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Bianchi type-V Dark Energy Modified $f(R,T)$ Gravity Model in the Presence of Massive Scalar Field in Lyra Geometry

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Abstract

Here, we investigate Bianchi type-V modified $f(R,T)$ gravity cosmological models in Lyra geometry. To acquire the deterministic solution of the field equations with $f(R,T)$ gravity based on Lyra geometry, we consider $f(R,T) = f_1(R) + f_2(T)$, R indicates Ricci Scalar, T indicates trace of the energy momentum tensor in Lagrangian matter. Presence of Massive scalar field and Displacement vector of Lyra played an important role. The physical as well as the geometrical nature of the $f(R,T)$ model is also discussed.

Keywords: Lyra geometry, Bianchi type-V, Dark energy, $f(R,T)$ gravity

Introduction

Dark energy and dark matter, these two modern concepts are the main hidden tools in discussion of present cosmological models of the universe and they illustrate how the model behaves. Many researchers have been searching for dark energy and have proposed a variety of theoretical models in the presence of Quintessence, Phantom etc., as a result, they have found that in the universe approximately composed of 4% ordinary matter, 23 % dark matter, 73 % dark energy [1] and from that result, we can say that our universe is filled with dark energy. The rate of expansion of our universe is positive, that is the discovery of the present acceleration of our universe is unanimously, theoretically and geometrically proved by the astronomical observations like Supernova, Cosmic Microwave Background Radiation (CMBR) data [as refs. 2, 3, 4 and therein], while the fact of this acceleration is still very uncertain and this matter is suggests researchers trying to investigate such a remarkable behavior of the present universe of our model. Several physicists and authors have been effort to determine this unexpected occurrence, including modifying theory of gravity the current universe of the major conceivable existence of dark energy in different context [5-9]. As a result, from this experiment, we obtained that the dark energy, which construct repulsive force that give rise to the current accelerating expansion of the universe, also we found that many researchers have taken new assumption, modifications of existing model to explain this mysterious behavior of the present cosmological model.

Modifying gravity, which appears more appealing in exploring the behavior of dark energy (Negative Pressure), is one of the finest approaches to establish the cosmic acceleration (Late time) in cosmology.

Several authors have made remarkable contribution to the explanation of present universe of our accelerating phase and bring out further theories for relevant theories. The theories are $f(R), f(G), f(T), f(R,T)$ as theoretically and geometrically observations from many researchers, we found to be $f(R,T)$ gravity is very attractive and suitable among these modify theories for the present cosmological model, in which the Einstein–Hilbert action is replaced by a function of Ricci scalar [10-17]. However, Harko *et al.* [18] perceived new modification of $f(R)$, which term as $f(R,T)$ gravity theory, where gravitational Lagrangian is given by an arbitrarily function of Ricci-Scalar R and Trace T of the energy momentum tensor.

To reveal relevant facts about dark energy, two models of ideas have been considered: one is to create negative pressure (which is called Dark Energy), and the other is to modify Einstein's theory of gravitation to discuss the corresponding anisotropic of negative pressure (DE), and we have found that significant modifications have been suggested by several authors [19-22] in various contexts In general, modified theories of gravitation are one of the most useful tools for studying the possibility of anisotropic behavior of dark energy models and the consequences of current cosmological models of the evolution of the universe in the presence of various physical contents in late time acceleration. Here, we are focusing in the field of $f(R,T)$ gravity model in the frame work of Lyra Geometry in Bianchi type-V cosmological models in presence of an attractive massive scalar field. Subsequent behavior of the models is explored by many researchers, among them Singh and Rani [23], Mohanty and Pradhan [24], Naidu *et al.* [25], they examined a spatially homogenous and anisotropic cosmological models of the universe based on Lyra Geometry and General Relativity in presence of an attractive massive scalar field in different physical context, in which special law of variation for Hubble's parameter [Proposed by Berman (1983)] was applied to examined the constant deceleration parameter. In present decades, we have also obtained several authors who have examined in different cosmological models of the type with different physical features in the frame work of an attractive massive scalar field based in different context so far [26-36].

As physicists and researchers discovered that in our universe, there are mainly I-IX Bianchi type models, but, here in this paper, we are interested in type-V cosmological models, which being the natural generalization of FRW model, that preserves homogeneous and anisotropic universe of the model. Several authors [37-46] have examined the cosmological models in Bianchi type-V in different physical context with dark energy component. So far, we have been obtained several noteworthy modifications of Riemannian geometry as well as EFE in different physical features, Significantly, Weyl [47] one of the first who try to formulated the Riemannian geometry, as a result of non-integrability of length transfer his modification theory was never taken seriously. Subsequently, similar result was again reconstructed this modification by introducing gauge function into structure less manifold by Lyra [48] in order to geometrize gravitation and electromagnetism. In the same way, Sen [49], Sen and Dunn [50] formulated an analogue of the EFE in the frame work of Lyra geometry with new scalar tensor theory. In view of this fact, Halford [51,52] has pointed out the constant displacement vector field ϕ plays the vital role compared to the cosmological constant in the general relativity and as in Einstein theory of relativity, which predicts the same effects within observational limits. Many authors [53-55] have presented cosmological model with a constant displacement field vector and time displacement field based on Lyra geometry in the late time universe of the model.

Metric and the $f(R,T)$ gravity

Let us consider the Bianchi type-V line element in the form

$$ds^2 = -dt^2 + A^2dx^2 + e^{-2mx}(B^2dy^2 + C^2dz^2) \tag{1}$$

where A, B and C are functions of cosmic time t alone and m is constant. In Lyra geometry [56], the action of $f(R,T)$ gravity is given by $(G = 1)$

$$S = \int \sqrt{-g}\left(\frac{1}{16\pi} f\left(\tilde{R}, T\right) + L_m\right) d^4x \tag{2}$$

where,

$\tilde{R}$ = Function of Ricci scalar R

T = Trace of the energy momentum tensor

L_m = Matter lagrangian density

Also, the Riemannian curvature and the stress- energy tensor of the matter is defined by

$$\tilde{R} = R + 3\nabla_i\phi^i + \frac{3}{2}\phi^i\phi_i \tag{3}$$

$$T_{ij} = -\frac{2}{\sqrt{-g}}\frac{\delta\sqrt{-g}L_m}{\delta g^{ij}} \tag{4}$$

in which its trace is given by $T = g^{ij}T_{ij}$

Assuming that the Lagrangian density L_m of matter depends only on the metric tensor components g_{ij} rather than its derivatives which leads to

$$T_{ij} = g_{ij}L_m - 2\frac{\partial L_m}{\partial g^{ij}} \tag{5}$$

Now, by varying the action S in Eq. (2) with respect to metric tensor g_{ij}, the gravitational field equations of $f(\tilde{R},T)$ gravity is obtained [18] as

$$f_{\tilde{R}}(\tilde{R},T)R_{ij} - \frac{1}{2}f(\tilde{R},T)g_{ij} + (g_{ij}\nabla^i\nabla_i - \nabla_i\nabla_j)f_{\tilde{R}}(\tilde{R},T) = \frac{8\pi G}{c^2}T_{ij} - f_T(\tilde{R},T)T_{ij} - f_T(\tilde{R},T)\ominus_{ij} \quad (6)$$

where $f_{\tilde{R}}(\tilde{R},T) = \frac{\partial f(\tilde{R},T)}{\partial \tilde{R}}, f_T(\tilde{R},T) = \frac{\partial f(\tilde{R},T)}{\partial T}$ and ∇_i indicates covariant derivative in $f(\tilde{R},T)$ gravity model, and $\ominus_{ij}$ is given by

$$\ominus_{ij} = -2T_{ij} + g_{ij}L_m - 2g^{lm}\frac{\partial^2 L_m}{\partial g^{ij}\partial g^{lm}} \quad (7)$$

Here, we assumed that the standard stress-energy tensor for a perfect fluid matter Lagrangian given by

$$T_{ij} = (\rho + p)u_i u_j + pg_{ij} \quad (8)$$

Here, ρ and p denotes the energy density and pressure of the matter. Also, $u^i = (0, 0, 0, 1)$ is the 4-velocity vector in the co-moving coordinate system satisfying the condition $u_i u^i = -1$.
An attractive massive scalar field, whose energy momentum tensor is assumed as,

$$T_{ij}^{\varphi} = \varphi_{,i}\,\varphi_{,j} - \frac{1}{2}\left(\varphi_{,k}\varphi^{,k} - M^2\varphi^2\right) \quad (9)$$

where mass of that scalar field is represented by M and which satisfies the Klein- Gordon equation

$$g^{ij}g_{,ij} + M^2\varphi = 0 \quad (10)$$

The symbols comma and a semicolon represent respectively the ordinary and covariant differentiation, where φ is a function of time t.

$$L_m = -p, L_{\varphi} = \frac{1}{2}(M^2\varphi^2 - \dot{\varphi}^2) \quad (11)$$

Then the tensor Eq. (7) is obtained as bellow, where the physical nature of model depends on this tensor $(\ominus_{ij})$ as

$$\ominus_{ij} = -2T_{ij} - \frac{1}{2}\,g_{ij}(2p + \dot{\varphi}^2 - M^2\varphi^2) \quad (12)$$

It is worth to know that the several researchers have researched theoretical models and discussed in different cases corresponding to choose of Harko *et al.* [18] frame for the $f(\tilde{R},T)$ model, which shows the arbitrary function of Ricci scalar R. Here in this study, we assumed the second case of the model of Harko *et al.* [18] as

$$f(\tilde{R},T) = f_1(\tilde{R}) + f_2(T) \quad (13)$$

Now by contracting Eq. (6), we obtained as

$$\begin{aligned} f_1{}'(\tilde{R},T)\tilde{R}_{ij} - \frac{1}{2}f_1(\tilde{R})g_{ij} + (g_{ij}\nabla^i\nabla_i - \nabla_i\nabla_j)f_1{}'(\tilde{R}) \\ = -\frac{8\pi}{c^2}T_{ij} + f_2{}'(T)T_{ij} + \left[f_2{}'(T)\,p + \frac{1}{2}\,f_2(T)\right]g_{ij} \end{aligned} \quad (14)$$

Let us assume that $f_1(\tilde{R}) = \mu\,\tilde{R}$ and $f_2(T) = \mu T$, where μ is an arbitrary constant. For a perfect fluid matter source, the field equations of $f(\tilde{R},T)$ gravity is obtained in Eq. (14) as

$$\tilde{R}_{ij} - \frac{1}{2}\tilde{R}\,g_{ij} = -\left(\frac{8\pi - \mu c^2}{c^2}\right)T_{ij} + \left[p + \frac{1}{2}T\right]g_{ij} \quad (15)$$

According to Maurya [56], by applying eq (3) in (14), the field equations for Lyra Geometry is obtained as

$$R_{ij} - \frac{1}{2}Rg_{ij} + \frac{3}{2}\phi_i\phi_j - \frac{3}{4}g_{ij}\phi_i\phi^j = -\alpha\, T_{ij} + \left[p + \frac{1}{2}T\right]g_{ij} \tag{16}$$

Here $\emptyset_i = (0,0,0,\beta)$ is a displacement vector field and $\alpha = \left(\frac{8\pi - \mu C^2}{\mu C^2}\right)$ is a constant.

Field equations of the model in $f(\widetilde{R},T)$ gravity

For the metric Eq. (1), in view of Eq. (8), the Einstein field Eq. (16) reduces to the form as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{m^2}{A^2} + \frac{3}{4}\beta^2 + \frac{\dot{\varphi}^2}{2} + \frac{M^2\varphi^2}{2} = -\alpha p + \left(\frac{\rho - p}{2}\right) \tag{17}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{C}\dot{A}}{CA} - \frac{m^2}{A^2} + \frac{3}{4}\beta^2 + \frac{\dot{\varphi}^2}{2} + \frac{M^2\varphi^2}{2} = -\alpha p + \left(\frac{\rho - p}{2}\right) \tag{18}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{m^2}{A^2} + \frac{3}{4}\beta^2 + \frac{\dot{\varphi}^2}{2} + \frac{M^2\varphi^2}{2} = -\alpha p + \left(\frac{\rho - p}{2}\right) \tag{19}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{3m^2}{A^2} - \frac{3}{4}\beta^2 + \frac{\dot{\varphi}^2}{2} + \frac{M^2\varphi^2}{2} = \alpha\rho + \left(\frac{\rho - p}{2}\right) \tag{20}$$

$$\frac{\dot{B}}{B} + \frac{\dot{C}}{C} - 2\frac{\dot{A}}{A} = 0 \tag{21}$$

The Klein Gordon equation is as

$$\ddot{\varphi} + \dot{\varphi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + M^2\varphi = 0 \tag{22}$$

and the conservation law for the energy momentum tensor of dark energy fluid is

$$\dot{\varphi} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0 \tag{23}$$

Cosmological solutions of the field equations

In solving the above filed Eqs. (17) - (21), the following physical parameters are very important and these parameters are defined as follows:

The spatial volume (V) and the scale factor $a(t)$ are given by

$$V = a^3 = ABC \tag{24}$$

The generalized Hubble parameter (H) and the scalar expansion (θ) are defined as

$$H = \frac{\dot{a}}{a} = \left(H_x + H_y + H_z\right) \tag{25}$$

$$\theta = 3H = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \tag{26}$$

where, $H_x = \frac{\dot{A}}{A}, H_y = \frac{\dot{B}}{B}$ and $H_z = \frac{\dot{C}}{C}$ are the directional parameters in the direction of x, y and z axes.

The shear expansion $(\sigma)^2$ and the anisotropy parameter (Δ) are defined as

$$\sigma^2 = \frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2 - \left(\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA}\right)\right] \tag{27}$$

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \tag{28}$$

Integrating Eq. (21), we found that

$$A^2 = k_1 BC \tag{29}$$

where the constant of integration k_1 can be chosen as unity, so that we get

$$A^2 = BC \tag{30}$$

In the Einstein field Eqs. (17) - (21), there are five highly non-linear differential equations with seven unknown variables, namely $A, B, C, p, \rho, \beta, \varphi$. Thus, in order to find out these seven unknown constants, we need another condition to complete the field equations and hence for this we assumed the following conditions as:
(i) we consider. $\frac{\sigma}{\theta}$ =constant, which yields [57]

$$C = B^n \tag{31}$$

where $n \neq 1$ is a positive constant
(ii) We used power law relation between the scalar field (φ) and the average scale factor $a(t)$ to reduce the complexity of above field eqs. [17-21], where several authors have studied to construct the cosmological model of the scalar field.

$$3\frac{\dot{A}}{A} = -2\frac{\dot{\varphi}}{\varphi} \tag{32}$$

From Eqs. (22), (30) - (32), we obtained that the scalar field reduces to

$$\varphi = \frac{2Mexp^{Mt}}{\varphi_0 exp^{2Mt} - \varphi_1} \tag{33}$$

From Eqs. (24), (31) - (33), we obtained the dynamical parameters, which shows the physical nature of the model are as

$$A = \left[\frac{\varphi_0 e^{2Mt} - \varphi_1}{2Me^{Mt}}\right]^{\frac{2}{3}}, \quad B = \left[\frac{\varphi_0 e^{2Mt} - \varphi_1}{2Me^{Mt}}\right]^{\frac{4}{3(n+1)}}, \quad C = \left[\frac{\varphi_0 e^{2Mt} - \varphi_1}{2Me^{Mt}}\right]^{\frac{4n}{3(n+1)}} \tag{34}$$

where φ_0, φ_1 are constant of integration. using Eq. (34) in the metric (1), it reduces to

$$ds^2 = -dt^2 + \left[\frac{\varphi_0 e^{2Mt} - \varphi_1}{2Me^{Mt}}\right]^{\frac{2}{3}} dx^2 + \left[\frac{\varphi_0 e^{2Mt} - \varphi_1}{2Me^{Mt}}\right]^{\frac{8}{3(n+1)}} dy^2 + \left[\frac{\varphi_0 e^{2Mt} - \varphi_1}{2Me^{Mt}}\right]^{\frac{4n}{3(n+1)}} dz^2 \tag{35}$$

Physical and dynamical parameters of the model

The following are the dynamical parameters of the model, which are necessary for discussion in cosmology:

$$V = \left[\frac{\varphi_0 e^{2Mt} - \varphi_1}{2Me^{Mt}}\right]^2, H = \frac{2M}{3}\left[\frac{\varphi_0 e^{2Mt} + \varphi_1}{\varphi_0 e^{2Mt} - \varphi_1}\right], \ \theta = 2M\left[\frac{\varphi_0 e^{2Mt} + \varphi_1}{\varphi_0 e^{2Mt} - \varphi_1}\right] \tag{36}$$

and $$\sigma^2 = \frac{4M^2}{9}\left(\frac{n-1}{n+1}\right)^2 \left[\frac{\varphi_0 e^{2Mt} + \varphi_1}{\varphi_0 e^{2Mt} - \varphi_1}\right]^2, \ \Delta = \frac{2}{3}\left(\frac{n-1}{n+1}\right)^2 \tag{37}$$

The deceleration parameter is obtained as

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right) = -1 + \frac{6\varphi_0\varphi_1 e^{2Mt}}{(\varphi_0 e^{2Mt} + \varphi_1)^2} \tag{38}$$

Now, from Eq. (20), we obtained as

$$\alpha\rho = 3H^2 - \sigma^2 - \frac{3M^2}{A^2} - \frac{3}{4}\beta^2 - \frac{\dot{\varphi}^2}{2} + \frac{M^2\varphi^2}{2} - \left(\frac{\rho - p}{2}\right) \tag{39}$$

Adding Eqs. (17) - (19), it becomes

$$\alpha p = H^2(2q-1) - \sigma^2 + \frac{m^2}{A^2} - \frac{3}{4}\beta^2 + \left(\frac{\rho - p}{2}\right) - \frac{\dot{\varphi}^2}{2} - \frac{M^2\varphi^2}{2} \quad (40)$$

Applying Eq. (39) in Eq. (40), we get

$$K_1\rho = -\frac{8}{3}M^2\left(\frac{\varphi_0 e^{2Mt}+\varphi_1}{\varphi_0 e^{2Mt}-\varphi_1}\right)^2 + \frac{48}{9}M^2\frac{\varphi_0\varphi_1 e^{2Mt}}{(\varphi_0 e^{2Mt}-\varphi_1)^2} + 4m^2\left(\frac{2Me^{Mt}}{\varphi_0 e^{2Mt}-\varphi_1}\right)^{\frac{4}{3}} - 4M^2\frac{e^{2Mt}}{(\varphi_0 e^{2Mt}-\varphi_1)^2} \quad (41)$$

where $K_1 = (\alpha\gamma - \alpha - 1 + \gamma)$ is constant.

Apart from that, we assumed the relation $p = \gamma\rho$ and using this condition in the above Eq. (39), the pressure of the model is obtained as

$$p = \frac{\gamma}{K_1}\left[-\frac{8}{3}M^2\left(\frac{\varphi_0 e^{2Mt}+\varphi_1}{\varphi_0 e^{2Mt}-\varphi_1}\right)^2 + \frac{48}{9}M^2\frac{\varphi_0\varphi_1 e^{2Mt}}{(\varphi_0 e^{2Mt}-\varphi_1)^2} + 4m^2\left(\frac{2Me^{Mt}}{\varphi_0 e^{2Mt}-\varphi_1}\right)^{\frac{4}{3}} - 4M^2\frac{e^{2Mt}}{(\varphi_0 e^{2Mt}-\varphi_1)^2}\right] \quad (42)$$

Now from (39), the displacement vector field is obtained as

$$(1-\gamma)\frac{3}{4}\beta^2 = -\frac{4}{3}M^2\left(\frac{\varphi_0 e^{2Mt}+\varphi_1}{\varphi_0 e^{2Mt}-\varphi_1}\right)^2 + \frac{16}{3}M^2\frac{\varphi_0\varphi_1 e^{2Mt}}{(\varphi_0 e^{2Mt}-\varphi_1)^2} - \frac{4}{3}\gamma M^2\left(\frac{\varphi_0 e^{2Mt}+\varphi_1}{\varphi_0 e^{2Mt}-\varphi_1}\right)^2 -$$
$$\frac{4}{9}M^2(1-\gamma)\left(\frac{n-1}{n+1}\right)^2\left(\frac{\varphi_0 e^{2Mt}+\varphi_1}{\varphi_0 e^{2Mt}-\varphi_1}\right)^2 + (1+3\gamma)m^2\left(\frac{2Me^{Mt}}{\varphi_0 e^{2Mt}-\varphi_1}\right)^{\frac{4}{3}} + (\gamma-1)\frac{\varphi_0^2 M^4 e^{6Mt}}{(\varphi_0 e^{2Mt}-\varphi_1)^4} +$$
$$(\gamma-1)\frac{M^4\varphi_1^2 e^{2Mt}}{(\varphi_0 e^{2Mt}-\varphi_1)^4} + (\gamma-1)\frac{2\varphi_0\varphi_2 M^4 e^{4Mt}}{(\varphi_0 e^{2Mt}-\varphi_1)^4} - 2(\gamma+1)\frac{M^4 e^{2Mt}}{(\varphi_0 e^{2Mt}-\varphi_1)^2} -$$
$$\frac{4}{3}\frac{(1+\gamma)(1-\gamma)M^2}{K_1}\left(\frac{\varphi_0 e^{2Mt}+\varphi_1}{\varphi_0 e^{2Mt}-\varphi_1}\right)^2$$
$$\frac{24}{9}\frac{(1+\gamma)(1-\gamma)M^2}{K_1}\frac{\varphi_0\varphi_1 e^{2Mt}}{(\varphi_0 e^{2Mt}-\varphi_1)^2} + 2m^2\frac{(1+\gamma)(1-\gamma)}{K_1}\left(\frac{2Me^{Mt}}{\varphi_0 e^{2Mt}-\varphi_1}\right)^{\frac{4}{3}} - 2M^2\frac{(1+\gamma)(1-\gamma)}{K_1}\frac{e^{Mt}}{(\varphi_0 e^{2Mt}-\varphi_1)^2} \quad (43)$$

We obtained the Trace of the model $(T = \rho - 3p)$ from Eqs. (41) and (42) as

$$T = \left(\frac{1-3\gamma}{K_1}\right)\left[-\frac{8}{3}M^2\left(\frac{\varphi_0 e^{2Mt}+\varphi_1}{\varphi_0 e^{2Mt}-\varphi_1}\right)^2 + \frac{48}{9}M^2\frac{\varphi_0\varphi_1 e^{2Mt}}{(\varphi_0 e^{2Mt}-\varphi_1)^2} + 4m^2\left(\frac{2Me^{Mt}}{\varphi_0 e^{2Mt}-\varphi_1}\right)^{\frac{4}{3}} - 4M^2\frac{e^{2Mt}}{(\varphi_0 e^{2Mt}-\varphi_1)^2}\right] \quad (44)$$

Subsequently, the Riemannian curvature scalar is obtained as

$$R = \frac{6m^2}{A^2} - 2\left(\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C}\right) - 2\left(\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C}\right)$$
$$= 6m^2\left(\frac{2Me^{Mt}}{\varphi_0 e^{2Mt}-\varphi_1}\right)^{\frac{4}{3}} + \frac{16}{9}\left(\frac{n^2-14n+1}{(n+1)^2}\right)\left(\frac{M\varphi_0 e^{2Mt}}{\varphi_0 e^{2Mt}-\varphi_1}\right)^2 + \frac{64}{9}\left(\frac{n^2+4n+1}{(n+1)^2}\right)\left(\frac{M^2\varphi_0 e^{2Mt}}{\varphi_0 e^{2Mt}-\varphi_1}\right) \quad (45)$$

Also, the statefinder parameter of the model is obtained as

$$r = 1 - \frac{18\varphi_0^2\varphi_1 e^{2Mt}}{(\varphi_0 e^{2Mt}+\varphi_1)^3} \quad (46)$$

$$s = \frac{2\varphi_0^2\varphi_1 e^{2Mt}}{(\varphi_0 e^{2Mt}-\varphi_1)^2(\varphi_0 e^{2Mt}+\varphi_1)} \quad (47)$$

Discussion of the behavior of the $f(\tilde{R}, T)$ model

The main behavior of the model is obtained from the above equations, with the proper choice of the values of the constants, which are the suitable values of this model as we found in the frame work of Lyra Geometry in $f(\tilde{R}, T)$ gravity model.

It is observed from the Eq. (36) that the spatial volume does not zero at $t = 0$, while as it diverges, which shows that in this $f(\tilde{R}, T)$ model the universe does not starts evolving with zero volume at the initial epoch i.e., the universe starts with an infinite rate of expansion. Also, the spatial volume does not tend to zero at the initial epoch, so that the model is free from initial singularity at this initial epoch. Subsequently, the Hubble's parameter (H), Expansion Scalar (θ) and Scalar field (φ) are also decreases when $t \to \infty$. From the **Figure 1**, we observed that the scalar field is finally decreasing function of cosmic time t and this result represents the corresponding kinetic energy increases.

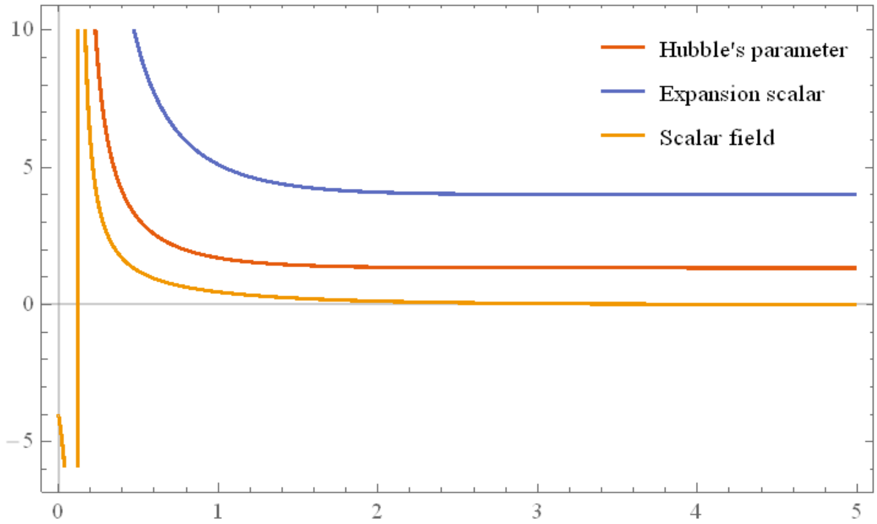

Figure 1 H, θ and φ vs. time (t).

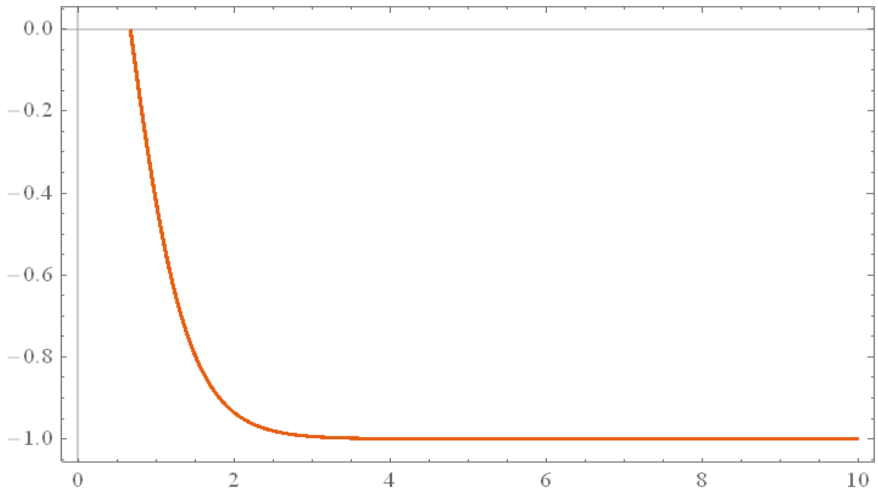

Figure 2 q vs. time (t).

- Here in this model as Eq. (38) and **Figure 2**, we found that the deceleration parameter always lies in the range –1 to 0, and this result indicates that the deceleration parameter satisfies observational data like Ia Supernova [58, 59], (CMB) [60,61], BAO [62,63] and thus for this the present $f(\tilde{R}, T)$ model expands at an accelerated rate.
- Further from Eqs. (36) and (37) we found that $\frac{\sigma^2}{\theta^2} \neq 0$ and this condition shows that this model does not approach isotropy with the cosmic time t with current observations.
- From the **Figure 3**, we found that the behavior of the model of pressure vs. cosmic time t. It is clear from that by the choice of the values $\varphi_0 = 3, \varphi_1 = 4, \alpha = 1, M = 1.2, m = 2, n = 3, \gamma = -1.5$, the

pressure remains negative throughout the evolution of time and this condition satisfy the present observational data that there is dark energy in this $f(\tilde{R},T)$ gravity model. While, we observed that the density of the model decreases with the increases in time and it has finally a minimum positive value.

- It is observed from the **Figure 4** that the displacement vector (β^2) gradually decreases in the beginning but increases with the evolution of cosmic time t.
- From Eq. (33) it is found that the scalar field (φ) and the kinetic energy (φ^2) both are non singular, as this model does not tend to zero with the evolution of time.

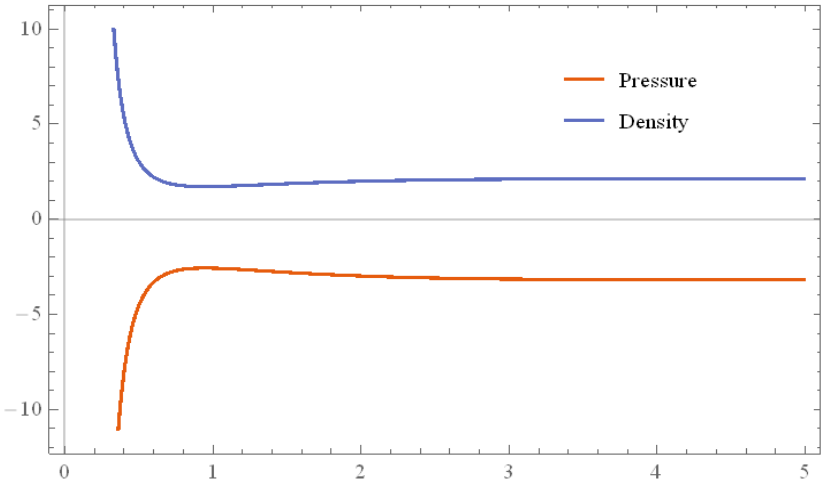

Figure 3 p and ρ vs. time (t).

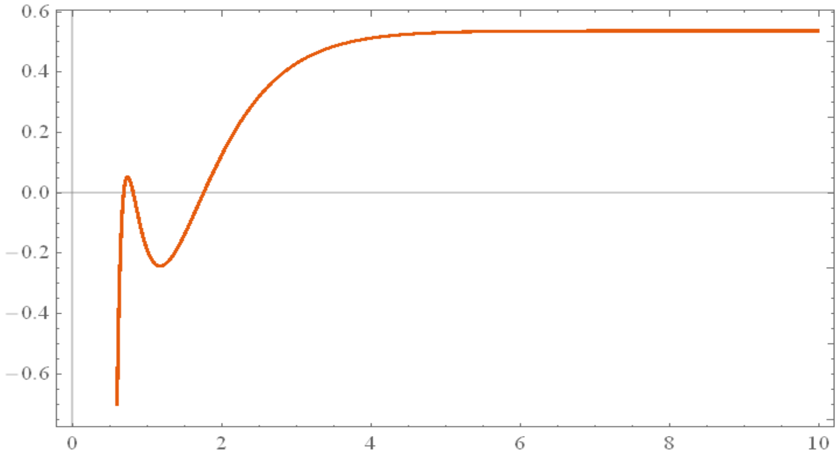

Figure 4 β^2 vs. time (t).

- It is clear from the **Figure 5** that the Riemannian curvature R and the trace of the model T respectively decreases with the evolution of time and finally it has constant negative value; however, trace of the model gradually increases, which have a constant positive value with cosmic time t in our accelerated expansion of the universe in $f(\tilde{R},T)$ model in the frame work of Lyra geometry.
- **Figure 6** represents that the nature of the statefinder parameters r and s, which does not tend to 0 and 1, and this suggest us that $f(\tilde{R},T)$ gravity in this case does not satisfy the ΛCDM model.

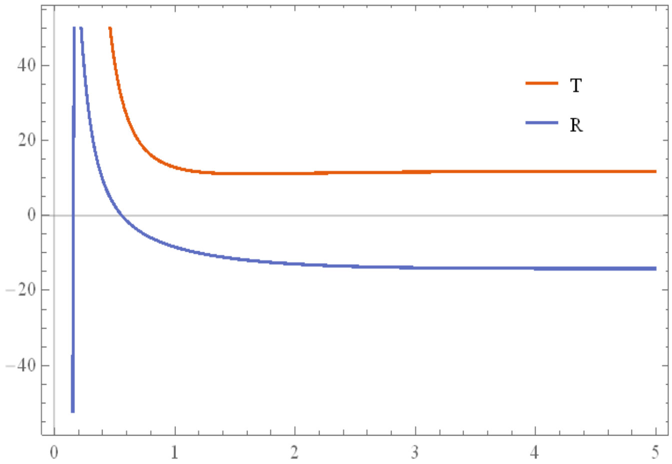

Figure 5 R and T vs. time t.

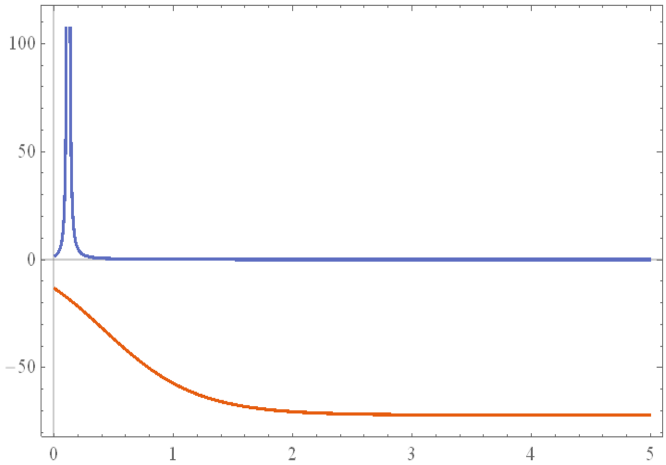

Figure 6 r and s vs. time t.

Conclusions

In this work we have studied Bianchi Type-V Modified Lyra Geometry based $f(R,T)$ Gravity Model with Massive scalar field. The Eq. (35) indicates the spatially homogenous and anisotropic cosmological model with an attractive massive scalar field. The model is nonsingular which starts with a finite volume. The displacement vector field β^2 in Eq. (43) is gradually increases from small negative value to positive value at initial time and it reaches a positive value for $\gamma = -1.5$. This model found is an expanding with acceleration. A significant role played by $f(R,T)$ gravity along with displacement vector field (β), massive scalar field (φ) can be seen.

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