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Fuzzy Bayesian Estimation of Linear (Circular) Consecutive k-out-of-n: F System Reliability

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Abstract

The consecutive k-out-of-n: F structure is broadly applicable in industrial and military frameworks. Without lifetime information about the entire design, it is appealing to use fuzzy earlier beliefs on its segments. In this paper the fuzzy Bayesian reliability assessment of the linear (circular) consecutive k-out-of-n: F system is proposed under the squared error loss function. The parameters are known as fuzzy random variables in the process of obtaining fuzzy Bayesian reliability. The conventional strategy for estimating Bayesian reliability will be utilized to construct the fuzzy Bayesian point estimator of the proposed system by applying the Resolution Identity theorem. A comparative study of the derived result with the literature is presented.

Keywords: Fuzzy Bayesian reliability, Consecutive k-out-of-n:F system, Squared error loss function, Gamma distribution, Confidence interval

Introduction

The lifetime and failure of a component cannot be accurately calculated. We may simply say that there are "around s" failures over the lifetime of "around t" time units when m items are put on the test for this circumstance. The failure rate will thus be referred to as "around s/t". It can represent the expression "around s/t" by using a fuzzy set.

Assuming the component's failure time having an exponential distribution, the component reliability will simply say "around" $r = e^{-\mu t}$ at a time t. That means the reliability of the component is also seen as a fuzzy real number. Even, under certain unexpected incidents, the lifetimes of objects now and then can certainly not be measured. However, the lifetimes of objects can be seen as fuzzy random variables. In this paper, from the fuzzy variable, we will estimate the fuzzy Bayesian reliability of the cons/k-n:F system.

Zadeh [1] introduced the concept of fuzzy sets and established various properties related to them. And also applied the resolution identity theorem which states that for a fuzzy sub-set $\tilde{a}$ of $\mathbb{R}$,

$$\xi_{\tilde{a}}(x) = \sup_{\alpha \in [0,1]} \alpha \cdot 1_{\tilde{a}_\alpha}(x)$$

Where $\xi_{\tilde{a}}$ is membership function and 1_A is a characteristic function of set A.

Zadeh [2] defined and demonstrated some of the elementary concepts of probability theory in which fuzzy events are allowed. Utkin [3] described the general approach of representing information as fuzzy sets. Madan and Dan [4] defined the concepts of the fuzzy random variable. He derived some properties based on these defined concepts. Almond [5] compared the fuzzy and probability approaches to understand their strengths and weaknesses.

Peköz and Ross [6] derived the exact reliability formula for linear and circular cons-k-out-of-n:F systems with the same component reliability. Lambiris and Papastavridis [7] derived 2 different exact reliability formulas for cons/k-n:Fsystem. To replace the expensive experimental process, Rezaeianjouybari *et al.* [8] used the Bayesian optimization method in nanorefrigerant. Sheikholeslami *et al.* [9] reviewed flat plate solar collector system, photovoltaic systems progress, and their recent development. Sheikholeslami and Farshad [10] investigated the thermal efficiency of the solar collector system.

To analyze fuzzy cons/k-n:Fsystem reliability, Cheng [11] proposed a new method using fuzzy graphical evaluation and review technique, and the triangular fuzzy number has been used to fuzzify the probabilities. Elghamry *et al.* [12] obtained the reliability model of fuzzy cons/k-n:F system with independent, un-repairable, and non-identical components under the assumption that the component lifetime follows the Rayleigh distribution. Sharma and Krishna [13] estimated the Bayesian reliability measure of a k-out-of-m system and its confidence interval. Madhumitha and Vijayalakshmi [14] obtained the exact equation of Bayesian system reliability for the cons/k-n:F system and its confidence interval.

Karen *et al.* [15] presented an algorithm to compute the posterior probability by incorporating fuzzy-set theory into Baye's theorem. Tavassoli *et al.* [16] proposed a fuzzy differential equation for reliability and the concept of fuzzy mean time to failure. Li *et al.* [17] introduced a novel method based on fuzzy probability and he applied the proposed concepts to analyze the reliability measure of a series system, parallel system, and compound system. Wu [18] developed the construction of fuzzy estimators of fuzzy parameters using the Resolution identity theorem and also the membership values of the fuzzy estimates were evaluated. Wu [19] proposed the B.P.E under fuzzy concepts. Using fuzzy prior distributions he applied the conventional Bayes estimation procedure to fuzzy random variables. Görkemli and Ulusoy [20] determined the fuzzy Bayesian reliability and availability measures of production systems using a new modeling and analysis approach. Under the assumption of 2-parameter exponential distribution as prior, Gholizadeh *et al.* [21] obtained the fuzzy Bayesian estimation under different loss functions. For any given B.P.E, he evaluated the membership degree. Kabir and Papadopoulos [22] presented a review on fuzzy sets and their applications to safety and reliability engineering.

This paper is organized as follows. In section 2, the background details are explained with the help of various definitions and theorem statements. The calculation of fuzzy Bayes point estimate of L(cons/k-n:F) system and C(cons/k-n:F) system are derived in section 3. In section 4, Numerical computations are illustrated, a comparison has been made with Wu *et al.* [19] and Gholizadeh *et al.* [21], and the results are summarized and concluded in section 5.

Background

Notations

r– common component reliability
μ – failure time
t – mission time
T_i – $i$failures up to time $T_i, i = 1, 2, \ldots,$
n– total number of components in the proposed system
k– number of failing components
m– number of components tested (under experiment)
v – total time on testing m components
s – number of failures upto time$v$
B.P.E – Bayesian point estimate
SELF – squared error loss function
Prior belief :βnumber of failures in θtime
cons/k-n:F – Consecutive k-out-of-n: F system
L(cons/k-n:F) – Linear cons/k-n:F system
C(cons/k-n:F) – Circular cons/k-n:F system
M – random variable of μ
$\hat{r}$–B.P.E of L(cons/k-n:F) system
$\hat{r_c}$–B.P.E of C(cons/k-n:F) system
MTTF – Mean time to failure
$\widetilde{M^*}$ - fuzzyBayesian estimate of MTTF for L(cons/k-n:F)
$\widetilde{M_c^*}$ - fuzzy Bayesian estimate of MTTF for C(cons/k-n:F)

Assumptions

cons/k-n:F system permits higher fault tolerance ability and reliability.

There are n working homogeneous components.
The links and connections are assumed to be perfect.
The component is either working or in a failure state.
The components are mutually independent and identically distributed.
The failure time of the component is exponentially distributed with parameter μ
The system fails when a minimum of k consecutive components fail where k ≤ n

For a mission time t, the reliability is $r(t) = e^{-\mu t}$
Let the total time on testing m units be $V = \sum_{j=1}^{s} T_j + (m-s)T_s$
Where $T_1 \leq T_2 \leq \cdots \leq T_s$– ordered first s failure times and $T_j, (j = 1, 2, \dots, s)$ – independent and identical exponential distributed with the unknown $\mu$and v – the observed values of V

Bayesian approach

Assume that the prior distribution of $M$is gamma (β, θ) with p.d.f,

$$\pi_M(\mu) = \frac{\theta^\beta \mu^{\beta-1} e^{-\mu\theta}}{\Gamma(\beta)} \tag{1}$$

where θ – scale parameter, β – shape parameter

Then the posterior distribution of μgiven s failures upto time v is

$$\pi_M(\mu/s, v) = \frac{f(s/\mu)\pi_M(\mu)}{\int_0^\infty f(s/\mu)\pi_M(\mu)dy} \tag{2}$$

$$\pi_M(\mu/s, v) = \frac{(v+\theta)^{s+\beta}\mu^{s+\beta-1}e^{-(v+\theta)\mu}}{\Gamma(s+\beta)} \tag{3}$$

Which is a gamma distribution $g(s+\beta, v+\theta)$. Since the relation between r and μ is one-to-one, there exists the unique inverse $\mu = \frac{-\ln(r)}{t}$ for $0 < r < 1, t > 0$

The prior distribution of $\bar{R} = \bar{R}(t)$for fixed t is given by

$$\pi_{\bar{R}}(\bar{r}) = \pi_M\left(\frac{-\ln(r)}{t}\right)\left|\frac{d\mu}{d\bar{r}}\right| \tag{4}$$

$$\pi_{\bar{R}}(\bar{r}) = \pi_M\left(\frac{-\ln(r)}{t}\right)\left(\frac{1}{\bar{r}t}\right) \tag{5}$$

$$\pi_{\bar{R}}(\bar{r}) = \frac{\left(\frac{\theta}{t}\right)^\beta \bar{r}^{\left(\frac{\theta}{t}\right)-1}\left(-\ln(\bar{r})\right)}{\Gamma(\beta)} \tag{6}$$

The posterior distribution of $\bar{R}$is given by a negative-log-gamma distribution

$$\pi_{\bar{R}}(\bar{r}\,/s, v) = \frac{\left(\frac{(v+\theta)}{t}\right)^{s+\beta} \bar{r}^{\left(\frac{(v+\theta)}{t}\right)-1}\left(-\ln(\bar{r})\right)^{s+\beta-1}}{\Gamma(s+\beta)} \tag{7}$$

The Mellin transform of the posterior of $\bar{R}$ is

$$Me(\pi_{\bar{R}}(\bar{r}\,/s,v);u) = \left[\frac{v+\theta}{v+\theta+(u-1)t}\right]^{s+\beta} \tag{8}$$

Where $Re(u) > \left\{1 - \frac{(v+\theta)}{t}\right\}$

Resolution Identity: (Zadeh, 1965)
Let $\tilde{a}$ be a fuzzy sub-set of $\mathbb{R}$ with membership function $\xi_{\tilde{a}}$.

Then $\xi_{\tilde{a}}(x) = \underset{\alpha\,\in\,[0,1]}{sup}\ \alpha\,.\,1_{\tilde{a}_\alpha}(x)$ where a characteristic function of set A is 1_A.

Let $N = \left\lfloor\frac{n}{k+1}\right\rfloor; N1 = \left\lfloor\frac{n}{k+1} - 1\right\rfloor; N2 = n - lk; N3 = n - lk - k; N4 = lk;$
$N5 = lk + k; N6 = n - lk - k - 1$

Fuzzy Bayesian Point Estimation
Fuzzy B.P.E for L(cons/k-n:F)
For a L(cons/k-n:F) system, the system reliability is

$$R = \sum_{l=0}^{N} C_{N2}^{l}\,(-1)^l \bar{R}^l \bar{Q}^{N4} - \sum_{l=0}^{N} C_{N3}^{l}\,(-1)^l \bar{R}^l \bar{Q}^{N5} \tag{9}$$

Where $\bar{R}$- component reliability and $\bar{Q} = 1 - \bar{R}$

The B.P.E of the system reliability R under a SELF is

$$\hat{r} = E[R\,/s,v] = Me(\pi_R(\bar{r}\,/s,v);u=2) \tag{10}$$

$$\hat{r} = \sum_{l=0}^{N} C_{N2}^{l}\,(-1)^l \int_0^1 \bar{R}^l \bar{Q}^{N4} \pi_R(\bar{r}\,/s,v)\,d\bar{r} - \sum_{l=0}^{N} C_{N3}^{l}\,(-1)^l \int_0^1 \bar{R}^l \bar{Q}^{N5} \pi_R(\bar{r}\,/s,v)\,d\bar{r} \tag{11}$$

$$\hat{r} = \sum_{l=0}^{N}\sum_{i=0}^{N4} C_{N2}^{l} C_{N4}^{i}\,(-1)^{(l+i)} \left[\frac{v+\theta}{v+\theta+tl+ti}\right]^{s+\beta} - \sum_{l=0}^{N}\sum_{i=0}^{N5} C_{N3}^{l} C_{N5}^{i}\,(-1)^{(l+i)} \left[\frac{v+\theta}{v+\theta+tl+ti}\right]^{s+\beta} \tag{12}$$

Based on our earlier assumptions related to fuzzy,

For all$\alpha_1 \in [0,1]$, the B.P.Es of$\widehat{\tilde{r}_{\alpha_1}^{L}}$ and$\widehat{\tilde{r}_{\alpha_1}^{U}}$

$$\widehat{\tilde{r}_{\alpha_1}^{L}} = \sum_{l=0}^{N}\sum_{i=0}^{N4} C_{N2}^{l} C_{N4}^{i}\,(-1)^{(l+i)} \left[\frac{\tilde{v}_{(\alpha_1)}^{L} + \tilde{\theta}_{(\alpha_1)}^{L}}{\tilde{v}_{(\alpha_1)}^{L} + \tilde{\theta}_{(\alpha_1)}^{L} + tl + ti}\right]^{s+\tilde{\beta}_{(\alpha_1)}^{L}} - \sum_{l=0}^{N}\sum_{i=0}^{N5} C_{N3}^{l} C_{N5}^{i}\,(-1)^{(l+i)} \left[\frac{\tilde{v}_{(\alpha_1)}^{L} + \tilde{\theta}_{(\alpha_1)}^{L}}{\tilde{v}_{(\alpha_1)}^{L} + \tilde{\theta}_{(\alpha_1)}^{L} + tl + ti}\right]^{s+\tilde{\beta}_{(\alpha_1)}^{L}} \tag{13}$$

$$\widehat{\tilde{r}^{U}_{\alpha_1}} = \sum_{l=0}^{N}\sum_{i=0}^{N4} C_{N2}^{l} C_{N4}^{i} (-1)^{(l+i)} \left[\frac{\tilde{v}^{U}_{(\alpha_1)} + \tilde{\theta}^{U}_{(\alpha_1)}}{\tilde{v}^{U}_{(\alpha_1)} + \tilde{\theta}^{U}_{(\alpha_1)} + tl + ti}\right]^{s+\tilde{\beta}^{U}_{(\alpha_1)}} - \sum_{l=0}^{N}\sum_{i=0}^{N5} C_{N3}^{l} C_{N5}^{i} (-1)^{(l+i)} \left[\frac{\tilde{v}^{U}_{(\alpha_1)} + \tilde{\theta}^{U}_{(\alpha_1)}}{\tilde{v}^{U}_{(\alpha_1)} + \tilde{\theta}^{U}_{(\alpha_1)} + tl + ti}\right]^{s+\tilde{\beta}^{U}_{(\alpha_1)}} \tag{14}$$

Where

$\tilde{v}^{L}_{(\alpha_1)} = \sum_{j=1}^{s} (\tilde{t}_j)^{L}_{(\alpha_1)} + (m-s)(\tilde{t}_s)^{L}_{(\alpha_1)}$ and $\tilde{v}^{U}_{(\alpha_1)} = \sum_{j=1}^{s} (\tilde{t}_j)^{U}_{(\alpha_1)} + (m-s)(\tilde{t}_s)^{U}_{(\alpha_1)}$

Let $A_{(\alpha_1)} = \left[min\left\{\underset{\alpha_1 \leq b \leq 1}{min} \widetilde{r_b}^{L}, \underset{\alpha_1 \leq b \leq 1}{min} \widetilde{r_b}^{U}\right\}, max\left\{\underset{\alpha_1 \leq b \leq 1}{max} \widetilde{r_b}^{L}, \underset{\alpha_1 \leq b \leq 1}{max} \widetilde{r_b}^{U}\right\}\right]$

Then $A_{(\alpha_1)}$contain all B.P.Es for all $r \in \left[\tilde{r}^{L}_{(\alpha_1)}, \tilde{r}^{U}_{(\alpha_1)}\right]$
The membership function of the fuzzy B.P.E of $\tilde{r}$ is at time t,

$\xi_{\hat{\tilde{r}}} = \underset{0 \leq \alpha_1 \leq 1}{sup} (\alpha_1) \,.\, 1_{A_{(\alpha_1)}}(r)$, via the form of resolution identity.

According to the same assumptions mentioned in Bayesian Approach, the posterior distribution of $\bar{R}_c$ is

$$\pi_{\bar{R}_c}(\bar{r}_c/s, v) = \frac{\left(\frac{(v+\theta)}{t}\right)^{s+\beta} \bar{r}_c^{\left(\frac{(v+\theta)}{t}\right)-1} \left(-\ln(\bar{r}_c)\right)^{s+\beta-1}}{\Gamma(s+\beta)} \tag{15}$$

Fuzzy B.P.E for L(cons/k-n:F)
For a C(cons/k-n:F) system, the system reliability is

$$R_c = \sum_{l=0}^{N} C_{N2}^{l} (-1)^{l} \bar{R}_c{}^{l} \bar{Q}_c{}^{N4} - k \sum_{l=0}^{N1} C_{N6}^{l} (-1)^{l+1} \bar{R}_c{}^{l} \bar{Q}_c{}^{N5} - [\bar{Q}_c]^{n} \tag{16}$$

Where $\bar{R}_c$ - component reliability and $\bar{Q}_c = 1 - \bar{R}_c$

The B.P.E of $\bar{r}_c$ under a SELF is

$$\widehat{r_c} = E[R_c/s, v] = Me\left(\pi_{R_c}(\overline{r_c}/s, v); u = 2\right) \tag{17}$$

$$\widehat{r_c} = \sum_{l=0}^{N} C_{N2}^{l} (-1)^{l} \int_{0}^{1} \bar{R}^{l} \bar{Q}^{N2} \pi_{\bar{R}}(\overline{r_c}/s, v)\, d\overline{r_c} - k \sum_{l=0}^{N1} C_{N6}^{l} (-1)^{l+1} \int_{0}^{1} \bar{R}^{l} \bar{Q}^{N5} \pi_{\bar{R}}(\overline{r_c}/s, v)\, d\overline{r_c} - \int_{0}^{1} \bar{Q}^{n} \pi_{\bar{R}}(\overline{r_c}/s, v)\, d\overline{r_c} \tag{18}$$

$$\widehat{r_c} = \sum_{l=0}^{N}\sum_{i=0}^{N4} C_{N2}^{l} C_{N4}^{i} (-1)^{(l+i)} \left[\frac{v+\theta}{v+\theta+tl+ti}\right]^{s+\beta} - k\sum_{l=0}^{N1}\sum_{i=0}^{N5} C_{N6}^{l} C_{N5}^{i} (-1)^{(l+i)} \left[\frac{v+\theta}{v+\theta+tl+ti}\right]^{s+\beta} - \sum_{i=0}^{n} C_n^i (-1)^i \left[\frac{v+\theta}{v+\theta+ti}\right]^{s+\beta} \tag{19}$$

Based on our earlier assumptions related to fuzzy,

For all $\alpha_2 \in [0,1]$, the B.P.Es of $\widehat{\tilde{r}_{\alpha_2}^{L}}$ and $\widehat{\tilde{r}_{\alpha_2}^{U}}$

$$\widehat{\tilde{r}_{\alpha_2}^{L}} = \sum_{l=0}^{N}\sum_{i=0}^{N4} C_{N2}^{l} C_{N4}^{i} (-1)^{(l+i)} \left[\frac{\tilde{v}_{(\alpha_2)}^{L}+\tilde{\theta}_{(\alpha_2)}^{L}}{\tilde{v}_{(\alpha_2)}^{L}+\tilde{\theta}_{(\alpha_2)}^{L}+tl+ti}\right]^{s+\tilde{\beta}_{(\alpha_2)}^{L}} - k\sum_{l=0}^{N1}\sum_{i=0}^{N5} C_{N6}^{l} C_{N5}^{i} (-1)^{(l+i)} \left[\frac{\tilde{v}_{(\alpha_2)}^{L}+\tilde{\theta}_{(\alpha_2)}^{L}}{\tilde{v}_{(\alpha_2)}^{L}+\tilde{\theta}_{(\alpha_2)}^{L}+tl+t(i+1)}\right]^{s+\tilde{\beta}_{(\alpha_2)}^{L}} - \sum_{i=0}^{n} C_n^i (-1)^i \left[\frac{\tilde{v}_{(\alpha_2)}^{L}+\tilde{\theta}_{(\alpha_2)}^{L}}{\tilde{v}_{(\alpha_2)}^{L}+\tilde{\theta}_{(\alpha_2)}^{L}+ti}\right]^{s+\tilde{\beta}_{(\alpha_2)}^{L}} \tag{20}$$

$$\widehat{\tilde{r}_{\alpha_2}^{U}} = \sum_{l=0}^{N}\sum_{i=0}^{N4} C_{N2}^{l} C_{N4}^{i} (-1)^{(l+i)} \left[\frac{\tilde{v}_{(\alpha_2)}^{U}+\tilde{\theta}_{(\alpha_2)}^{U}}{\tilde{v}_{(\alpha_2)}^{U}+\tilde{\theta}_{(\alpha_2)}^{U}+tl+ti}\right]^{s+\tilde{\beta}_{(\alpha_2)}^{U}} - k\sum_{l=0}^{N1}\sum_{i=0}^{N5} C_{N6}^{l} C_{N5}^{i} (-1)^{(l+i)} \left[\frac{\tilde{v}_{(\alpha_2)}^{U}+\tilde{\theta}_{(\alpha_2)}^{U}}{\tilde{v}_{(\alpha_2)}^{U}+\tilde{\theta}_{(\alpha_2)}^{U}+tl+t(i+1)}\right]^{s+\tilde{\beta}_{(\alpha_2)}^{U}} - \sum_{i=0}^{n} C_n^i (-1)^i \left[\frac{\tilde{v}_{(\alpha_2)}^{U}+\tilde{\theta}_{(\alpha_2)}^{U}}{\tilde{v}_{(\alpha_2)}^{U}+\tilde{\theta}_{(\alpha_2)}^{U}+ti}\right]^{s+\tilde{\beta}_{(\alpha_2)}^{U}} \tag{21}$$

where $\tilde{v}_{(\alpha_2)}^{L} = \sum_{j=1}^{s}\left(\tilde{t}_j\right)_{(\alpha_2)}^{L} + (m-s)(\tilde{t}_s)_{(\alpha_2)}^{L}$

and $\tilde{v}_{(\alpha_2)}^{U} = \sum_{j=1}^{s}\left(\tilde{t}_j\right)_{(\alpha_2)}^{U} + (m-s)(\tilde{t}_s)_{(\alpha_2)}^{U}$

Let $A_{(\alpha_2)} = \left[min\left\{\underset{\alpha_2 \le b \le 1}{min}\tilde{r_b}^{L}, \underset{\alpha_2 \le b \le 1}{min}\tilde{r_b}^{U}\right\}, max\left\{\underset{\alpha_2 \le b \le 1}{max}\tilde{r_b}^{L}, \underset{\alpha_2 \le b \le 1}{max}\tilde{r_b}^{U}\right\}\right]$

Then $A_{(\alpha_2)}$contain all B.P.Es for all $r_c \in \left[\tilde{r}_{(\alpha_2)}^{L}, \tilde{r}_{(\alpha_2)}^{U}\right]$

At time t, the membership function of the fuzzy B.P.E of $\tilde{r_c}$ is

$\xi_{\widehat{r_c}}(r_c) = \underset{0 \le \alpha_2 \le 1}{sup}(\alpha_2) \,.\, 1_{A_{(\alpha_2)}}(r_c)$, via the form of resolution identity.

Computational techniques

We use the following terms in the method of finding the membership degree of the fuzzy B.P.E as defined in the above equations,

$A_\alpha = [f(\alpha), g(\alpha)] = [min\{f_1(\alpha), f_2(\alpha)\}, min\{g_1(\alpha), g_2(\alpha)\}]$

where $f_1(\alpha) = \inf_{\alpha \le b \le 1} \widehat{\tilde{\delta}_b^L}, f_2(\alpha) = \inf_{\alpha \le b \le 1} \widehat{\tilde{\delta}_b^U}$

and $g_1(\alpha) = \sup_{\alpha \le b \le 1} \widehat{\tilde{\delta}_b^L},\ g_2(\alpha) = \sup_{\alpha \le b \le 1} \widehat{\tilde{\delta}_b^U}$

The membership function of the fuzzy B.P.E $\hat{\tilde{\delta}}$ is

$$\xi_{\hat{\tilde{\delta}}} = \sup_{0 \le \alpha \le 1} \alpha \,.\, 1_{A_\alpha}(r) = \ sup\{\alpha : f(\alpha) \le r \le g(\alpha), 0 \le \alpha \le 1\}$$

Therefore, we need to solve the following non-linear programming problem (NLPP):

$$\begin{array}{ll} max & \alpha_1 \\ subject\ to & \max\{g_1(\alpha), g_2(\alpha)\} \ge r \\ & \min\{f_1(\alpha), f_2(\alpha)\} \le r \\ & \alpha \ge 0, \alpha \le 1 \end{array}$$

It is further divided into 4 sub problems:

I:

$$\begin{array}{ll} max & \alpha \\ subject\ to & f_1(\alpha) \le r \\ & g_1(\alpha) \ge r \\ & 0 \le \alpha \le 1 \end{array}$$

II:

$$\begin{array}{ll} max & \alpha \\ subject\ to & f_1(\alpha) \le r \\ & g_2(\alpha) \ge r \\ & 0 \le \alpha \le 1 \end{array}$$

III:

$$\begin{array}{ll} max & \alpha \\ subject\ to & f_2(\alpha) \le r \\ & g_1(\alpha) \ge r \\ & 0 \le \alpha \le 1 \end{array}$$

IV:

$$\begin{array}{ll} max & \alpha \\ subject\ to & f_2(\alpha) \le r \\ & g_2(\alpha) \ge r \\ & 0 \le \alpha \le 1 \end{array}$$

Let Z^* be the objective value of the original NLPP with optimal solution $a^*(a^* = Z^*)$and Z_{I}, Z_{II}, Z_{III}, and Z_{IV} be the objective values of sub problems I, II, III, and IV respectively. Now let $Z = \max\{Z_{\mathrm{I}}, Z_{\mathrm{II}}, Z_{\mathrm{III}}$, and $Z_{\mathrm{IV}}\}$ then by [19] $Z = Z^*$. Thus, it is enough to solve the above 4 sub problems I, II, III and IV to solve the original NLPP, to get the objective value Z^*.Since $f_1, f_2 \uparrow$ (increases) and $g_1, g_2 \downarrow$ (decreases) and $f(\alpha) \le g(\alpha)$ for all $\alpha \in [0,1]$
For a given r,

$$\begin{array}{rl} \xi(r) = max & \alpha \\ & f(\alpha) \le r \\ subject\ to & g(\alpha) \ge r \\ & 0 \le \alpha \le 1 \end{array}$$

Where r is fixed.

We note that the above assumptions, except for sub problem III, are fulfilled by the remaining 3 sub problems. The supplement procedure of solving sub problem III is proposed in [19]. For numerical illustrations, we will use a triangular fuzzy real number (TFN).
A fuzzy real number $\tilde{a}$ is said to be TFN if its membership function is given by

$$\xi_{\tilde{a}}(r) = \begin{cases} \dfrac{(r - a_1)}{(a_2 - a_1)} & if\ a_1 \le r \le a_2 \\ \dfrac{(a_3 - r)}{(a_3 - a_{21})} & if\ a_2 \le r \le a_3 \\ 0 & otherwise \end{cases}$$

A fuzzy real number $\tilde{a}$is denoted as $(a_1,\ a_2, a_3)$and its α- level set is
$\tilde{a}_\alpha = [(a_2 - a_1)\alpha + a_1, (a_2 - a_3)\alpha + a_3]$ for all $\alpha \in [0, 1]$

Fuzzy Bayesian estimate of Mean Time to Failure
For L(cons/k-n:F) :

$$\widetilde{M^*} = \int_0^\infty \hat{r}\, dt \tag{22}$$

$$\widetilde{M^*} = \int_0^\infty \left\{ \sum_{l=0}^{N} \sum_{i=0}^{N4} C_{N2}^{l} C_{N4}^{i} (-1)^{(l+i)} \left[\frac{v+\theta}{v+\theta+tl+ti} \right]^{s+\beta} - \sum_{l=0}^{N} \sum_{i=0}^{N5} C_{N3}^{l} C_{N5}^{i} (-1)^{(l+i)} \left[\frac{v+\theta}{v+\theta+tl+ti} \right]^{s+\beta} \right\} dt \tag{23}$$

$$\widetilde{M^*} = \sum_{l=0}^{N} \sum_{i=0}^{N4} C_{N2}^{l} C_{N4}^{i} (-1)^{(l+i)} \frac{(v+\theta)}{(v+\theta)(l+i)} \sum_{j=0}^{\infty} B\left(j+1, \frac{s+\beta-1}{s+\beta} \right) - \sum_{l=0}^{N} \sum_{i=0}^{N5} C_{N3}^{l} C_{N5}^{i} (-1)^{(l+i)} \frac{(v+\theta)}{(v+\theta)(l+i)} \sum_{j=0}^{\infty} B\left(j+1, \frac{s+\beta-1}{s+\beta} \right) \tag{24}$$

$$\left(\widetilde{M^*}\right)_{\alpha_1}^{L} = \sum_{l=0}^{N} \sum_{i=0}^{N4} C_{N2}^{l} C_{N4}^{i} (-1)^{(l+i)} \frac{\left(\tilde{v}_{(\alpha_1)}^{L} + \tilde{\theta}_{(\alpha_1)}^{L}\right)}{\left(\tilde{v}_{(\alpha_1)}^{L} + \tilde{\theta}_{(\alpha_1)}^{L}\right)(l+i)} \sum_{j=0}^{\infty} B\left(j+1, \frac{s+\tilde{\beta}_{(\alpha_1)}^{L}-1}{s+\tilde{\beta}_{(\alpha_1)}^{L}} \right) - \sum_{l=0}^{N} \sum_{i=0}^{N5} C_{N3}^{l} C_{N5}^{i} (-1)^{(l+i)} \frac{\left(\tilde{v}_{(\alpha_1)}^{L} + \tilde{\theta}_{(\alpha_1)}^{L}\right)}{\left(\tilde{v}_{(\alpha_1)}^{L} + \tilde{\theta}_{(\alpha_1)}^{L}\right)(l+i)} \sum_{j=0}^{\infty} B\left(j+1, \frac{s+\tilde{\beta}_{(\alpha_1)}^{L}-1}{s+\tilde{\beta}_{(\alpha_1)}^{L}} \right) \tag{25}$$

$$\left(\widetilde{M^*}\right)_{\alpha_1}^{U} = \sum_{l=0}^{N} \sum_{i=0}^{N4} C_{N2}^{l} C_{N4}^{i} (-1)^{(l+i)} \frac{\left(\tilde{v}_{(\alpha_1)}^{U} + \tilde{\theta}_{(\alpha_1)}^{U}\right)}{\left(\tilde{v}_{(\alpha_1)}^{U} + \tilde{\theta}_{(\alpha_1)}^{U}\right)(l+i)} \sum_{j=0}^{\infty} B\left(j+1, \frac{s+\tilde{\beta}_{(\alpha_1)}^{U}-1}{s+\tilde{\beta}_{(\alpha_1)}^{U}} \right) - \sum_{l=0}^{N} \sum_{i=0}^{N5} C_{N3}^{l} C_{N5}^{i} (-1)^{(l+i)} \frac{\left(\tilde{v}_{(\alpha_1)}^{U} + \tilde{\theta}_{(\alpha_1)}^{U}\right)}{\left(\tilde{v}_{(\alpha_1)}^{U} + \tilde{\theta}_{(\alpha_1)}^{U}\right)(l+i)} \sum_{j=0}^{\infty} B\left(j+1, \frac{s+\tilde{\beta}_{(\alpha_1)}^{U}-1}{s+\tilde{\beta}_{(\alpha_1)}^{U}} \right) \tag{26}$$

For C(cons/k-n:F) :

$$\widetilde{M_c^*} = \int_0^\infty \widehat{r_c}\, dt \tag{27}$$

$$\widetilde{M_c^*} = \int_0^\infty \left\{ \sum_{l=0}^{N} \sum_{i=0}^{N4} C_{N2}^{l} C_{N4}^{i} (-1)^{(l+i)} \left[\frac{v+\theta}{v+\theta+tl+ti} \right]^{s+\beta} - k \sum_{l=0}^{N1} \sum_{i=0}^{N5} C_{N6}^{l} C_{N5}^{i} (-1)^{(l+i)} \left[\frac{v+\theta}{v+\theta+tl+ti} \right]^{s+\beta} - \sum_{i=0}^{n} C_{n}^{i} (-1)^{i} \left[\frac{v+\theta}{v+\theta+ti} \right]^{s+\beta} \right\} dt \tag{28}$$

$$\widetilde{M_c^*} = \sum_{l=0}^{N}\sum_{i=0}^{N4} C_{N2}^{l} C_{N4}^{i} (-1)^{(l+i)} \frac{(v+\theta)}{(v+\theta)(l+i)} \sum_{j=0}^{\infty} B\left(j+1, \frac{s+\beta-1}{s+\beta}\right) \tag{29}$$

$$- k \sum_{l=0}^{N1}\sum_{i=0}^{N5} C_{N6}^{l} C_{N5}^{i} (-1)^{(l+i)} \frac{(v+\theta)}{(v+\theta)(l+i+1)} \sum_{j=0}^{\infty} B\left(j+1, \frac{s+\beta-1}{s+\beta}\right) - \sum_{i=0}^{n} C_n^i (-1)^i \frac{(v+\theta)}{(v+\theta)(i)} \sum_{j=0}^{\infty} B\left(j+1, \frac{s+\beta-1}{s+\beta}\right) \tag{30}$$

$$\left(\widetilde{M_c^*}\right)_{\alpha_2}^{L} = \sum_{l=0}^{N}\sum_{i=0}^{N4} C_{N2}^{l} C_{N4}^{i} (-1)^{(l+i)} \frac{\left(\tilde{v}_{(\alpha_2)}^{L} + \tilde{\theta}_{(\alpha_2)}^{L}\right)}{\left(\tilde{v}_{(\alpha_2)}^{L} + \tilde{\theta}_{(\alpha_2)}^{L}\right)(l+i)} \sum_{j=0}^{\infty} B\left(j+1, \frac{s+\tilde{\beta}_{(\alpha_2)}^{L}-1}{s+\tilde{\beta}_{(\alpha_2)}^{L}}\right) \tag{31}$$

$$-k \sum_{l=0}^{N1}\sum_{i=0}^{N5} C_{N6}^{l} C_{N5}^{i} (-1)^{(l+i)} \frac{\left(\tilde{v}_{(\alpha_2)}^{L} + \tilde{\theta}_{(\alpha_2)}^{L}\right)}{\left(\tilde{v}_{(\alpha_2)}^{L} + \tilde{\theta}_{(\alpha_2)}^{L}\right)(l+i+1)} \sum_{j=0}^{\infty} B\left(j+1, \frac{s+\tilde{\beta}_{(\alpha_2)}^{L}-1}{s+\tilde{\beta}_{(\alpha_2)}^{L}}\right)$$

$$- \sum_{i=0}^{n} C_n^i (-1)^i \frac{\left(\tilde{v}_{(\alpha_2)}^{L} + \tilde{\theta}_{(\alpha_2)}^{L}\right)}{\left(\tilde{v}_{(\alpha_2)}^{L} + \tilde{\theta}_{(\alpha_2)}^{L}\right)(i)} \sum_{j=0}^{\infty} B\left(j+1, \frac{s+\tilde{\beta}_{(\alpha_2)}^{L}-1}{s+\tilde{\beta}_{(\alpha_2)}^{L}}\right)$$

$$\left(\widetilde{M_c^*}\right)_{\alpha_2}^{U} = \sum_{l=0}^{N}\sum_{i=0}^{N4} C_{N2}^{l} C_{N4}^{i} (-1)^{(l+i)} \frac{\left(\tilde{v}_{(\alpha_2)}^{U} + \tilde{\theta}_{(\alpha_2)}^{U}\right)}{\left(\tilde{v}_{(\alpha_2)}^{U} + \tilde{\theta}_{(\alpha_2)}^{U}\right)(l+i)} \sum_{j=0}^{\infty} B\left(j+1, \frac{s+\tilde{\beta}_{(\alpha_2)}^{U}-1}{s+\tilde{\beta}_{(\alpha_2)}^{U}}\right)$$

$$-k \sum_{l=0}^{N1}\sum_{i=0}^{N5} C_{N6}^{l} C_{N5}^{i} (-1)^{(l+i)} \frac{\left(\tilde{v}_{(\alpha_2)}^{U} + \tilde{\theta}_{(\alpha_2)}^{U}\right)}{\left(\tilde{v}_{(\alpha_2)}^{U} + \tilde{\theta}_{(\alpha_2)}^{U}\right)(l+i+1)} \sum_{j=0}^{\infty} B\left(j+1, \frac{s+\tilde{\beta}_{(\alpha_2)}^{U}-1}{s+\tilde{\beta}_{(\alpha_2)}^{U}}\right)$$

$$- \sum_{i=0}^{n} C_n^i (-1)^i \frac{\left(\tilde{v}_{(\alpha_2)}^{U} + \tilde{\theta}_{(\alpha_2)}^{U}\right)}{\left(\tilde{v}_{(\alpha_2)}^{U} + \tilde{\theta}_{(\alpha_2)}^{U}\right)(i)} \sum_{j=0}^{\infty} B\left(j+1, \frac{s+\tilde{\beta}_{(\alpha_2)}^{U}-1}{s+\tilde{\beta}_{(\alpha_2)}^{U}}\right)$$

Numerical results and discussion

For a systematic computation, we have obtained all results by Matlab. We consider a L(cons/2-6:F) system with i.i.d exponentially distributed components. On testing 10 items, 3 failures occurred. The failure time of component 1, 2 and 3 are around 20, 30 and 40 h respectively. Based on past experiences, it is found that around 2 failures occurred in 10 h. The problem is to obtain fuzzy B.P.E of the proposed system at mission time 20 h.

By applying TFN and its $\alpha-$ level sets.

$t_1 = \widetilde{20};\ \widetilde{20}_{\alpha_1} = [15 + 5\alpha_1, 25 - 5\alpha_1]$
$t_2 = \widetilde{30};\ \widetilde{30}_{\alpha_1} = [25 + 5\alpha_1, 35 - 5\alpha_1]$
$t_3 = \widetilde{40};\ \widetilde{40}_{\alpha_1} = [35 + 5\alpha_1, 45 - 5\alpha_1]$
$\beta = \tilde{2};\ \tilde{2}_{\alpha_1} = [1 + \alpha_1, 3 - \alpha_1]$
$\theta = 10\ h; k = 2; n = 6; s = 3; m = 10$

From Eqs. (13) and (14), the B.P.E of $\widehat{\tilde{r}_{\alpha_1}^{L}}$ and $\widehat{\tilde{r}_{\alpha_1}^{U}}$ are

$$\widehat{\tilde{r}_{\alpha_1}^{L}} = \sum_{l=0}^{2}\sum_{i=0}^{2l} C_{6-2l}^{l} C_{2l}^{i} (-1)^{l+i} \left[\left(\frac{(15+5\alpha_1)+(25+5\alpha_1)+(35+5\alpha_1)+7(35+5\alpha_1)+10}{(15+5\alpha_1)+(25+5\alpha_1)+(35+5\alpha_1)+7(35+5\alpha_1)+10+20l+20i}\right)\right]^{3+(1+\alpha_1)}$$

$$- \sum_{l=0}^{2}\sum_{i=0}^{2l+2} C_{4-2l}^{l} C_{2l+2}^{i} (-1)^{l+i} \left[\left(\frac{(15+5\alpha_1)+(25+5\alpha_1)+(35+5\alpha_1)+7(35+5\alpha_1)+10}{(15+5\alpha_1)+(25+5\alpha_1)+(35+5\alpha_1)+7(35+5\alpha_1)+10+20l+20i}\right)\right]^{3+(1+\alpha_1)}$$

and

$$\widehat{\tilde{r}^{U}_{\alpha_1}} = \sum_{l=0}^{2}\sum_{i=0}^{2l} C^{l}_{6-2l} C^{i}_{2l}\,(-1)^{l+i}\left[\left(\frac{(25-5\alpha_1)+(35-5\alpha_1)+(45-5\alpha_1)+7(45-5\alpha_1)+10}{(25-5\alpha_1)+(35-5\alpha_1)+(45-5\alpha_1)+7(45-5\alpha_1)+10+20l+20i}\right)\right]^{3+(3-\alpha_1)} - \sum_{l=0}^{2}\sum_{i=0}^{2l+2} C^{l}_{4-2l} C^{i}_{2l+2}\,(-1)^{l+i}\left[\left(\frac{(25-5\alpha_1)+(35-5\alpha_1)+(45-5\alpha_1)+7(45-5\alpha_1)+10}{(25-5\alpha_1)+(35-5\alpha_1)+(45-5\alpha_1)+7(45-5\alpha_1)+10+20l+20i}\right)\right]^{3+(3-\alpha_1)}$$

respectively for all$\alpha_1 \in [0\,,1]$

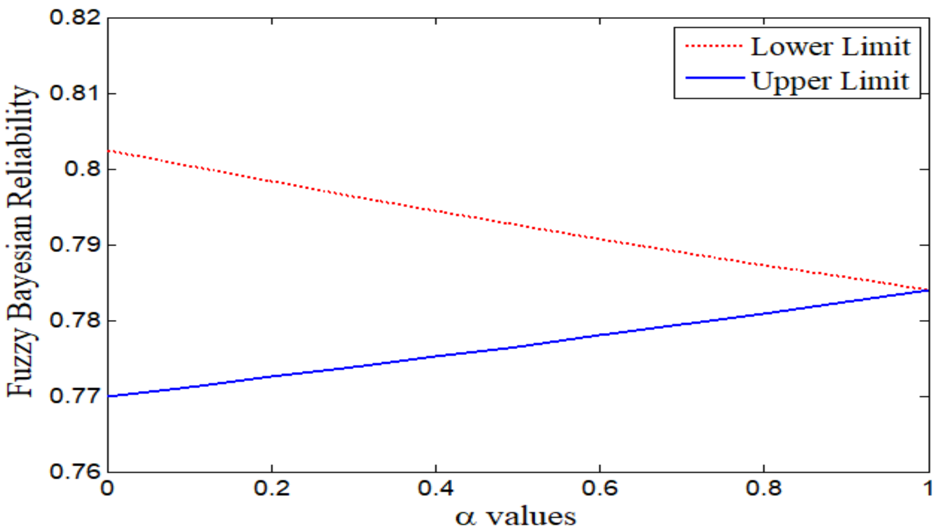

Figure 1 Fuzzy Bayesian Estimate of L(cons/k-n:F) System.

From **Figure 1**, it is clear that $\widehat{\tilde{r}^{U}_{\alpha_1}} \le \widehat{\tilde{r}^{U}_{1}} \le \widehat{\tilde{r}^{L}_{1}} \le \widehat{\tilde{r}^{L}_{\alpha_1}} \Longrightarrow A_{\alpha_1} = \left[\widehat{\tilde{r}^{U}_{\alpha_1}}, \widehat{\tilde{r}^{L}_{\alpha_1}}\right]$
From the above equations, we observe that $f(\alpha) = f_1(\alpha) = \widehat{\tilde{r}^{U}_{\alpha_1}}$ and $g(\alpha) = g_1(\alpha) = \widehat{\tilde{r}^{L}_{\alpha_1}}$
which implies to solve sub problem II ($Z^{*} = Z_{\mathrm{II}}$)
In the process of solving sub problem II with the help of supplemental procedure,
It is found that $\widehat{\tilde{r}^{L}_{\alpha_1}} = 0.7840 = \widehat{\tilde{r}^{U}_{\alpha_1}}$ when $\alpha_1 = 1$
The B.P.E of r will be 0.7840 when the fuzzy real numbers are reconsidered as the real numbers which proves our expectation that the B.P.E will have membership degree 1.
Since $A_0 = [0.7700\,, 0.8024]$, we are just interested in considering $r \in A_0$
For $r \in A_0$,

Case (i) when $r < 0.7840$
The problem is rephrased as

$$\xi_{\hat{r}} = max\left\{\alpha_1 \in [0,1]: f(\alpha_1) = f_1(\alpha_1) = \widehat{\tilde{r}^{U}_{\alpha_1}} \le r\right\}$$

Since $\widehat{\tilde{r}^{U}_{\alpha_1}}$ is increasing with respect to α_1
Case (ii) when $r > 0.7840$
The problem is rephrased as

$$\xi_{\hat{r}} = max\left\{\alpha_1 \in [0,1]: g(\alpha_1) = g_1(\alpha_1) = \widehat{\tilde{r}^{L}_{\alpha_1}} \ge r\right\}$$

Since $\widehat{\tilde{r}^{L}_{\alpha_1}}$ is decreasing with respect to α_1
Therefore, the membership degree of $\tilde{r}$ can be obtained for any given Bayes point estimate r can be obtained by using these formulas.
The reliability of the system lies in the confidence interval $A_{0.95} = [0.7832\,, 0.7848]$

Similarly, for a C(cons/2-6:F) system:
Using Eqs. (20) and (21), it is found that
$\widetilde{r}^{L}_{\alpha_2} = 0.7957 = \widetilde{r}^{U}_{\alpha_2}$ when $\alpha_2 = 1$
$A_{\alpha_2} = [0.7825\,, 0.8129]$ when $\alpha_2 = 0$
and $A_{\alpha_2} = [0.7949\,, 0.7964]$ when $\alpha_2 = 0.95$
For various values of α, the fuzzy Bayesian estimate for a C(cons/2-6:F) system are plotted in **Figure 2**.

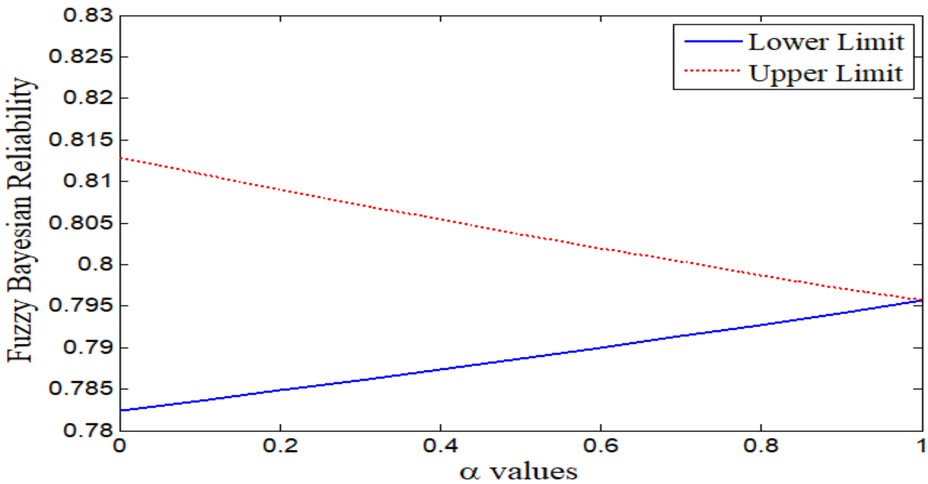

Figure 2 Fuzzy bayesian estimate of C(cons/k-n:F) system.

Table 1 Fuzzy Bayesian Reliability for different cons/k-n:F systems.

∝ values	Linear System			Circular System		
	L(cons/3-5:F)	**L(cons/3-6:F)**	**L(cons/4-6:F)**	**C(cons/3-5:F)**	**C(cons/3-6:F)**	**C(cons/4-6:F)**
0.0	[0.9705, 0.9796]	[0.9618, 0.9736]	[0.9917, 0.9944]	[0.9729, 0.9812]	[0.9649, 0.9757]	[0.9924, 0.9949]
0.1	[0.9708, 0.9791]	[0.9623, 0.9729]	[0.9918, 0.9942]	[0.9732, 0.9807]	[0.9654, 0.9750]	[0.9925, 0.9947]
0.2	[0.9712, 0.9785]	[0.9628, 0.9722]	[0.9919, 0.9941]	[0.9736, 0.9802]	[0.9658, 0.9744]	[0.9926, 0.9946]
0.3	[0.9716, 0.9780]	[0.9633, 0.9715]	[0.9920, 0.9939]	[0.9739, 0.9797]	[0.9663, 0.9738]	[0.9927, 0.9945]
0.4	[0.9720, 0.9775]	[0.9638, 0.9708]	[0.9922, 0.9938]	[0.9743, 0.9793]	[0.9668, 0.9732]	[0.9928, 0.9943]
0.5	[0.9724, 0.9770]	[0.9643, 0.9702]	[0.9923, 0.9936]	[0.9747, 0.9788]	[0.9672, 0.9726]	[0.9930, 0.9942]
0.6	[0.9729, 0.9765]	[0.9649, 0.9696]	[0.9924, 0.9935]	[0.9750, 0.9784]	[0.9677, 0.9720]	[0.9931, 0.9941]
0.7	[0.9733, 0.9760]	[0.9654, 0.9689]	[0.9925, 0.9934]	[0.9754, 0.9779]	[0.9682, 0.9714]	[0.9932, 0.9939]
0.8	[0.9737, 0.9755]	[0.9660, 0.9683]	[0.9927, 0.9932]	[0.9758, 0.9775]	[0.9687, 0.9709]	[0.9933, 0.9938]
0.9	[0.9741, 0.9751]	[0.9665, 0.9677]	[0.9928, 0.9931]	[0.9762, 0.9771]	[0.9692, 0.9703]	[0.9934, 0.9937]

From **Table 1**, we observed that the fuzzy Bayesian reliability increases as the k value increases. Hence the performance of the proposed system is improved when the components are arranged in cons/k-n:F Systems. In particular, the C(cons/k-n:F Systems) structure is superior to the L(cons/k-n:F Systems) based on the performance analysis.

Table 2 Fuzzy MTTF when $\alpha = 0.95$.

L(cons/k-n:F)			C(cons/k-n:F)		
r	$\left(\widetilde{M}^{*}\right)^{L}_{\alpha_1}$	$\left(\widetilde{M}^{*}\right)^{U}_{\alpha_1}$	r	$\left(\widetilde{M}^{*}_{c}\right)^{L}_{\alpha_2}$	$\left(\widetilde{M}^{*}_{c}\right)^{U}_{\alpha_2}$
0	542.49	510.47	0	671.66	632.01
1	303.42	294.78	1	375.66	370.20
2	210.12	206.81	2	260.14	258.06

Table 3 Fuzzy MTTF when $\alpha = 1$.

r	L(cons/k-n:F)	C(cons/k-n:F)
0	525.89	651.10
1	299.01	370.20
2	208.44	258.06

The fuzzy Bayesian estimates of MTTF are calculated and are displayed in **Table 2** and **Table 3**. It is observed that, for increasing value of r, the MTTF shows a decreasing trend.

Wu *et al.* [19] obtained the fuzzy Bayesian estimation for k-out-of-n system based on exponential distribution. Gholizadeh *et al.* [21] implemented the same method and estimated fuzzy Bayesian estimation for k-out-of-n system based on prior 2 parameter exponential distribution. We compared the results of Wu *et al.* [19] and Gholizadeh *et al.* [21] with our derived result in **Tables 4 - 7**.

Table 4 Fuzzy Bayesian reliability estimate of k-out-of-n versus L(cons/k-n:F).

n	k	k-n (Wu)	Middle value [k-n (Wu)]	L(cons/k-n:F)	Middle value [L(cons/k-n:F)]	Increase in Fuzzy Bayesian Reliability
6	2	[0.6601, 0.6646]	0.6624	[0.8408, 0.8431]	0.8420	0.1796
6	3	[0.5462, 0.5502]	0.5482	[0.9668, 0.9674]	0.9671	0.4189
6	4	[0.4975, 0.5002]	0.4989	[0.9929, 0.9930]	0.993	0.4941
5	3	[0.5401, 0.5434]	0.5418	[0.9744, 0.9748]	0.9746	0.4328
5	4	[0.3639, 0.3651]	0.3645	[0.9949, 0.9950]	0.9950	0.6305
						Average = 43 %

Table 5 Fuzzy Bayesian reliability estimate of k-out-of-n versus C(cons/k-n:F).

n	k	k-n (Wu)	Middle value [k-n (Wu)]	C(cons/k-n:F)	Middle value [C(cons/k-n:F)]	Increase in Fuzzy Bayesian Reliability
6	2	[0.6601, 0.6646]	0.6624	[0.8483, 0.8505]	0.8494	0.187
6	3	[0.5462, 0.5502]	0.5482	[0.9695, 0.9700]	0.9698	0.4498
6	4	[0.4975, 0.5002]	0.4989	[0.9935, 0.9936]	0.9936	0.4947
5	3	[0.5401, 0.5434]	0.5418	[0.9764, 0.9768]	0.9766	0.4348
5	4	[0.3639, 0.3651]	0.3645	[0.9970, 0.9971]	0.9971	0.6326
						Average = 44 %

From **Table 4** and **Table 5**, we observed that the fuzzy Bayesian reliability estimate of the L(cons/k-n:F) system shows an increase of 43 %, and the C(cons/k-n:F) system show an increase of 44 % in comparison with the k-out-of-n system obtained by Wu *et al.* [19].

Table 6 Fuzzy Bayesian reliability estimate of k-out-of-n versus L(cons/k-n:F).

n	k	k-n (Gholizadeh)	Middle value k-n [(Gholizadeh)]	L(cons/k-n:F)	Middle value [L(cons/k-n:F)]	Increase in Fuzzy Bayesian Reliability
6	2	[0.5570, 0.5615]	0.5593	[0.8408, 0.8431]	0.842	0.2827
6	3	[0.5462, 0.5502]	0.5482	[0.9668, 0.9674]	0.9671	0.4189
6	4	[0.4975, 0.5002]	0.4989	[0.9929, 0.9930]	0.993	0.4941
5	3	[0.4647, 0.4680]	0.4664	[0.9744, 0.9748]	0.9746	0.5082
5	4	[0.3373, 0.3384]	0.3379	[0.9949, 0.9950]	0.995	0.6571
						Average = 47 %

Table 7 Fuzzy Bayesian reliability estimate of k-out-of-n versus C(cons/k-n:F).

n	k	**k-n (**Gholizadeh**)**	**Middle value [k-n** (Gholizadeh)**]**	C(cons/k-n:F)	Middle value [C(cons/k-n:F)]	Increase in Fuzzy Bayesian Reliability
6	2	[0.5570, 0.5615]	0.5593	[0.8483, 0.8505]	0.8494	0.2901
6	3	[0.5462, 0.5502]	0.5482	[0.9695, 0.9700]	0.9698	0.4216
6	4	[0.4975, 0.5002]	0.4989	[0.9935, 0.9936]	0.9936	0.4947
5	3	[0.4647, 0.4680]	0.4664	[0.9764, 0.9768]	0.9766	0.5102
5	4	[0.3373, 0.3384]	0.3379	[0.9970, 0.9971]	0.9971	0.6592
						Average = 48 %

Tables 6 and **7** indicate that the fuzzy Bayesian reliability estimate of the proposed system is greater than that of the k-out-of-n system derived by Gholizadeh *et al.* [21]. In particular, when compared to L(cons/k-n:F) the reliability of the k-out-of-n system is decreased by 47 %, and with C(cons/k-n:F) the value is decreased by 48 %.

Conclusions

The main purpose of applying this method is to overcome the difficulty of handling imprecise data. For the proposed system, the fuzzy set theory was successfully extended to the Bayesian system reliability. To overcome the impreciseness of the data, the fuzzy concept has been used. The fuzzy Bayesian reliability assessment of cons/k-n:F system under the SELF and the fuzzy Bayesian estimate of MTTF are calculated. The membership degree of $\tilde{r}$ and its confidence interval of degree 0.95 are obtained. The fuzzy Bayesian estimate obtained in this paper is significantly higher reliability when compared with the fuzzy Bayesian estimates of Wu *et al.* [19], and Gholizadeh *et al.* [21].

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