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Analysis of Free Vibrations in Homogeneous Isotropic Rigidly Fixed Thermoelastic Circumferential Spherical Curved Plates

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Abstract

The 1st and 2nd class vibrations of rigidly fixed thermoelastic spherical curved plates have been presented in this article. After mathematical modeling, the obtained equations have been solved by applying normal mode analysis. The uncoupled equation has been considered as 1st class vibrations named toroidal vibrations, remains independent of temperature variations. The coupled system of equations which is considered as 2nd class vibrations named spheroidal vibrations. The series solution of matrix Fröbenius method has been applied to coupled system of ordinary differential equations to obtain potentials and temperature. Fixed point iteration numerical technique has been applied to analytical results with the assistance of MATLAB software tools. Computational techniques have been applied to analytical results and numerically generated data has been represented graphically for frequencies, dissipation factor, displacements and temperature.

Keywords: Dissipation factor, Lowest frequency, Matrix Fröbenius method, Time harmonics, Vibrations

Introduction

Spherical curved plates have been widely used in the fields of spherical domes and pressure vessels of power plants. For the purpose of non-destructive testing and evaluation, it is necessary to understand the characteristics of waves and vibrations in respect of spherical and circular curved plates. Structural analysis of elastic curved plate structures has received much attention regarding theories of thermoelasticity. The theory of thermoelasticity forecasts two considerations, first one is the equation of heat conduction, which does not contain any elastic expressions. Second one is the heat conduction equation of parabolic type having infinite speed of transmission for heat waves. Therefore, the conventional heat conduction theory considers the thermal disturbances which propagate at infinite speed. In recent years, considerable research efforts have been made by researchers to understand the characteristics of waves and vibrations in respect of circular curved plate structures. Cohan *et al.* [1] developed the analysis of traction free vibrations of hollow sphere which is spherically isotropic, and presented the analysis for 2nd class vibrations by using series solution of Fröbenius method. Ding *et al.* [2] used Fröbenius method for series solutions in the form of matrix to solve the governing equations analytically and obtained the solution of free vibration analysis. Towfighi *et al.* [3] worked on a research problem which is based on anisotropic cylindrical curved plate with elastic wave propagation in circumferential direction. Towfighi and Kundu [4] worked on spherical curved plates for both isotropic and anisotropic materials for guided wave propagation. Yu and Ma [5] explored the harmonic waves propagating in piezoelectric and piezo-magnetic functionally graded spherical curved plates subjected to stress free and magneto electrically open boundary conditions.

Sharma and Sharma [6] developed a model for trans-radial isotropic thermoelastic solid sphere with free vibrations. Sharma and coworkers described the stress-free vibration analysis of 3-dimensional hollow spheres [7] and analyzed the results for damping factor and lowest frequency. The 3-dimensional hollow sphere was studied by Sharma *et al*. [8] in the reference of generalized viscothermoelasticity when boundaries were rigidly fixed thermally insulated/isothermal. Yu *et al*. [9] explored toroidal wave vibrations with the improvement of method of conventional series solution for multilayered spherical curved plates. Abbas [10] presented the analytical solutions for free vibration analysis for isothermal stress free thermoelastic hollow sphere with outer/inner surfaces. Tripathi *et al.* [11] presented the 2-dimensional thermoelastic cylindrical plates with diffusion in an axisymmetric heat supply using Laplace and Hankel transforms. Sharma [12] used series solution of Fröbenius method to present the analysis of free vibrations of thermoelastic spherical curved plates. Sharma *et al.* [13] presented transient wave analysis in functionally graded thermoelastic spherical cavity in radial direction using series solution. Singh and Muwal [14] presented the analytical results for thermoelastic stress and stain fields due to spherical inclusion for exterior and interior points. Sharma *et al.* [15] studied generalized thermoelastic hollow cylinder analytically with the effect of 3-phase-lag model and analyzed the results graphically. Sharma *et al.* [16,17] explored the free vibration analysis of electro-magneto nonlocal elastic cylinder and sphere with voids in the context of generalized thermoelasticity.

Keeping in view the above facts, this article is devoted to study the analysis of homogeneous isotropic, thermoelastic spherical curved plate subjected to rigidly fixed, isothermal and thermally insulated boundary conditions. Mathematical modeling for the problem has been carried out using the theories of thermoelasticity given by Lord-Shulman (LS) [18] and Green Lindsay (GL) [19]. To implement the potential function technique, the decoupled equation has been separated from the rest of the equations which remains independent of thermal variations. These vibrations are taken as 1st class (toroidal) vibrations. The coupled system of equations is taken for 2nd class (spheroidal) vibrations which have been further solved by using series solution. To compute frequency vibrations the iteration numerical method has been applied with the help of MATLAB software tools.

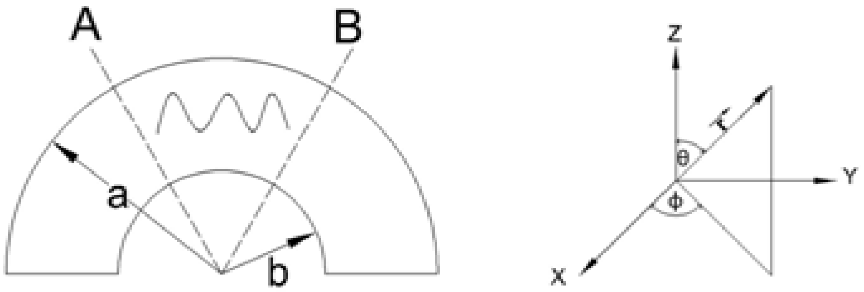

Figure 1 Geometry of the model of spherical curved plate.

Materials and methods

Formulation of problem

We consider thermoelastic spherical curved plate having outer radius $R_2 = b$, inner radius $R_1 = a$ and thickness h which is homogeneous isotropic thermally conducting, initially at uniform temperature T_0 in undisturbed state. The geometry of the problem assumed to be a section of a spherical structure (shown in **Figure 1**). For toroidal wave front (Towfighi and Kundu [4]) the 2 points A and B of spherical plate section can always be arranged in a line by making adjustment of the positions of north and south poles along the equator of the sphere. Therefore, for spherical curved plate section, the governing equations are considered from Sharma *et al.* [8] for $\theta = \frac{\pi}{2}$ only, by taking viscoelastic constants zero, and wave propagation to be independent of θ. The basic governing equations of isotropic, homogeneous thermoelastic body in tensor form are as follows:

$$e_{ij} = \frac{u_{i,j} + u_{j,i}}{2}\ ;\ i,\ j = r, \theta, \phi , \tag{1}$$

$$\sigma_{ij} = 2\mu e_{ij} + \lambda e - \beta\left(1 + t_1 \delta_{2k} \frac{\partial}{\partial t}\right) T , \tag{2}$$

$$\sigma_{ij,j} = \rho \frac{\partial^2 u_i}{\partial t^2} \; ; \quad (i,\; j = r,\, \theta,\, \phi), \tag{3}$$

$$K\,T,_{ij} - \rho C_e \frac{\partial}{\partial t}\left(1 + t_0 \frac{\partial}{\partial t}\right) T = T_0\, \beta \frac{\partial}{\partial t}\left(1 + t_0 \delta_{1k} \frac{\partial}{\partial t}\right) e\,, \tag{4}$$

where $e = \frac{\partial u_r}{\partial r} + \frac{2u_r}{r} + \frac{1}{r}\frac{\partial u_\varphi}{\partial \phi}$ is cubical dilatation, $\beta = (3\lambda + 2\mu)\,\alpha_T$ is thermoelastic coupling .

Here $u_i\,,\, T, \sigma_{ij}\,, e_{ij}\,;\, (i,\, j = r\,,\, \theta\,,\, \phi)$ are the displacement, temperature, stress and strain components. t_0 and t_1 are the thermal relaxation times, K is the thermal conductivity, λ and μ are Lame's parameters, C_e, ρ and α_T are the specific heat, density and coefficient of thermal expansion. The quantity δ_{ik} is Kronecker's delta. The comma notion is used for spatial derivatives and superposed dots are used for time derivatives.

Introducing the non-dimensional quantities:

$$\begin{cases} (\zeta, R_1, R_2) = \dfrac{(r\,,a\,,b)}{R}\,,\; (\tau, \tau_0\,, \tau_1) = \dfrac{c_1}{R}(t\,, t_0\,, t_1)\;,\; (U_\zeta\,,\; U_\theta\,,\, U_\phi) = \dfrac{(u_r\,, u_\theta\,, u_\phi)}{R} \\ \Theta = \dfrac{T}{T_0}\,,\; \tau_{ij} = \dfrac{\sigma_{ij}}{\rho c_1^2}\;,\; \overline{\beta} = \dfrac{\beta T_0}{\lambda + 2\mu}\;\;,\; \varepsilon_T = \dfrac{T_0 \beta^2}{\rho C_e\,(\lambda + 2\mu)}\;,\; \Omega^* = \dfrac{\omega^* R}{c_1}\;,\; \delta^2 = \dfrac{c_2^2}{c_1^2} \end{cases}\;, \tag{5}$$

where $c_1 = \sqrt{\dfrac{(\lambda + 2\mu)}{\rho}}\;,\; c_2 = \sqrt{\dfrac{\mu}{\rho}}\;,\; \omega^* = \dfrac{C_e\,(\lambda + 2\mu)}{K}\;,\; R = \dfrac{a+b}{2}\,,$

And the potentials ψ , G and w are

$$U_\theta = -\frac{\partial \psi}{\partial \varphi} \quad , \qquad U_\varphi = -\frac{\partial G}{\partial \varphi} \quad , \qquad U_\zeta = w \;; \tag{6}$$

Plugging the Eqs. (5) and (6) in Eqs. (1) to (4) we get following governing equations:

$$\left.\begin{array}{l} \left(\left(\nabla_1^2 - \dfrac{2}{\zeta^2}\right) + \delta^2 \dfrac{1}{\zeta^2}\dfrac{\partial^2}{\partial \varphi^2} - \dfrac{\partial^2}{\partial^2 \tau}\right) w - \left(\left(1 - \delta^2\right)\dfrac{1}{\zeta}\dfrac{\partial}{\partial \zeta} - \dfrac{1}{\zeta^2}\right) G - \overline{\beta}\left(1 + \tau_1 \delta_{2k}\dfrac{\partial}{\partial \tau}\right)\dfrac{\partial \Theta}{\partial \zeta} = 0 \\ -\left((1-\delta^2)\left(\dfrac{1}{\zeta}\dfrac{\partial}{\partial \zeta}\right) + \dfrac{1}{\zeta^2}\right) w + \left(\delta^2\left(\nabla_1^2 - \dfrac{2}{\zeta^2}\right) + \dfrac{1}{\zeta^2}\dfrac{\partial^2}{\partial \varphi^2} - \dfrac{\partial^2}{\partial \tau^2}\right) G + \overline{\beta}\left(1 + \tau_1 \delta_{2k}\dfrac{\partial}{\partial \tau}\right)\dfrac{\Theta}{\zeta} = 0 \\ -\dfrac{\rho\, \varepsilon_T\, \Omega^*}{\overline{\beta}} \nabla_{\tau_{01}}\left(\dfrac{\partial}{\partial \zeta} + \dfrac{2}{\zeta}\right) w + \dfrac{\rho\, \varepsilon_T \Omega^*}{\overline{\beta}} \nabla_{\tau_0} \dfrac{1}{\zeta}\dfrac{\partial^2 G}{\partial \varphi^2} + \left(\nabla_1^2 + \dfrac{1}{\zeta^2}\dfrac{\partial^2}{\partial \varphi^2} - \Omega^* \nabla_{\tau_0}\right)\Theta = 0 \end{array}\right\}, \tag{7}$$

$$\left[\left(\nabla_1^2 - \frac{2}{\zeta^2} + \frac{1}{\zeta^2}\frac{\partial^2}{\partial \varphi^2}\right) - \frac{1}{\delta^2}\frac{\partial^2}{\partial \tau^2}\right] \psi = 0\,, \tag{8}$$

where $\nabla_1^2 = \dfrac{\partial^2}{\partial \zeta^2} + \dfrac{2}{\zeta}\dfrac{\partial}{\partial \zeta}$, $\nabla_{\tau_{01}} = \left(\dfrac{\partial}{\partial \tau} + \tau_0 \delta_{1k} \dfrac{\partial^2}{\partial\, \tau^2}\right)$, $\nabla_{\tau_0} = \left(\dfrac{\partial}{\partial \tau} + \tau_0 \dfrac{\partial^2}{\partial\, \tau^2}\right)$

The quantities c_1 , c_2 , ε_T and ω^* are longitudinal, shear wave velocities, thermo-mechanical coupling constant and characteristic frequency of the medium.

Initial and boundary conditions

For the analysis of homogeneous thermoelastic spherical curved plate, the non-dimensional initial conditions when medium being supposed to be at rest and undisturbed, both mechanically and thermally, are given as:

$$\begin{cases} w(\zeta\ ,\phi\ ,0)=0=\dfrac{\partial\, w(\zeta\ ,\phi\ ,0)}{\partial\tau}\ ;G(\zeta\ ,\phi\ ,0)\ =0=\dfrac{\partial\, G(\zeta\ ,\phi\ ,0)}{\partial\tau}\ ,\ \text{at}\ \ R_2\le\zeta\le R_1 \\ \psi(\zeta\ ,\phi\ ,0)=0=\dfrac{\partial\,\psi(\zeta\ ,\phi\ ,0)}{\partial\tau}\ ;\ \Theta(\zeta\ ,\phi\ ,0)=0=\dfrac{\partial\,\Theta(\zeta\ ,\phi\ ,0)}{\partial\tau}\ ,\ \text{at}\ \ R_2\le\zeta\le R_1 \end{cases}, \tag{9}$$

The boundary conditions for rigidly fixed, thermally insulated and rigidly fixed isothermal thermoelastic spherical curved plate (at $\zeta = R_1$ (outer radius) and $\zeta = R_2$ (inner radius)) are considered and can be written in non dimensional form as:

$$\begin{cases} U_\zeta = 0\ ,\quad U_\theta = 0\ ,\quad U_\phi = 0\ ,\quad \dfrac{\partial\Theta}{\partial r}=0 \\ U_\zeta = 0\ ,\quad U_\theta = 0\ ,\quad U_\phi = 0\ ,\quad \Theta = 0 \end{cases},\ \text{at}\ \ \zeta = R_1\ \text{ etc.} \tag{10}$$

Solution of the problem

We take wave solution as given by Towfighi and Kundu [4]:

$$\{\psi,\ w,\ G,\ \Theta\}\,(\zeta,\varphi,t)=\zeta^{-\frac{1}{2}}\sum_{n=1}^{\infty}\psi_m(\zeta),\ \ w_m(\zeta),\ \ G_m(\zeta),\ \ \Theta_m(\zeta)\ \ e^{-i(\Omega\tau+m\varphi)} \tag{11}$$

where m is the wave number around the circumference, and $\Omega=\dfrac{R\,\omega}{c_1}$ is circular frequency.

Using Eq. (11) into Eqs. (7) to (8) we have

$$\begin{pmatrix} \left(\nabla_2^2-\dfrac{b_3^2}{\xi^2}+\Omega^2\right) & m^2\left(\dfrac{(1-\delta^2)}{\xi}\dfrac{\partial}{\partial\xi}-\dfrac{b_1^2}{\xi^2}\right) & c_3^*\left(\dfrac{\partial}{\partial\xi}-\dfrac{1}{2\xi}\right) \\ -\left((1-\delta^2)\ \dfrac{1}{\xi}\dfrac{\partial}{\partial\xi}+\dfrac{b_1^2}{\xi^2}\right) & \left(\delta^2\,\nabla_2^2-\dfrac{b_2^2}{\xi^2}+\Omega^2\right) & -\dfrac{c_3^*}{\xi} \\ c_4^*\left(\dfrac{\partial}{\partial\xi}+\dfrac{3}{2\xi}\right) & \dfrac{c_4^* m^2}{\xi} & \left(\nabla_2^2-\dfrac{b_4^2}{\xi^2}+\hat{\tau}_0^*\right) \end{pmatrix}\begin{pmatrix} w_m \\ G_m \\ \Theta_m \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \tag{12}$$

$$\left(\nabla_2^2+\frac{\Omega^2}{\delta^2}-\frac{\eta^2}{\xi^2}\right)\psi_m=0\,, \tag{13}$$

where

$$\begin{cases} \nabla_2^2=\dfrac{1}{\xi^2}\dfrac{\partial}{\partial\,\xi}\left(\xi^2\,\dfrac{\partial}{\partial\,\xi}\right),\ \xi=\zeta\,\Omega\ \ ,\ c_3^*=i\Omega\bar{\beta}\hat{\tau}\ ,\ c_4^*=\dfrac{\varepsilon_T\ \Omega^2\hat{\tau}_0^{*\prime}}{\bar{\beta}}\ ,\ \ \eta^2=\dfrac{9}{4}+m^2 \\ b_1^2=\dfrac{3+\delta^2}{2}\ \ ,\ \ b_2^2=\dfrac{9}{4}\delta^2+m^2\ ,\ \ b_3^2=\dfrac{9}{4}+m^2\,\delta^2\ ,\ \ \ b_4^2=\dfrac{1}{4}+m^2 \end{cases}, \tag{14}$$

$$\hat{\tau}_0=i\Omega^{-1}+\tau_0\ ,\ \ \hat{\tau}_0'=i\Omega^{-1}+\tau_0\delta_{1k}\ ,\ \hat{\tau}_1=i\Omega^{-1}+\tau_1\delta_{2k}\ ,\ \hat{\tau}_0^*=\hat{\tau}_0\Omega^*,\ \ \hat{\tau}_0^{*\prime}=\hat{\tau}_0'\Omega^*$$

The Eq. (13) for the potential ψ_m and coupled system in matrix Eq. (12) for w_m , G_m , Θ_m has been indicated the existence of 2 different classes of vibrations: toroidal and spheroidal respectively, in the circumferential curved plate. The solution of the spherical Bessel Eq. (13) is given by

$$\psi_m(\xi)=\sum_{m=1}^{\infty}\left[\ B_{m1}\ J_\eta\left(\frac{\xi}{\delta}\right)+B_{m2}\ Y_\eta\left(\frac{\xi}{\delta}\right)\right], \tag{15}$$

where arbitrary constants B_{m1} and B_{m2} are to be determined.

Solution for spheroidal vibrations

Applying matrix Fröbenius method in Eq. (12), we suppose series solution of the type given below in the interval $\xi_2 \le \xi \le \xi_1$ as:

$$\begin{pmatrix} w_m \\ G_m \\ \Theta_m \end{pmatrix} = \sum_{k=0}^{\infty} \begin{pmatrix} L_k \\ M_k \\ N_k \end{pmatrix} \xi^{s+k} , \tag{16}$$

where $\xi_1 = R_1\Omega$ and $\xi_2 = R_2\Omega$. Here s is the characteristic value and the unknowns $\left(L_k \quad M_k \quad N_k \right)'$ are to be calculated. Using Eq. (16) in Eq. (12), following equation has been obtained in matrix form as:

$$\sum_{k=0}^{\infty} \Big[\left(G(s+k) \right)_{3\times3} \xi^{-2} + \left(G'(s+k) \right)_{3\times3} \xi^{-1} + diag(\Omega^2 , \Omega^2 , \hat{\tau}_0^{*'}) \Big] \begin{pmatrix} L_k \\ M_k \\ N_k \end{pmatrix} \xi^{s+k} = 0 , \tag{17}$$

where

$$G(s+k) = \begin{pmatrix} G_{11}(s+k) & G_{12}(s+k) & 0 \\ G_{21}(s+k) & G_{22}(s+k) & 0 \\ 0 & 0 & G_{33}(s+k) \end{pmatrix}, G'(s+k) = \begin{pmatrix} 0 & 0 & G'_{13}(s+k) \\ 0 & 0 & G'_{23}(s+k) \\ G'_{31}(s+k) & G'_{32}(s+k) & 0 \end{pmatrix},$$

$$G_{11}(s+k) = \left((s+k)^2 - b_3^2 \right) \ , \quad G_{12}(s+k) = m^2 \left((1-\delta^2)(s+k) - b_1^2 \right);$$

$$G_{21}(s+k) = -\left((1-\delta^2)(s+k) + b_1^2 \right) \ , \quad G_{22}(s+k) = \left(\delta^2 (s+k)^2 - b_2^2 \right);$$

$$G_{33}(s+k) = \left((s+k)^2 - b_4^2 \right) \ , \quad G'_{13}(s+k) = c_3^* \left(s+k-\frac{1}{2} \right) \ ;$$

$$G'_{23}(s+k) = -c_3^* \ , \quad G'_{31}(s+k) = c_4^* \left(s+k+\frac{3}{2} \right), \quad G'_{32}(s+k) = c_4^* m^2 ;$$

Equating to zero the coefficients of lowest powers of ξ (i.e. $\xi^{s-2} = 0$) in Eq. (17), we obtain

$$\begin{pmatrix} \left(s^2 - b_3^2 \right) & m^2 \left((1-\delta^2)s - b_1^2 \right) & 0 \\ -\left((1-\delta^2)s + b_1^2 \right) & \left(\delta^2 s^2 - b_2^2 \right) & 0 \\ 0 & 0 & \left(s^2 - b_4^2 \right) \end{pmatrix} \begin{pmatrix} L_0 \\ M_0 \\ N_0 \end{pmatrix} = 0 . \tag{18}$$

Indicial equation for matrix Eq. (18) having non-trivial solution is

$$(s^4 - D s^2 + B)(s^2 - b_4^2) = 0 \tag{19}$$

where $D = \dfrac{(b_2^2 + \delta^2 b_3^2) - m^2 \ (1-\delta^2)^2}{\delta^2}$, $B = \dfrac{b_2^2 b_3^2 - m^2 b_1^4}{\delta^2}$

The roots of Eq. (19) are $s = \pm s_r \ (r = 1, 2, 3)$

$$s_1 = \sqrt{\left(\frac{D + \sqrt{D^2 - 4B}}{2} \right)}, \quad s_2 = \sqrt{\left(\frac{D - \sqrt{D^2 - 4B}}{2} \right)}, \quad s_3 = b_4 \tag{20}$$

And $s_r \ (r = 1 \ to \ 6)$ as $s_4 = -s_1$, $s_5 = -s_2$, $s_6 = -s_3$. Obviously s_3 , s_6 are real and s_1 , s_2 , s_4 , s_5 might be complex. Hence the series in the solution (17) can be written as:

$$\begin{pmatrix} L_0 \\ M_0 \\ N_0 \end{pmatrix} \xi^{s} = \begin{pmatrix} L_0 \\ M_0 \\ N_0 \end{pmatrix} \xi^{s_R + i s_I} = \begin{pmatrix} L_0 \\ M_0 \\ N_0 \end{pmatrix} \xi^{s_R} \left[\cos\left(s_I \log \xi\right) + i \sin\left(s_I \log \xi\right)\right]. \tag{21}$$

Using any one of the complex root of the indicial equation we get two independent real solutions of the system of equations. Therefore, Eq. (18) leads to following characteristic vectors

$$\begin{pmatrix} \boldsymbol{Y}_0(s_1) \\ \boldsymbol{Y}_0(s_2) \\ \boldsymbol{Y}_0(s_3) \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ Q_B(s_1) & Q_B(s_2) & 0 \\ 0 & 0 & 1 \end{pmatrix} Q_0 \quad , \tag{22}$$

$$\left(\boldsymbol{Y}_0(s_4) \quad \boldsymbol{Y}_0(s_5) \quad \boldsymbol{Y}_0(s_6)\right)^T = \left(\boldsymbol{Y}_0(-s_1) \quad \boldsymbol{Y}_0(-s_2) \quad \boldsymbol{Y}_0(-s_3)\right)^T$$

where $Q_B\left(s_r\right) = -\dfrac{\left(s_r^2 - b_3^2\right)}{m^2\left((1-\delta^2)s_r - b_1^2\right)} = \dfrac{(1-\delta^2)s_r + b_1^2}{\delta^2 s_r^2 - b_2^2}$, $\left(r = 1\,,\,2\right)$;

$$Q_B\left(-s_r\right) = \frac{\left(s_r^2 - b_3^2\right)}{m^2\left((1-\delta^2)s_r + b_1^2\right)} = \frac{-(1-\delta^2)s_r + b_1^2}{\delta^2 s_r^2 - b_2^2} \;,\; \left(r = 1\,,\,2\right);$$

and Q_0 is a constant vector. This suggest us to have

$$L_0(s_r) = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \;,\; M_0(s_r) = \begin{pmatrix} Q_B(s_r) \\ Q_B(s_r) \\ 0 \end{pmatrix} \;,\; N_0(s_r) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \;;\; r = 1,2,3\,, \tag{23}$$

where $L_0(s_r)$, $M_0(s_r)$, $N_0(s_r)$; $r = 4,5,6$ can be written from (23) by replacing s_r with $-s_r$, $(r = 1, 2, 3)$. Equating the coefficients of next lowest degree term ξ^{s-1} to zero, which corresponds to $k = 1$, and utilizing Eq. (17) we obtain:

$$\left(L_1 \quad M_1 \quad N_1\right)^T = D_1^*\left(L_0 \quad M_0 \quad N_0\right)^T \,, \tag{24}$$

where D_1^* is shown in Appendix A1.1.

Now for like powers of ξ^{s+k}, the recurrence relation has been obtained as:

$$\begin{pmatrix} L_{k+2} \\ M_{k+2} \\ N_{k+2} \end{pmatrix} = -\left(G_{ij}(s+k+2)\right)_{3\times3}^{-1} \left(\left(G'_{ij}(s+k+1)\right)_{3\times3}^{-1} \begin{pmatrix} L_{k+1} \\ M_{k+1} \\ N_{k+1} \end{pmatrix} + diag\left(\Omega^2\,,\,\Omega^2\,,\,\hat{\tau}_0^{*\prime}\right) \begin{pmatrix} L_k \\ M_k \\ N_k \end{pmatrix} \right), \tag{25}$$

Now substituting $k = 0, 1, 2, 3...$ in Eq. (25), we obtain

$$\begin{pmatrix} L_{k+2} \\ M_{k+2} \\ N_{k+2} \end{pmatrix} == D_{k+2}^* \begin{pmatrix} L_0 \\ M_0 \\ N_0 \end{pmatrix}, \tag{26}$$

where $D_0^* = I$,

$$D_{k+2}^* = -\left(G_{ij}(s+k+2)\right)_{3\times3}^{-1} \left(\left(G'_{ij}(s+k+1)\right)_{3\times3}^{-1} \begin{pmatrix} L_{k+1} \\ M_{k+1} \\ N_{k+1} \end{pmatrix} + diag\left(\Omega^2\,,\,\Omega^2\,,\,\hat{\tau}_0^{*\prime}\right) \begin{pmatrix} L_k \\ M_k \\ N_k \end{pmatrix} \right); \tag{27}$$

$$k = 0,\ 1,\ 2,\ 3, \ldots$$

Results of matrix D^*_{k+2} has comparable form to that of $G_{ij}(s_r+k+2)$ for even values of k and is alike $G'_{ij}(s_r+k+1)$ for odd values of k. Thus, we have

$$\begin{pmatrix} L_{2k+2} \\ M_{2k+2} \\ N_{2k+2} \end{pmatrix} == \left(D^*_{2k+2}\right)_{3\times 3} \begin{pmatrix} L_0 \\ M_0 \\ N_0 \end{pmatrix} \quad , \quad k=0,\ 1,\ 2,\ 3\ldots, \tag{28}$$

$$\begin{pmatrix} L_{2k+1} \\ M_{2k+1} \\ N_{2k+1} \end{pmatrix} == \left(D^*_{2k+1}\right)_{3\times 3} \begin{pmatrix} L_0 \\ M_0 \\ N_0 \end{pmatrix} \quad , \quad k=0,\ 1,\ 2,\ 3\ldots. \tag{29}$$

Simplifications of Eq. (27) we have

$$D^*_{2k+2} = \begin{bmatrix} H_{11} & H_{12} & 0 \\ H_{21} & H_{22} & 0 \\ 0 & 0 & H_{31} \end{bmatrix} \approx o\left(k^{-2}\right) E^*,\ D^*_{2k+1} = \begin{bmatrix} 0 & 0 & H'_{13} \\ 0 & 0 & H'_{23} \\ H'_{31} & H'_{32} & 0 \end{bmatrix} \approx o\left(k^{-1}\right) E^{**}, \tag{30}$$

where

$$E^* == c_4^* \begin{pmatrix} i\Omega\bar{\beta}\,\hat{\tau}_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 4 \end{pmatrix} \text{ and } E^{**} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{ as } k \to \infty,$$

$$H_{11} = \frac{G'_{31}(s_r+2k)G'_{13}(s_r+2k+1)}{G_{33}(s_r+2k+1)} + \Omega^2 \frac{G_{22}(s_r+2k)}{H^*(s_r+2k)} \quad ;$$

$$H_{12} = \frac{G'_{32}(s_r+2k)G'_{13}(s_r+2k+1)}{G_{33}(s_r+2k+1)} - \Omega^2 \frac{G_{12}(s_r+2k)}{H^*(s_r+2k)} \quad ;$$

$$H_{21} = \frac{G'_{31}(s_r+2k)G'_{23}(s_r+2k+1)}{G_{33}(s_r+2k+1)} + \Omega^2 \frac{G_{21}(s_r+2k)}{H^*(s_r+2k)} \ ;$$

$$H_{22} = \frac{G'_{32}(s_r+2k)G'_{23}(s_r+2k+1)}{G_{33}(s_r+2k+1)} + \Omega^2 \frac{G_{11}(s_r+2k)}{H^*(s_r+2k)} \ ;$$

$$H_{33} = \frac{G'_{31}(s_r+2k+1)K_1^*}{H^*(s_r+2k+1)} + \frac{G'_{32}(s_r+2k+1)K_2^*}{H^*(s_r+2k+1)} + \frac{\hat{\tau}_0^{*\prime}}{G_{33}(s_r+2k)} \ ;$$

$$K_1^* = \left(G_{22}(s_r+2k+1)G'_{13}(s_r+2k) - G_{12}(s_r+2k+1)G'_{23}(s_r+2k)\right);$$

$$K_2^* = \left(G_{21}(s_r+2k+1)G'_{13}(s_r+2k) + G_{11}(s_r+2k+1)G'_{23}(s_r+2k)\right);$$

$$H^*(s_r+2k) = G_{11}(s_r+2k)G_{22}(s_r+2k) - G_{21}(s_r+2k)G_{12}(s_r+2k);$$

$$H'_{13} = \frac{G'_{13}(s_r+2k)}{G_{33}(s_r+2k)} + \Omega^2 \frac{\left(G_{22}(s_r+2k-1)\ G'_{13}(s_r+2k+1) - G_{12}(s_r+2k+1)\ G'_{23}(s_r+2k+1)\right)}{H^*(s_r+2k+1)} \ ;$$

$$H'_{23} = \frac{G'_{23}(s_r+2k)}{G_{33}(s_r+2k)} + \Omega^2 \frac{\left(G_{21}(s_r+2k+1)G'_{13}(s_r+2k+1) - G_{11}(s_r+2k+1)G'_{23}(s_r+2k+1)\right)}{H^*(s_r+2k+1)} \ ;$$

$$H'_{31} = \frac{\left(-G'_{31}(s_r+2k)G_{22}(s_r+2k) + G'_{31}(s_r+2k)G_{12}(s_r+2k)\right)}{H^*(s_r+2k)} - \frac{G'_{31}(s_r+2k+1)\hat{\tau}_0^{*\prime}}{G_{33}(s_r+2k+1)};$$

$$H'_{32} = \frac{\left(-G'_{31}(s_r+2k)\,G_{21}(s_r+2k) + G'_{31}(s_r+2k)\ G_{11}(s_r+2k)\right)}{H^*(s_r+2k)} + \frac{G'_{32}(s_r+2k+1)\hat{\tau}_0^{*\prime}}{G_{33}(s_r+2k+1)};$$

Matrices $D^{*}_{2k+2} \rightarrow 0$, $D^{*}_{2k+1} \rightarrow 0$, as $k \rightarrow \infty$. Thus the potential functions w , G , Θ and ψ from Eq. (11) are given by

$$\left\{w, G, \Theta\right\}(\zeta,\varphi,\tau) = (\zeta)^{-\frac{1}{2}} \sum_{m=0}^{\infty}\sum_{j=1}^{6}\sum_{k=0}^{\infty} C_{mjk}\left\{a_{jk}(s_r),\, b_{jk}(s_r),\, d_{jk}(s_r)\right\}(\zeta\,\Omega)^{s_r+k} e^{-i(m\varphi+\Omega\,\tau)}, \tag{31}$$

$$\psi(\zeta,\ \varphi,\ \tau) = (\zeta)^{-\frac{1}{2}}\left[\sum_{m=1}^{\infty} B_{m1}\, J_{\eta}\left(\frac{\zeta\,\Omega}{\delta}\right) + \sum_{m=1}^{\infty} B_{m2}\, Y_{\eta}\left(\frac{\zeta\,\Omega}{\delta}\right)\right] e^{-i(m\varphi+\Omega\,\tau)}\ , \tag{32}$$

Here the quantities $\left\{a_{jk}(s_r),\, b_{jk}(s_r),\, d_{jk}(s_r)\right\}$ are characteristic vectors corresponding to the characteristic values $s_r\,;(r=1\ ,\ 2\ ,\ 3,\ 4\ ,\ 5,\ 6)$ and integer k. Here arbitrary constants $C_{mjk}\ ; i,\, j=1,2,3$ are to be calculated by using considered boundary conditions. The quantities $a_{jk}(s_r)$, $b_{jk}(s_r)$ and $d_{jk}(s_r)$ are defined as under:

$$a_{jk}(s_r) = \begin{cases} d^{k}_{11}(s_r) + d^{k}_{12}(s_r) Q_B(s_r)\ , & r=1\ ,\ 2\ ;\ k = \text{even} \\ d^{k}_{13}(s_r) & ,\ r=3\quad ;\ k=\text{odd} \end{cases},\text{with } a_{j0}=1\ ;\ j=1,2\ ,$$

$$b_{jk}(s_r) = \begin{cases} d^{k}_{21}(s_r) + d^{k}_{22}(s_r) Q_B(s_r)\ , & r=1\ ,\ 2\ ;\ k = \text{even} \\ d^{k}_{23}(s_r) & ,\ r=3\quad ;\ k=\text{odd} \end{cases},\text{ with } b_{j0}=1\ ;\ j=1\ ,\ 2\ ,$$

$$d_{jk}(s_r) = \begin{cases} d^{k}_{31}(s_r) + d^{k}_{32}(s_r) Q_B(s_r)\ , & r=1\ ,\ 2\ ;\ k = \text{odd} \\ d^{k}_{33}(s_r) & ,\ r=3\quad ;\ k=\text{even} \end{cases},\text{ with } d_{j0}=1;\ j=3\ ,$$

where $d^{k}_{ij}(s_r)\ ;\ (i,\, j=1,2,3)$ are the elements of determinant D^{*}_{k}. Using Eqs. (31) - (32) in Eq. (7), we obtain

$$U_{\zeta} = (\zeta)^{-\frac{1}{2}} \sum_{m=0}^{\infty}\left[\sum_{r=1}^{6}\sum_{k=0}^{\infty} C_{mjk}\, a_{jk}(s_r)(\zeta\,\Omega)^{s_r+k}\right]\exp\left(-i\,(m\varphi+\Omega\,\tau)\right), \tag{33}$$

$$U_{\theta} = (\zeta)^{-\frac{1}{2}}\left[im\sum_{m=1}^{\infty}\left[B_{m1} J_{\eta}\left(\frac{\zeta\,\Omega}{\delta}\right) + B_{m2} Y_{\eta}\left(\frac{\zeta\,\Omega}{\delta}\right)\right]\right]\exp\left(-i\,(m\varphi+\Omega\,\tau)\right) \tag{34}$$

$$U_{\phi} = (\zeta)^{-\frac{1}{2}} \sum_{m=0}^{\infty}\left[im\left(\sum_{r=1}^{6}\sum_{k=1}^{\infty} C_{mjk}\ b_{jk}\ (s_r)(\zeta\,\Omega)^{s_r+k}\right)\right]\exp\left(-i\,(m\,\phi+\Omega\,\tau)\right)\ , \tag{35}$$

$$\Theta,_{\zeta} = \zeta^{-\frac{1}{2}}\left\{\sum_{m=0}^{\infty}\sum_{k=0}^{\infty}\sum_{r=1}^{6} C_{mjk}\left(s_r + k - \frac{1}{2}\right) d_{j(k+1)}(s_r)\ (\zeta\Omega)^{s_r+k-1}\,\Omega\right\}\exp\left(-i\,(m\,\varphi+\Omega\,\tau)\right). \tag{36}$$

Frequency equations

Using boundary conditions in above equations, the determinant of the coefficients B_{n1}, B_{n2} and C_{mjk} ,$(j=1,\ 2,\ 3,\ 4,\ 5,\ 6)$ vanishes if and only if the coupled system of eight algebraic linear equations has a non-trivial solution. This non–trivial solution leads to determinant equations for thermally insulated and isothermal spherical curved plate which gives 2 different classes of vibrations as discussed by Ding *et al.* [17] as given below

For breathing mode $(k=0,\ m=0)$

$$\det\ (F_{ij}) = 0\ ,\quad (i,\, j=1,\ 2,\ 3,\ 4,\ 5,\ 6) \tag{37}$$

$$F_{1j} = a_{j0}(s_r)(R_1\Omega)^{s_j} \qquad ; \qquad r=1\ \ to\ \ 6\ ,$$

$$F_{3j} = 0 \qquad\qquad ; \qquad j=1\ \ to\ \ 6\ ,$$

$$F_{5j} = \begin{cases} \left(s_r - \frac{1}{2}\right) d_{j(0+1)}(s_r)\left(R_1\Omega\right)^{s_r-1} \text{; thermally insulated} \\ d_{j0}(s_r)\left(R_1\Omega\right)^{s_r} \quad \text{; isothermal} \end{cases} \; ; \; r, \; j = 1 \; \text{ to } \; 6 \tag{38}$$

For non-breathing mode $(k > 0, \; m > 0)$

$$\det(F'_{ij}) = 0 \;, \quad (i, j = 1, 2, 3, 4, 5, 6) \tag{39}$$

$$J\left(\frac{R_1\Omega}{\delta}\right) Y\left(\frac{R_2\Omega}{\delta}\right) - Y\left(\frac{R_1\Omega}{\delta}\right) J\left(\frac{R_2\Omega}{\delta}\right) = 0, \tag{40}$$

where

$$\begin{cases} F'_{1j} = a_{jk}(s_r)(R_1\Omega)^{s_r+k} \quad ; \quad r, \; j = 1 \; \textit{to} \; 6 \\ F'_{3j} = b_{jk}(s_r)(R_1\Omega)^{s_r+k} \quad ; \quad r, \; j = 1 \; \textit{to} \; 6 \end{cases}, \tag{41}$$

$$F'_{5j} = \begin{cases} \left(s_r + k - \frac{1}{2}\right) d_{j(k+1)}(s_r)\left(R_1\Omega\right)^{s_j+k-1} \text{; thermally insulated} \\ d_{jk}(s_r)\left(R_1\Omega\right)^{s_r+k} \quad \text{; isothermal} \end{cases} \; ; \; r, \; j = 1 \; \textit{to} \; 6 \;, \tag{42}$$

Here F_{2j}, F'_{2j}, F_{4j}, F'_{4j}, F_{6j}, F'_{6j} can be obtained by changing R_1 with R_2 in the above quantities F_{1j}, F'_{1j}, F_{3j}, F'_{3j}, F_{5j}, F'_{5j}.

1st class vibrations

Simplifying Eq. (40) by using asymptotic expansion we get

$$\frac{\tan\left((\xi/\delta)\Omega h\right)}{(\xi/\delta)\Omega h} \cong \frac{4\eta^2 - 1}{8ab\Omega^2 - 2\eta^4 + \eta^2 - 1/8}, \tag{43}$$

Substituting $\eta^2 = m^2 + 9/4$ asymptotically, the Eq. (43) reduces to

$$\frac{\tan\left((\xi/\delta)\Omega h\right)}{(\xi/\delta)\Omega h} \cong \frac{1}{ab\Omega^2 - 1} \quad \text{for} \quad m = 0, \tag{44}$$

Eqs. (43) and (44) govern the motion of vibrations which corresponds to toroidal (T-modes) vibrations called as 1st class vibrations. The Eq. (44) has complete agreement with the Ding *et al.* [20] of chapter 10, Eq. (10.2.16) for toroidal (T-modes) vibrations of elastic spherical curved plates.

2nd class vibrations

The frequency Eqs. (37) and (39) for rigidly fixed case governs spheroidal (S-modes) vibrations called as 2nd class vibrations.

Elastic spherical curved plate

If we take $\varepsilon_T = 0 = T$, $t_0 = 0 = t_1$, then the above analysis reduces to 1st class and 2nd class vibrations of coupled elastic spherical curved plates. On conversion of isotropic elastic material to transversely isotropic material i.e. $\lambda + 2\mu = c_{11} = c_{33}$, $\lambda = c_{12} = c_{13}$, $\mu = c_{44}, \beta = \beta_1 = \beta_2$, the system of governing equations are transformed into transversely isotropic elastic spherical curved plate and the reduced results are quantitatively consistent with Sharma [12] and Ding *et al.* [20] in case of 2nd class vibrations.

Results and discussion

Here we compute rigidly fixed vibrations of toroidal (T-mode) and spheroidal (S-modes) in the spherical curved plate made up of copper material for thermally insulated boundary conditions. The numerically analyzed results have been taken for T-modes and S-modes of vibrations for $k > 0$, $m > 0$ by using fixed point iteration numerical technique for thickness to mean radius ratio. All the numerical programs have been performed in MATLAB software tools. Due to the occurrence of dissipation factor in equation (4), the frequency equations give us complex values of Ω and hence ω. Writing $\omega = \omega_R + i\omega_I$, then for fixed values of m and k we have real part i.e. lowest frequency $\Omega_R = \text{Re}(\Omega/\delta) = R\omega_R / c_2$ and imaginary part i.e. dissipation factor $D = \text{Im}(\Omega/\delta) = R\omega_I / c_2$. By taking positive integer values of k the numerical computations have been done with the purpose to get the converged values of lowest frequency (Ω_R) and dissipation factor (D) (up to convergence level achieved). The simulated results are shown in **Figures 2** to **7** for generalized thermoelastic (GTE) material and coupled thermoelastic (CTE) material of spherical curved plate. In order to demonstrate the mathematical modeling, the physical data for copper material has been used from Tripathi *et al*. [11].

Here the values of thermal relaxation time parameters τ_1 and τ_0 have been estimated from the Eq. (2.5) with reference to Chandrasekharaiah [21]. The value of τ_1 is taken proportional to τ_0 (i.e. $\tau_1 = (\text{constant})\ \tau_0$) (the equation has been shown in Appendix). The variations of lowest frequency (Ω_R) and dissipation factor (D) versus thickness to mean radial ratio t^* *i.e.* $t^* = h/R$, $R = (a+b)/2$) and $h = a - b$ have been presented in **Figure 2** for modes $(m = 1, 2, 3)$ for generalized thermoelasticity (GTE) theory and coupled thermoelasticity (CTE) theory. **Figure 2 (a)** and **(b)** shows that the vibrations are low initially, attains maximum amplitude at $(t^* = 0.2)$, start decreasing to become linear after $t^* = 0.5$. It has been noted that the behavior in variation of vibrations has wobbling and humps in between is due to coupling of elastic field with thermal and mechanical forces. The effect of relaxation times shows that variation in generalized thermoelasticity is higher than coupled thermoelasticity for all modes $(m = 1, 2, 3)$ which indicates the effect of generalized thermoelasticity.

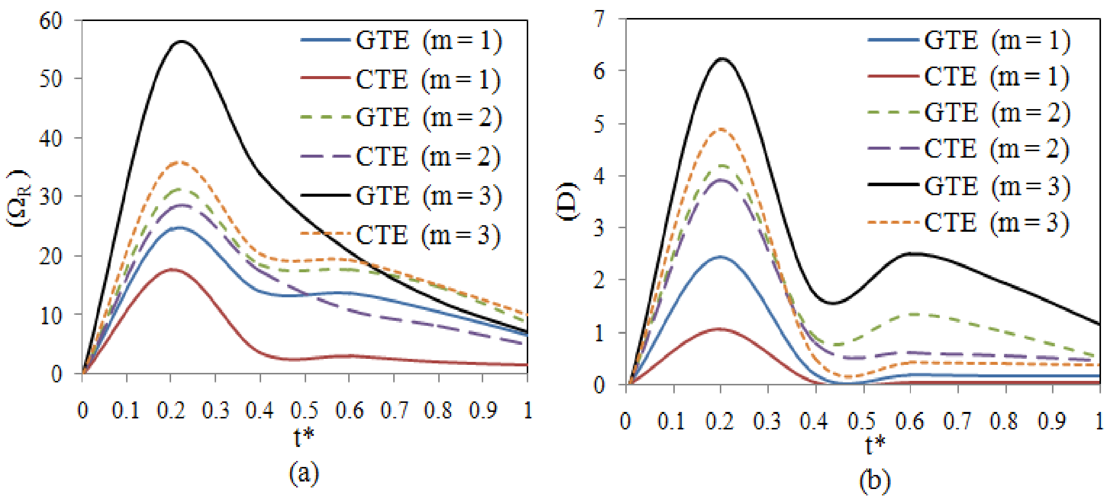

Figure 2 Comparison of (a) lowest frequency (Ω_R) (b) (D) versus thickness to mean radial ratio (t^*) of S-mode for GTE with CTE.

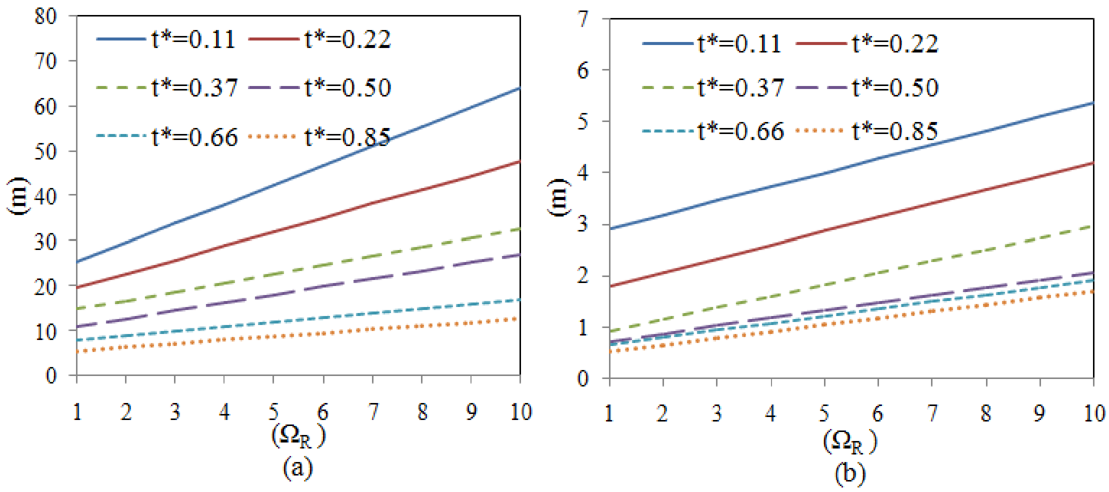

Figure 3 Wave number (m) versus frequency (Ω_R) at time (a) $\tau = 0.1$ (b) $\tau = 0.3$ for spheroidal vibrations.

Figure 3 has been presented for the vibration of wave number (m) versus lowest frequency (Ω) for $(t^* = 0.11, 0.22, 0.37, 0.50, 0.66, 0.85)$ at non-dimensional time (a) $\tau = 0.1$, (b) $\tau = 0.3$. It has been concluded that the vibrations go on increasing with increase in modes number (m), and further there is decrease in vibrations as thickness to mean radial ratio (t^*) increases. The variations of lowest frequency (Ω_R) and dissipation factor (D) for thermally insulated spherical curved plate versus wave number (m) have been presented in **Figure 4** at different values of thickness to mean radial ratio $(t^* = 0.11, 0.22, 0.37, 0.50, 0.66, 0.85)$. **Figure 4(a)** and **4(b)** shows that the variation of lowest frequency and dissipation factor vibrations keeps on increasing with increasing wave number. As the thickness to mean radial ratio increases the variation of frequency vibrations go on decreasing. Further, dissipation factor (D) increases with wave number (m) and the behavior of vibrations go on decreasing as the thickness to mean radial ratio (t^*) increases in both the cases. Also, the behavior of vibrations in **Figures 3** and **4** are quantitatively consistent with Sharma [12].

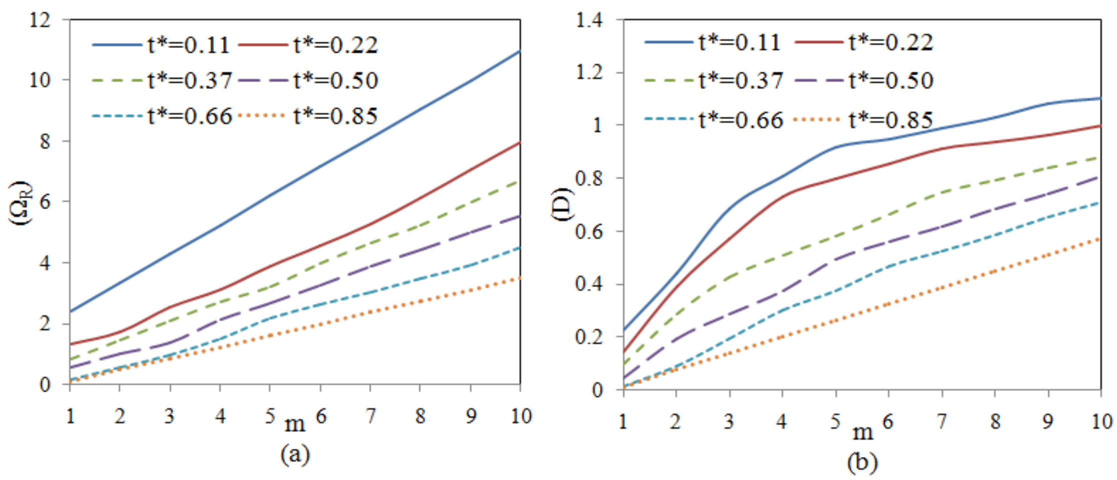

Figure 4 (a) Lowest frequency (Ω_R) (b) dissipation factor (D) versus wave number (m) for S-mode vibrations.

In **Figure 5**, the Lowest frequency (Ω_R) versus wave number (m) of 1st class vibrations has been presented graphically for distinct values of $t^* = 0.11$, 0.22, 0.37, 0.50, 0.66, 0.85 which shows that there is gradual increase in values of Ω_R up to $m = 2$, and with increasing values of m, the vibrations become linear and stable. But the dissipation factor in 1st class vibrations of (T-mode) is of the order 10^{-12} which is very low in the instantaneous case which is insignificant. From the trends of lowest frequency (Ω_R) and dissipation factor (D) of S-mode, this is to be observed that the thermal changes, the thermal relaxation times considerably affect the distinctiveness and behavior, and its magnitudes in contrast to the T-modes of vibrations with no impact of temperature variations as expected. **Figure 6(a)** and **6(b)** have been presented for the variations of temperature change (Θ) and radial displacement (U_ζ) versus standardized thickness (X^*) for the modes $(m = 1, 2, 3)$ for rigidly fixed and thermally insulated surface of the spherical curved plate (where $X^* = ((\zeta - R_2)/h)$ i.e. $0 \le X^* \le 1$)). It has been revealed from **Figure 6(a)** that the magnitude of temperature change (Θ) is higher in beginning at $X^* = 0.0$, decreases with increasing standardized thickness (X^*) but become steady at the value of $X^* = 1$ for all values of *m* = 1, 2, 3. **Figure 6(b)** shows (U_ζ) is initially lowest for all $(m = 1, 2, 3)$ and its magnitude is maximum between $0.3 \le X^* \le 0.6$, with increase in the value of X^* the variations go on decreasing and die out at $X^* = 1$. The mode $m = 3$ shows higher variation in contrast to $m = 1, 2$ due to coupling between thermal and elastic fields.

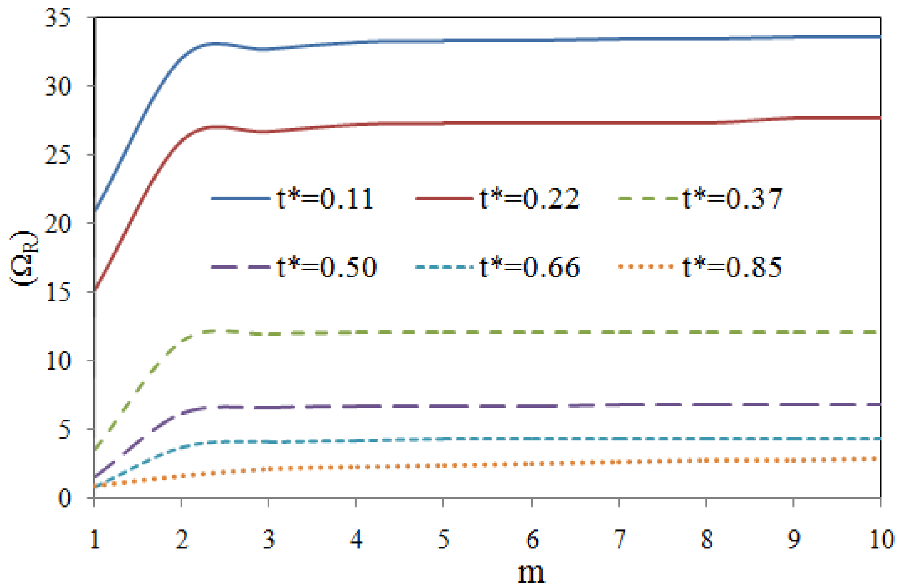

Figure 5 Lowest frequency (Ω_R) versus wave number (m) for toroidal vibrations.

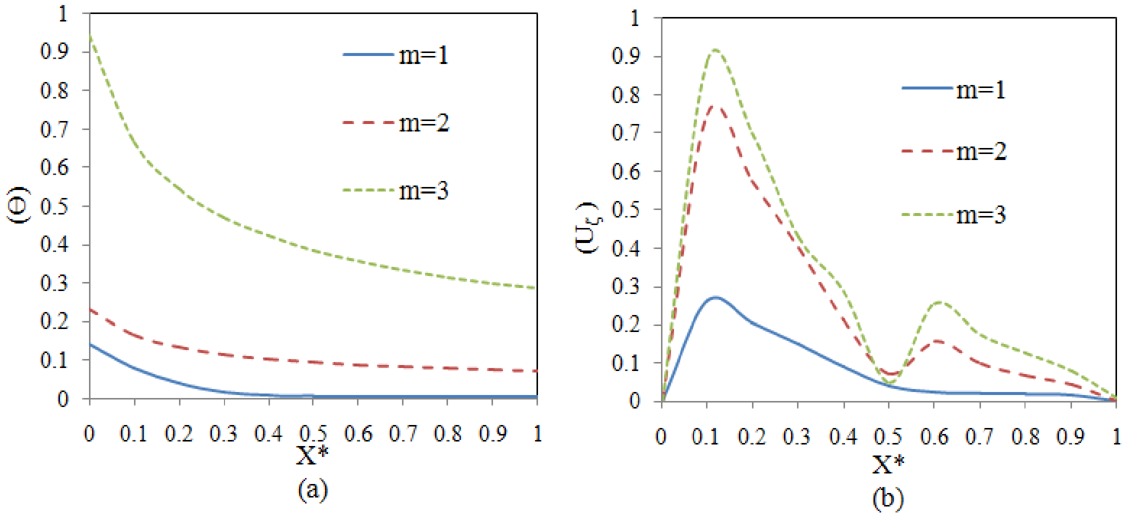

Figure 6 (a) Temperature (Θ) (b) radial displacement (U_{ς}) versus standardized thickness (X^*).

Figure 7(a) and **7(b)** present the variation of meridian (latitude) displacement U_{θ} and azimuthal (Longitude) displacement U_{ϕ} versus standardized thickness (X^*) for $m=1, 2, 3$. This is concluded from **Figure 7(a)**, that the variation of meridian displacement (U_{θ}) is initially low, attains its maximum amplitude between $(0.2 \leq X^* \leq 0.6)$ decreases to be died out at $X^* = 1$. **Figure 7(b)** reckons that the variation of U_{ϕ} is initially low, increases its magnitude up to $X^* = 0.8$ die out at $X^* = 1$. The magnitudes of these quantities are initially low, attain its maximum values between $(0.2 \leq X^* \leq 0.6)$ in **Figure 7(a)**, in contrast to $(0.6 \leq X^* \leq 0.8)$ in **Figure 7(b)**. Further, with increase in the value of X^* the variations go on decreasing and die out at $X^* = 1$. It can also be noted from the trends of **Figure 7(a)** and **7(b)** that the magnitude of meridian (latitude) displacement U_{θ} obeys the inequality $\left\{ U_{\theta}^{(m=1)} > U_{\theta}^{(m=2)} > U_{\theta}^{(m=3)} \right\}$ in **Figure 7(a)** in contrast to the azimuthal (Longitude) displacement U_{ϕ} in **Figure 7(b)** that obeys the inequality $\left\{ U_{\varphi}^{(m=1)} < U_{\varphi}^{(m=2)} < U_{\varphi}^{(m=3)} \right\}$ due to coupling between thermal and elastic fields. This shows that in all the figures representing displacements the maximum variation lies between $(0.2 \leq X^* \leq 0.8)$.

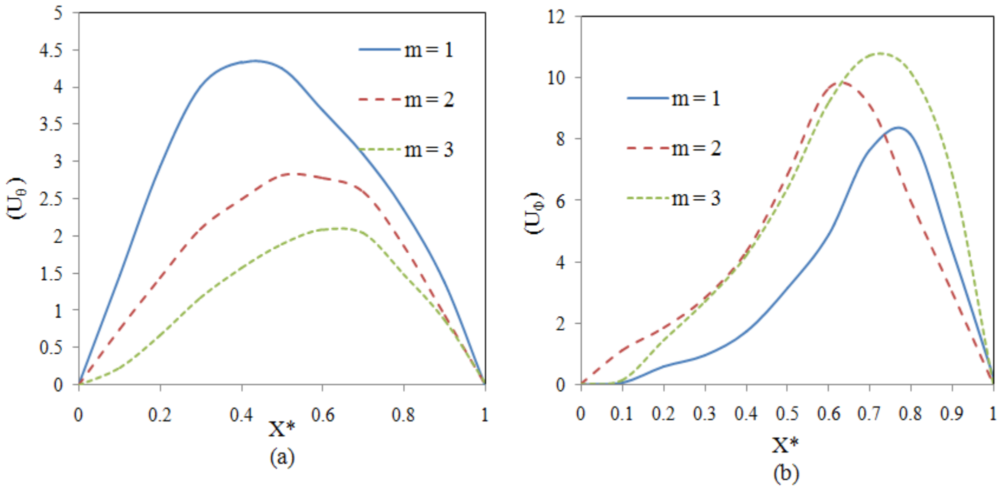

Figure 7 Variation of (a) Meridian displacement (U_θ) (b) azimuthal displacement (U_ϕ) versus standardized thickness (X^*).

Conclusions

1. The governing equations of isotropic generalized thermoelastic spherical curved plates have been presented with the facilitation of Helmholtz decomposition theorem in Equation (6) and the series solution is productively implemented to solve system of equations so obtained.

2. It is also observed that T-modes get decoupled from the rest of the motion and remain independent of thermal effects.

3. This is noticed that the variations in generalized thermoelasticity are higher in contrast to coupled thermoelasticity for all modes of wave numbers, because of coupling of elastic field with thermal and mechanical forces.

4. As thickness to mean radial ratio increases, the variation of vibrations of lowest frequency and dissipation factor go on deceasing due to resonance of frequency and the frequency generated at rigidly fixed boundary conditions for fixed values of thickness to mean radial ratio. The behavior of vibrations is consistent with reference to Sharma [12].

5. The displacement of spherical curved plate depends on rigidly fixed boundaries and the nature of tractions applied.

6. The results reported in this article might be useful for engineers and researchers who are working on mathematical physics, material science, and thermodynamics with low temperatures as well as on the development of the hyperbolic thermoelasticity theory.

7. Due to thermal variations and elastic nature of the material, the dissipation energy is caused which significantly affects the characteristics of the vibrations of field functions such as frequencies, dissipation, stresses, displacements and temperature change for the material.

8. The work reported here is more general, with the application to spherical plate structures of arbitrary thickness, from thin shells to extremely thick ones. This can be employed in applications involving pressure vessels, submarine constructions, aerospace, offshore, etc.

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Appendix

$$D_1^* = \begin{pmatrix} 0 & 0 & \dfrac{G_{22}(s_r+1)G'_{13}(s_r) - G'_{12}(s_r+1)H'_{23}(s_r)}{G_{11}(s_r+1)G_{22}(s_r+1) - G_{12}(s_r+1)G_{21}(s_r+1)} \\ 0 & 0 & \dfrac{G_{21}(s_r+1)G'_{13}(s_r) - G_{11}(s_r+1)G'(s_r)}{G_{11}(s_r+1)H_{22}(s_r+1) - G_{12}(s_r+1)G_{21}(s_r+1)} \\ \dfrac{G'_{31}(s_r)}{G_{33}(s_r+1)} & \dfrac{G'_{32}(s_r)}{G_{33}(s_r+1)} & 0 \end{pmatrix}, \quad \text{(A1)}$$

The value of thermal relaxation time parameter τ_0 is;

$$\tau_0 = \frac{3K}{\rho C_e c_1}, \quad \text{(A2)}$$

where K is thermal conductivity; ρ stands for mass density; C_e is specific heat; c_1 is longitudinal wave velocity (speed of the ordinary sound i.e. 1st sound)