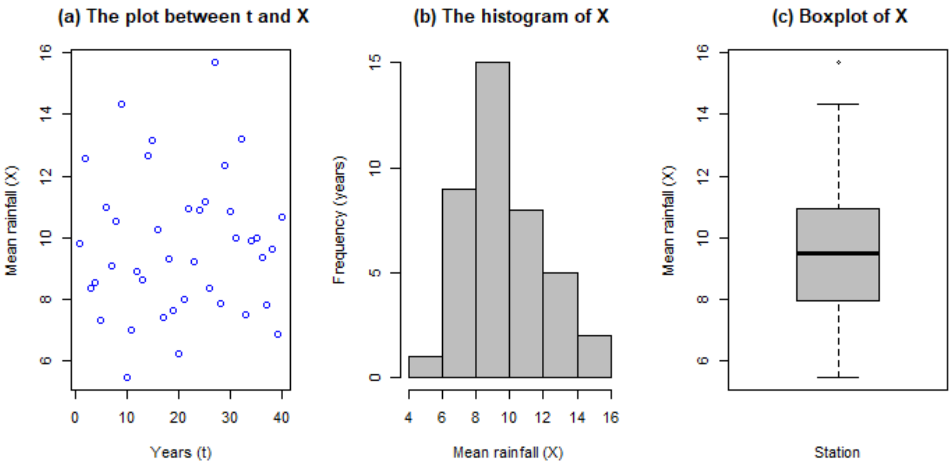

Figure 6 Plots of maximum mean rainfall values from 1982 to 2021 at Lopburi Station.

Table 2 Extreme value analyses for maximum values of mean rainfall from 1982 to 2021 in Lopburi Province, Thailand.

Distribution	ML estimates					KS test (p-value)
	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\xi}$	$\hat{\lambda}$	$\hat{\gamma}$	
Buachum Station						
GEV ($\xi = 0$)	8.3873	2.3808	-	-	-	0.0719 (0.9763)
GEV ($\xi \neq 0$)	8.3050	2.9128	0.0232	-	-	0.0749 (0.9660)
Go-GEV ($\xi = 0$)	**7.8757**	**2.2084**	**-**	**0.7822**	**0.0041**	**0.0618 (0.9955)**
Go-GEV ($\xi \neq 0$)	5.6824	1.7285	0.0640	0.0640	3.3930	0.0661 (0.9900)
Lopburi Station						
GEV ($\xi = 0$)	8.6070	1.9020	-	-	-	0.0629 (0.9974)
GEV ($\xi \neq 0$)	8.7740	1.9943	-01137	-	-	0.0665 (0.9945)
Go-GEV ($\xi = 0$)	**8.2400**	**1.8010**	**-**	**0.6931**	**0.2329**	**0.0568 (0.9995)**
Go-GEV ($\xi \neq 0$)	5.6000	0.5970	1.0304	0.0133	2.3708	0.0625 (0.9976)

Table 3 Return level estimations for maximum mean rainfall (millimeters) in Lopburi Province, Thailand.

Station	Model	Year				
		2	**5**	**10**	**15**	**20**
Buachum	**Go-GEV** ($\xi = 0$)	**9.34**	**12.61**	**14.56**	**15.58**	**16.27**
	Go-GEV ($\xi \neq 0$)	9.26	12.25	14.43	15.43	16.24
Lopburi	**Go-GEV** ($\xi = 0$)	**9.41**	**11.43**	**12.59**	**13.19**	**13.58**
	Go-GEV ($\xi \neq 0$)	9.41	11.48	12.62	13.20	13.58

In this study, the Kolmogorov-Smirnov (KS) test was used to determine the criteria for the goodness of fit test. The model gave a smaller value of KS that best fitted the data. The parameter estimates and the goodness of fit test for these data sets are given in **Table 2**. For both stations, the Go-GEV$_{\xi=0}$ distribution gave a lower KS value than the other distributions, i.e., Go-GEV$_{\xi\neq0}$, GEV$_{\xi=0}$ and GEV$_{\xi\neq0}$ respectively. The Go-GEV$_{\xi=0}$ distribution was, therefore, appropriate to fit this data (KS = 0.0618, p-value = 0.9955 at Buachum Station and KS = 0.0568, p-value = 0.9995 at Lopburi Station).

The extreme value theory was used to derive the Go-GEV distribution and a sample of extreme rainfall value was fitted to the Go-GEV$_{\xi=0}$ distribution to obtain the parameters that best explained the probability distribution of the extremes, i.e., $\hat{\mu} = 7.8757, \hat{\sigma} = 2.2084, \hat{\lambda} = 0.7822,$ and $\hat{\gamma} = 0.0041$ at Buachum Station, and $\hat{\mu} = 8.2400, \hat{\sigma} = 1.8010, \hat{\lambda} = 0.6931,$ and $\hat{\gamma} = 0.2329$, at Lopburi Station. The model gave the best fit to describe the maximum rainfall data and was better than the GEV distribution. Return level estimations of rainfall value in Lopburi Province are presented in **Table 3**.

Conclusions

Extreme value modeling for extreme rainfall is one of the most critical processes in hydrology. Most precipitation data tend to have extreme values with a heavy-tailed distribution. Developing a fitted distribution to the actual data increases the accuracy of the model. Previous studies related to a new extension of generalized extreme value distribution (GEV). This paper presented the Gompertz-generated (Go-G) family of distributions, with the aim of improving a new extension of generalized extreme value distribution that was more flexible and comprehensive for extreme data. Constructing this new distribution with the Go-G family made the existing distribution more flexible. The developed distribution was called the Gompertz-general extreme value: Go-GEV distribution with 4 parameters. The suitability of the models was compared by applying a new extension of the GEV distribution to the actual data, namely rainfall in Lopburi Province, and comparing the developed model with the traditional GEV distribution. The developed Go-GEV model described the maximum rainfall data better than the GEV distribution because the new extension distribution was more flexible and the probability density function of the Go-GEV distribution had a more diverse shape. i.e., both left-skewed, right-skewed and close to symmetrical. Moreover, the Go-GEV distribution had 2 shape parameters, with an alternative flexibility in describing the extreme data. This new extension of generalized extreme value distribution, Go-GEV, offers an alternative option for those interested in applying the model to extreme value data and developing new methods to determine extreme value distributions.

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