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Fixed Points for α-$\mathcal{F}_G(\xi,\lambda,\theta)$-Generalized Suzuki Contraction with C_G-Class Functions in $b_v(s)$-Metric Spaces

Leta Bekere Kumssa

Department of Mathematics, Madda Walabu University, Bale Robe, Ethiopia

(Corresponding author's e-mail: letabekere@yahoo.com)

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Abstract

The primary goal of this research is to derive generalized C_G-class functions and prove that a fixed point exists for α-$\mathcal{F}_G(\xi,\lambda,\theta)$-generalized Suzuki contraction on $b_v(s)$ -metric spaces. In our study, we use some properties of different control functions. Our findings broaden and unify a number of previous results in the literature. The conclusions are supported by examples.

Keywords: Fixed point, Suzuki contraction, C-class function, C_G-class function, $b_v(s)$-metric spaces

Introduction

Banach's contraction principle [1] provides the basis for the development of the fixed point theorem, which in turn leads to the further development of nonlinear analysis. There are several generalizations of the Banach result in the literature. Some examples are Ciric [2], Rhoades [3], Tasković [4], Edelstein [5], Popescu [6] and Bogin [7]. Also there are more generalizations of Banach's principle by modifying the space, changing the concept of slef-mapping to non self-mapping and so on [8-10].

Suzuki [11] proposed the generalization theorem of Banach's theory in 2007 which is known as Suzuki type contraction. Ansari proposed C-class functions in 2014 to establish fixed point theorems for certain contractive mappings with respect to such class of functions.

In this article, we define a new class of functions called the generalized C_G -class function, and establish a fixed point theorem for Suzuki contractions of rational type under the $b_v(s)$-metric space setting.

Throughout this study, $\mathbb{R}_+$ represents all sets of non-negative real numbers and $\mathbb{Z}^+$ represents all sets of positive integers.

Materials and methods

In this work, the main method we use in proving our main result is implementing the properties of $b_v(s)$-metric spaces and C_G-class functions. In the proof of our main result, we also need the properties of 2 auxiliary functions namely altering distance and ultra altering distance functions. Triangular α-admissibility and α-admissibility are also important tools in our proof. We begin this section by the definition of b-metric space.

Definition 2.1 [12] Let $M \neq \emptyset$ is a set and $s \geq 1$ is a real number. Suppose that for all $p,q,r \in M$ the mapping $\delta: M \times M \to \mathbb{R}_+$ satisfies the following conditions:

1. $\delta(p,q) \geq 0$;
2. $\delta(p,q) = 0$ if and only if $p = q$;
3. $\delta(p,q) = \delta(q,p)$;
4. $\delta(p,r) \leq s[\delta(p,q) + \delta(q,r)]$ (b-triangular inequality).

If δ satisfies conditions (δ_1)-(δ_4), then δ is known as b- metric on M. The couple (M,δ) is named as b-metric space.

After the introduction of b-metric spaces, generalized versions were introduced. This includes extended b-metric space, rectangular b-metric space, $b_v(s)$-metric space, etc.

Definition 2.2 [13] Let $M \neq \emptyset$ is a set and $s \geq 1$ be a fixed real number. Let $\delta: M \times M \to \mathbb{R}_+$ be a mapping such that for all $p,q \in M$ and distinct points $r,t \in M$, each distinct from p and q:

1. $\delta(p,q)=\delta(q,p)=0$ if and only if $p=q$;
2. $\delta(p,q)=\delta(q,p)$;
3. $\delta(p,q)\leq s[\delta(p,r)+\delta(r,t)+\delta(t,q)]$ (b-rectangular inequality).

In that case, δ is called a b-rectangular metric, and the couple (M,δ) is called a b-rectangular metric space.

In the year 2017, Mitrovic and Radenovic [14] introduced a more general version of b-metric space called $b_v(s)$-metric space.

Definition 2.3 [14] Let $\mathrm{M}\neq\emptyset$ is a set and $\delta:\mathrm{M}\times\mathrm{M}\to\mathbb{R}_+$ be a mapping with $\mathrm{v}\in\mathbb{Z}^+,\mathrm{s}\geq 1$. Then (M,δ) is called the $b_v(s)$-metric space for all $\mathrm{p},\mathrm{q}\in\mathrm{M}$ and all distinct points $\mathrm{u}_1,\mathrm{u}_2,\ldots,\mathrm{u}_\mathrm{v}\in\mathrm{M}$, each is different from p and q, the following hold:

1. $\delta(p,q)=\delta(q,p)=0$ if and only if $p=q$;
2. $\delta(p,q)=\delta(q,p)$;
3. $\delta(p,q)\leq s[\delta(p,u_1)+\delta(u_1,u_2)+\ldots+\delta(u_v,q)]$ ($b_v(s)$-metric inequality).

They discovered that:

$b_1(1)$-metric is the normal metric,
$b_1(s)$-metric is b-metric with a factor of $s\geq 1$ (Czerwik[12]; Bakhtin [15]),
$b_2(1)$-metric is the generalized metric,
$b_2(s)$-metric is the rectangular b-metric with the George et al. coefficient s [13],
$b_v(1)$-metric is v-generalized metric by Branciari [16].

Definition 2.4 [14] Let the couple (M,δ) be the $b_v(s)$-metric space, $(\mathrm{p_k})$ the sequence of M, and $\mathrm{p}\in\mathrm{M}$. Then:

1. sequence (p_k) converges to p in (M,δ) if for any $\gamma>0$ there is $N_0=N_0(\gamma)\in\mathbb{Z}^+$ such that $\delta(p_k,p)\leq\gamma$ for all $k\geq N_0$ and this fact is expressed as $\lim_{k\to\infty}p_k=p$;

2. sequence (p_k) is Cauchy if for any $\gamma>0$ there is $N_0=N_0(\gamma)\in\mathbb{Z}^+$ such that $\delta(p_k,p_l)\leq\gamma$ for all $k,l>N_0$;

3. couple (M,δ) is called complete $b_v(s)$-metric space if every Cauchy sequence in M converges to a point in it.

Some of the recent fixed point results in $b_v(s)$ metric spaces can be found on [17-20], and references therein. Here, we give an instance of a $b_v(s)$-metric space.

Example 2.5 Let $\mathrm{M}=\{\frac{1}{\mathrm{w}}:\mathrm{w}\in\{2,3,4,5,\ 6,7,\ 8,\ 9,10\}\}$. We define $\delta:\mathrm{M}\times\mathrm{M}\to\mathbb{R}_+$ by:

$$\delta\left(\frac{1}{\mathrm{w}},\frac{1}{\mathrm{u}}\right)=\begin{cases}0 & \text{if } \mathrm{w}=\mathrm{u},\\ 6 & \text{if } \mathrm{w}=2,\quad \mathrm{v}=3 \text{ or } \mathrm{w}=3,\quad \mathrm{v}=2,\\ 1 & \text{if } \mathrm{w}\in\{2,3\},\mathrm{u}\in\{4\} \text{ or}\\ & \text{if } \mathrm{w}\in\{4\},\mathrm{u}\in\{2,3\},\\ \frac{1}{2} & \text{if } \mathrm{w}\in\{2,3,4\},\mathrm{t}\in\{5\} \text{ or}\\ & \text{if } \mathrm{w}\in\{5\},\mathrm{u}\in\{2,3,4\},\\ \frac{1}{4} & \text{if } \mathrm{w}\in\{2,3,4,5\},\mathrm{u}\in\{6\} \text{ or}\\ & \text{if } \mathrm{w}\in\{6\},\mathrm{u}\in\{2,3,4,5\},\\ \frac{1}{8} & \text{if } \mathrm{w} \text{ or } \mathrm{u}\notin\{2,3,4,5,6\}.\end{cases}$$

Conditions $(\delta_{\ 1})$ and $(\delta_{\ 2})$ of Definition 2.3 are obvious. We verify $b_v(s)$-metric inequality.

$$\delta\left(\frac{1}{2},\frac{1}{3}\right)=6\leq 3\left[1+\frac{1}{2}+\frac{1}{4}+\frac{1}{4}\right]=3\left[\delta\left(\frac{1}{2},\frac{1}{4}\right)+\delta\left(\frac{1}{4},\frac{1}{5}\right)+\delta\left(\frac{1}{5},\frac{1}{6}\right)+\delta\left(\frac{1}{6},\frac{1}{3}\right)\right].$$

Therefore, (M,δ) is a $b_3(3)$-metric space.

Definition 2.6 [21] A mapping $S: M \to M$ is called α-admissible if for each $p, q \in M$ we have:

$\alpha(p, q) \geq 1$ implies $\alpha(Sp, Sq) \geq 1$

where $\alpha: M \times M \to \mathbb{R}$ is a given function.

Recently, Afshari [22] reconsidered the method of Samet for F-contraction mappings in D-metric spaces.

Definition 2.7 [23] The mapping $S: M \to M$ is called triangular α-admissible if for all $p, q, r \in M$ we have:

1. S is α-admissible;
2. $\alpha(p, q) \geq 1$ and $\alpha(q, r) \geq 1$ implies $\alpha(p, r) \geq 1$ where $\alpha: M \times M \to \mathbb{R}$ is a given function.

Lemma 2.8 [23] Let $S: M \to M$ be a triangular α-admissible mapping. Assume that there exists $p_0 \in M$ such that $\alpha(p_0, Sp_0) \geq 1$ and $\alpha(Sp_0, p_0) \geq 1$. If $p_n = S^n p_0$, then $\alpha(p_m, p_n) \geq 1$ for all $m, n \in \mathbb{Z}^+$.

In 2014, Ansari [24] introduced the concept of C- class functions as follows:

Definition 2.9 [24] The continuous mapping $\mathcal{F}: \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$ is called a C-class function if for all $p, q \in \mathbb{R}_+$

1. $\mathcal{F}(p, q) \leq p$;
2. $\mathcal{F}(p, q) = p$ implies that either $p = 0$ or $q = 0$.

$\mathcal{C}$ represents the family of all C-class functions.

Definition 2.10 [25] The function $\xi: \mathbb{R}_+ \to \mathbb{R}_+$ is called the altering distance function if the following properties are met:

1. ξ is non-decreasing and continuous;
2. $\xi(s) = 0$ if and only if $s = 0$.

Ξ represents the family of all altering distance functions.

Altering distance function has been generelized and redefined by altering some of the properties in its axioms [26-30].

Definition 2.11 [31] The function $\theta: \mathbb{R}_+ \to \mathbb{R}_+$ is called the ultra altering distance function if the following properties are met:

1. θ is continuous;
2. $\theta(s) > 0$ for all $s > 0$.

Θ_u represents the class of all ultra-altering distance functions.

Throughout this work, we represent the class of functions $\{\lambda: \mathbb{R}_+ \to \mathbb{R}_+$ such that λ is non-decreasing, upper semi-continuous from the right and $\lambda(t) = 0$ only when $t = 0\}$ by Λ.

Results and discussion

Definition 3.1 A mapping $\mathscr{F}_G: \mathbb{R}_+^3 \to \mathbb{R}$ is called a generalized C_G-class function if for all $p, q, r \in \mathbb{R}_+$ 1. $\mathscr{F}_G$ is continuous;

2. $\mathscr{F}_G(p, q, r) \leq \max\{p, q\}$;
3. $\mathscr{F}_G(p, q, r) = p$ or q implies that either of p, q or r is zero.

C_G represents the family of all generalized C_G-class functions.

Example 3.2 In the following, we give some members of C_G where $\mathscr{F}_G: \mathbb{R}_+^3 \to \mathbb{R}$ is a mapping:

1. $\mathscr{F}_G(p, q, r) = q - p - r$, whenever $\frac{1}{2}(q - r) > p$. This implies $p < q$ and $\mathscr{F}_G(p, q, r) = q$ implies $p = r = 0$.
2. $\mathscr{F}_G(p, q, r) = q - \frac{p}{1+r}$, whenever $p < \frac{q+qr}{2+r}$ $\mathscr{F}_G(p, q, r) = q$ implies $p = 0$.
3. $\mathscr{F}_G(p, q, r) = q - \beta(q) - \beta(p)$, where $\beta: \mathbb{R}_+ \to \mathbb{R}_+$ is continuous, whenever $q - \beta(q) > p + \beta(p)$, $\beta(q) = 0$ and $\beta(p) = 0$ only when $p = q = 0$.

$\mathscr{F}_G(p, q, r) = q$ implies $\beta(q) + \beta(p) = 0$ if and only if $p = q = 0$.

Remark 3.3 Every C-class function is C_G-class function. But the converse may not hold.

Example 3.4 Define $\mathscr{F}_G: \mathbb{R}_+^3 \to \mathbb{R}$ by:

$$\mathscr{F}_G(p, q, r) = \mathscr{F}(p, q) = q - \frac{p}{1+q},$$

where $p < \frac{q+q^2}{2+q}$. So, $\mathscr{F}(p, q) = p$ may not imply either $p = 0$ or $q = 0$. On the other hand, $\mathcal{F}_G(p, q, r) = q$ implies $p = 0$. Hence $\mathcal{F}_G \in \mathcal{C}_G$, but $\mathcal{F}_G \not\subset C$. Therefore, C_G-class function is a generalization of C-class functions.

Lemma 3.5 Let (M, δ) be the $b_v(s)$-metric space and let $\{p_n\}$ be the sequence of M with different elements ($p_n \neq p_m$ with $n \neq m$). Suppose that $\lim_{n\to\infty}\delta(p_n, p_{n+h}) = 0$ for all $h \in \{1,2,\dots v\}$ and $\{p_n\}$ is not a $b_v(s)$- Cauchy sequence. Then there exist $\gamma > 0$ and sequences $\{m_k\}$ and $\{n_k\}$ of positive integers such that $m_k > n_k + v, n_k \geq k$ and:

$$\gamma \leq \liminf_{k\to\infty} \delta(p_{n_k}, p_{m_k}) \leq \limsup_{k\to\infty} \delta(p_{n_k}, p_{m_k}) \leq s\gamma, \tag{1}$$

$$\frac{\gamma}{s} \leq \liminf_{k\to\infty} \delta(p_{n_k}, p_{m_k+1}) \leq \limsup_{k\to\infty} \delta(p_{n_k}, p_{m_k+1}) \leq s\gamma, \tag{2}$$

$$\frac{\gamma}{s} \leq \liminf_{k\to\infty} \delta(p_{n_k-1}, p_{m_k}) \leq \limsup_{k\to\infty} \delta(p_{n_k-1}, p_{m_k}) \leq s\gamma, \tag{3}$$

$$\frac{\gamma}{s^2} \leq \liminf_{k\to\infty} \delta(p_{n_k-1}, p_{m_k+1}) \leq \limsup_{k\to\infty} \delta(p_{n_k-1}, p_{m_k+1}) \leq s^2\gamma. \tag{4}$$

Proof. If $\{p_n\}$ is not a Cauchy sequence, then there exist $\gamma > 0$ and subsequences $\{m_k\}$ and $\{n_k\}$ of $\{p_n\}$ such that:

$$m_k > n_k \geq k \text{ and } \delta(p_{n_k}, p_{m_k}) \geq \gamma. \tag{5}$$

That is,

$$\delta(p_{n_k}, p_{n_k+1}) < \gamma, \delta(p_{n_k}, p_{n_k+2}) < \gamma, \dots, \delta(p_{n_k}, p_{m_k-1}) < \gamma. \tag{6}$$

Since $\lim_{n\to\infty}\delta(p_n, p_{n+h}) = 0$ for all $h \in \{1,2,\dots v\}$, we can assume $m_k \geq n_k + v$. By applying Eqs. (1) and (6) and $b_v(s)$-metric inequality, we have:

$$\gamma \leq \delta(p_{n_k}, p_{m_k}) \leq s[\delta(p_{n_k}, p_{m_k-1}) + \delta(p_{m_k-1}, p_{m_k})] < s[\gamma + \delta(p_{m_k-1}, p_{m_k})]. \tag{7}$$

Taking the lower limit and the upper limit as $k \to \infty$ in Eq. (7), we get:

$$\gamma \leq \liminf_{k\to\infty} \delta(p_{n_k}, p_{m_k}) \leq \limsup_{k\to\infty} \delta(p_{n_k}, p_{m_k}) \leq s\gamma.$$

Hence, Eq. (1) is proved.
Again by Eq. (5) and $b_v(s)$-metric inequality, we have:

$$\gamma \leq \delta(p_{n_k}, p_{m_k}) \leq s[\delta(p_{n_k}, p_{m_k+1}) + \delta(p_{m_k+1}, p_{m_k})] \tag{8}$$

Since $\lim_{n\to\infty}\delta(p_{n+h}, p_n) = 0$ for all $h \in \{1,2,\dots v\}$, taking the lower limit as $k \to \infty$ in Eq. (8), we obtain:

$$\frac{\gamma}{s} \leq \liminf_{k\to\infty} \delta(p_{n_k}, p_{m_k+1}). \tag{9}$$

Similarly, by using Eq. (5) and $b_v(s)$-metric inequality, we obtain:

$$\delta(p_{n_k}, p_{m_k+1}) \leq s[\delta(p_{n_k}, p_{m_k}) + \delta(p_{m_k}, p_{m_k+1})] < s[\gamma + \delta(p_{m_k}, p_{m_k+1})]. \tag{10}$$

Since $\lim_{n\to\infty} \delta(p_n, p_{n+h}) = 0$ for all $h \in \{1,2,\ldots v\}$, taking the lower limit as $k \to \infty$ in Eq. (10), we get:

$$\limsup_{k\to\infty} \delta(p_{n_k}, p_{m_k+1}) \leq s\gamma. \tag{11}$$

From Eqs. (9) and (11), we have:

$$\frac{\gamma}{s} \leq \liminf_{k\to\infty} \delta(p_{n_k}, p_{m_k+1}) \leq \limsup_{k\to\infty} \delta(p_{n_k}, p_{m_k+1}) \leq s\gamma.$$

Similarly, by using Eq. (5) and $b_v(s)$-metric inequality, we obtain:

$$\gamma \leq \delta(p_{n_k}, p_{m_k}) \leq s[\delta(p_{n_k}, p_{n_k-1}) + \delta(p_{n_k-1}, p_{m_k})] \tag{12}$$

Since $\lim_{n\to\infty} \delta(p_n, p_{n-1}) = 0$, taking the lower limit as $k \to \infty$ in Eq. (12), we get:

$$\frac{\gamma}{s} \leq \liminf_{k\to\infty} \delta(p_{n_k-1}, p_{m_k}). \tag{13}$$

By $b_v(s)$-metric inequality, we have:

$$\delta(p_{n_k-1}, p_{m_k}) \leq s[\delta(p_{n_k-1}, p_{n_k}) + \delta(p_{n_k}, p_{m_k})]. \tag{14}$$

Taking upper limit as $k \to \infty$ in Eq. (14), we get:

$$\limsup_{k\to\infty} \delta(p_{n_k-1}, p_{m_k}) \leq s\gamma. \tag{15}$$

From Eqs. (13) and (15), we have:

$$\frac{\gamma}{s} \leq \liminf_{k\to\infty} \delta(p_{n_k-1}, p_{m_k}) \leq \limsup_{k\to\infty} \delta(p_{n_k-1}, p_{m_k}) \leq s\gamma.$$

Again, by using Eq. (5) and $b_v(s)$-metric inequality, we obtain

$$\begin{aligned}\gamma \leq \delta(p_{n_k}, p_{m_k}) &\leq s[\delta(p_{n_k}, p_{n_k-1}) + \delta(p_{n_k-1}, p_{m_k})] \\ &\leq s[\delta(p_{n_k}, p_{n_k-1})] + s^2[\delta(p_{n_k-1}, p_{m_k+1}) + \delta(p_{m_k+1}, p_{m_k})].\end{aligned} \tag{16}$$

By taking the lower limit as $k \to \infty$ in Eq. (16), we have:

$$\frac{\gamma}{s^2} \leq \liminf_{k\to\infty} \delta(p_{n_k-1}, p_{m_k+1}). \tag{17}$$

Finally, by using $b_v(s)$ inequality and Eq. (5), we obtain:

$$\begin{aligned}\delta(p_{n_k-1}, p_{m_k+1}) &\leq s[\delta(p_{n_k-1}, p_{m_k}) + \delta(p_{m_k}, p_{m_k+1})] \\ &\leq s^2[\delta(p_{n_k-1}, p_{n_k}) + \delta(p_{n_k}, p_{m_k})] + s\delta(p_{m_k}, p_{m_k+1})\end{aligned} \tag{18}$$

By taking the upper limit as $k \to \infty$ in Eq. (18), we get:

$$\limsup_{k\to\infty} \delta(p_{n_k-1}, p_{m_k+1}) \leq s^2\gamma. \tag{19}$$

From Eqs. (17) and (19), it follows that:

$$\frac{\gamma}{s^2} \leq \liminf_{k\to\infty} \delta(p_{n_k-1}, p_{m_k+1}) \leq \limsup_{k\to\infty} \delta(p_{n_k-1}, p_{m_k+1}) \leq s^2\gamma. \tag{20}$$

Definition 3.6 Given that (M,δ) is a $b_v(s)$-metric space with $s \geq 1$. Let $S: M \to M$ be a given mapping.

S is called an α-$\mathcal{F}_G(\xi,\lambda,\theta)$ -generalized Suzuki contraction if there exists a mapping $\alpha: M \times M \to \mathbb{R}_+$ with $\alpha(p,q) \geq 1$ satisfying:

$$\frac{1}{2s}\delta(p,Sp) \leq \delta(p,q) \Rightarrow \xi(\alpha(p,q)\delta(Sp,Sq)) \leq \mathcal{F}_G(\xi(L(p,q)),\lambda(L(p,q)),\theta(L(p,q))) \tag{21}$$

for each $p,q \in M$, where:

$$L(p,q) = \max\{\delta(p,q),\delta(p,Sp),\delta(q,Sq)\},\ \xi \in \Xi, \lambda \in \Lambda, \theta \in \Theta, \text{ and } \mathcal{F}_G \in \mathcal{C}_G.$$

Theorem 3.7 Given that (M,δ) is a complete $b_v(s)$-metric space. Let $S: M \to M$ be an α-$\mathcal{F}_G(\xi,\lambda,\theta)$-generalized Suzuki contraction with $\xi(t) > \lambda(t)$ for all $t > 0$. Assume that the following conditions hold:

1. S is triangular α-admissible;
2. There is $p_0 \in M$ such that $\alpha(p_0,Sp_0) \geq 1$ and $\alpha(Sp_0,p_0) \geq 1$;
3. S is continuous.

Then S admits a fixed point. If $z,w \in F(S)$, where $F(S)$ is the set of all fixed points of S, such that $\alpha(z,w) \geq 1$, then S has only one fixed point.

Proof. By hypothesis 2, there exists $p_0 \in M$ satisfying $\alpha(p_0,Sp_0) \geq 1$ and $\alpha(Sp_0,p_0) \geq 1$. Define a sequence $\{p_n\}$ contained in M with $p_{n+1} = Sp_n$ for all $n \in \mathbb{Z}^+$.

Assuming that $p_{n_0} = p_{n_0+1}$ for some $n_0 > 0$, then p_{n_0} is a fixed point and we are through. Therefore, we assume that $p_n \neq p_{n+1}$, for all $n \in \mathbb{Z}^+ \cup \{0\}$. By the fact that S is α-admissible, we obtain:

$$\alpha(p_0,Sp_0) = (p_0,p_1) \geq 1,\ \alpha(Sp_0,Sp_1) \geq 1. \tag{22}$$

On extending this process, we get:

$$\alpha(p_n,p_{n+1}) \geq 1, \text{forall } n \in \mathbb{Z}^+. \tag{23}$$

For the reason that $\frac{1}{2s}\delta(p_n,Sp_n) = \frac{1}{2s}\delta(p_n,p_{n+1}) \leq \delta(p_n,p_{n+1})$, and $p_n \neq p_{n+1}$ for all n on substituting $p = p_n$ and $q = p_{n+1}$ in Eq. (21), we have:

$$\begin{aligned}\xi(\delta(p_{n+1},p_{n+2})) = \xi(\delta(Sp_n,Sp_{n+1})) &\leq \xi(\alpha(p_n,p_{n+1})\delta(Sp_n,Sp_{n+1}))\\ &\leq \mathcal{F}_G(\xi(L(p_n,p_{n+1})),\lambda(L(p_n,p_{n+1})),\theta(L(p_n,p_{n+1})))\\ &\leq \max\{\xi(L(p_n,p_{n+1})),\lambda(L(p_n,p_{n+1}))\}, \end{aligned} \tag{24}$$

where

$$\begin{aligned}L(p_n,p_{n+1}) &= \max\{\delta(p_n,p_{n+1}),\delta(p_n,Sp_n),\delta(p_{n+1},Sp_{n+1})\}\\ &= \max\{\delta(p_n,p_{n+1}),\delta(p_n,p_{n+1}),\delta(p_{n+1},p_{n+2})\}\\ &= \max\{\delta(p_n,p_{n+1}),\delta(p_{n+1},p_{n+2})\}.\end{aligned} \tag{25}$$

Suppose that $\max\{\delta(p_n,p_{n+1}),\delta(p_{n+1},p_{n+2})\} = \delta(p_{n+1},p_{n+2})$. Hence from Eq. (24), we obtain:

$$\begin{aligned}\xi(\delta(p_{n+1},p_{n+2})) &\leq \xi(\alpha(p_n,p_{n+1})\delta(p_{n+1},p_{n+2}))\\ &\leq \max\{\xi(\delta(p_{n+1},p_{n+2})),\lambda(\delta(p_{n+1},p_{n+2}))\}\\ &< \xi(\delta(p_{n+1},p_{n+2})).\end{aligned}$$

This implies that $\delta(p_{n+1},p_{n+2}) < \delta(p_{n+1},p_{n+2})$, which is a contradiction.

Therefore,

$\max\{\delta(p_n,p_{n+1}),\delta(p_{n+1},p_{n+2})\} = \delta(p_n,p_{n+1})$. Hence from Eq. (24), we obtain:

$$\xi(\delta(p_{n+1},p_{n+2})) \leq \xi(\alpha(p_n,p_{n+1})\delta(p_{n+1},p_{n+2}) \leq \max\{\xi(\delta(p_n,p_{n+1})),\lambda(\delta(p_n,p_{n+1})) < \xi(\delta(p_n,p_{n+1})).$$

From non-decreasing property of ξ, it follows that $\delta(p_{n+1}, p_{n+2}) \leq \delta(p_n, p_{n+1})$ for all $n \in \mathbb{Z}^+$. Hence $\{\delta_n\} = \{\delta(p_n, p_{n+1})\}$ is a decreasing positive sequence in M. and it converges to some real number $l \geq 0$. We now claim that $l = 0$. Now, taking the upper limit letting $n \to \infty$ in Eq. (24), we have:

$$\xi(l) \leq \mathcal{F}_G(\xi(l), \lambda(l), \theta(l)) \leq \max\{\xi(l), \lambda(l)\} = \xi(l). \tag{26}$$

From Eq. (26), we get:

$$\mathcal{F}_G(\xi(l), \lambda(l), \theta(l)) = \xi(l). \tag{27}$$

By the condition c of Definition 3.1 and Eq. (27), it can be deduced that either $\xi(l) = 0$ or $\lambda(l) = 0$. Hence, $l = 0$.

Next, we will show that $p_m \neq p_n$, for all $m \neq n$. Assume the contrary. i. e., $p_m = p_n$, for some $m > n$. Hence we have,

$$p_{m+1} = Sp_m = Sp_n = p_{n+1} \text{ and } \delta(p_m, p_{m+1}) < \delta(p_{m-1}, p_m) < \ldots < \delta(p_n, p_{n+1}) = \delta(p_m, p_{m+1}),$$

a contradiction. Therefore, $p_m \neq p_n$, for all $m \neq n$.

Since the sequence $\{\delta(p_n, p_{n+1})\}$ is decreasing, by applying $b_v(s)$ metric inequality for $h = 1,2,3,\ldots,v$, we get:

$$\begin{aligned}\delta(p_n, p_{n+h}) &\leq s[\delta(p_n, p_{n+1}) + \delta(p_{n+1}, p_{n+2}) + \ldots + \delta(p_{n+h-1}, p_{n+h})].\\ &< s[\delta(p_n, p_{n+1}) + \delta(p_n, p_{n+1}) + \ldots + \delta(p_n, p_{n+1})].\end{aligned} \tag{28}$$

From Eq. (28), we have:

$$\frac{1}{sh}\delta(p_n, p_{n+h}) < \delta(p_n, p_{n+1}). \tag{29}$$

For $h \geq 1$, we have:

$$\frac{1}{2s}\delta(p_n, p_{n+1}) \leq \frac{1}{sh}\delta(p_n, p_{n+h}) < \delta(p_n, p_{n+1}) \leq \delta(p_n, p_{n+h}). \tag{30}$$

Since S is triangular α-admissible, by Lemma 2.8, it is an easy task to show that $\alpha(p_n, p_{n+h}) \geq 1$ for $h = 1,2,3,\ldots,v$. Again, by replacing $p = p_n$, $q = p_{n+h}$ in (21), where $p_n \neq p_{n+h}$ for all n, we have:

$$\begin{aligned}\xi(\delta(p_{n+1}, p_{n+h+1})) = \xi(\delta(Sp_n, Sp_{n+h})) &\leq \xi(\alpha(p_n, p_{n+h})\delta(Sp_n, Sp_{n+h}))\\ &\leq \mathcal{F}_G(\xi(L(p_n, p_{n+h})), \lambda(L(p_n, p_{n+h})), \theta(L(p_n, p_{n+h})))\\ &\leq \max\{\xi(L(p_n, p_{n+h})), \lambda(L(p_n, p_{n+h}))\} = \xi(L(p_n, p_{n+h})),\end{aligned} \tag{31}$$

where,

$$\begin{aligned}L(p_n, p_{n+h}) &= \max\{\delta(p_n, p_{n+h}), \delta(p_n, Sp_n), \delta(p_{n+h}, Sp_{n+h})\}\\ &= \max\{\delta(p_n, p_{n+h}), \delta(p_n, p_{n+1}), \delta(p_{n+h}, p_{n+h+1})\}.\end{aligned} \tag{32}$$

We denote by:

$$a_n = \delta(p_n, p_{n+1}),\ b_n = \delta(p_n, p_{n+h}),\ b_{n+1} = \delta(p_{n+1}, p_{n+h+1}) \text{ and } c_n = \delta(p_{n+h}, p_{n+h+1}).$$

Hence,

$$c_n < c_{n-1} < c_{n-2} < \ldots < c_{n-h} = a_n = \delta(p_n, p_{n+1}).$$

Thus c_n can not be the maximum.

Again from Eq. (30), we have $a_n < b_n$. Hence from Eqs. (31) and (32), we get:

$\xi(b_{n+1}) \leq \xi(b_n)$.

From non-decreasing property of ξ, it follows that $b_{n+1} \leq b_n$ for all $n \in \mathbb{Z}^+$ and for $h = 1,2,3,\dots,v$. Hence $\{b_n\} = \{\delta(p_n, p_{n+h})\}$ is a decreasing positive sequence in M.

In the next step, we will prove that the sequence $\{\delta(p_n, p_{n+h})\} \to 0$ for $h = 1,2,3,\dots,v$. Since $\delta(p_n, p_{n+h}) > 0$, we have:

$$0 < \delta(p_n, p_{n+h}) \leq s[\delta(p_n, p_{n+1}) + \delta(p_{n+1}, p_{n+2}) + \dots + \delta(p_{n+h-1}, p_{n+h})] < sh[\delta(p_n, p_{n+1})]. \tag{33}$$

Letting $n \to \infty$ in Eq. (33), we obtain:

$\delta(p_n, p_{n+h}) \to 0$.

In the following, we show that $\{p_n\}$ is a $b_v(s)$-Cauchy sequence. Now, assuming the contrary, $\{p_n\}$ is not a $b_v(s)$-Cauchy sequence. From Lemma 3.5, there is $\gamma > 0$ and sequences $\{m_k\}$ and $\{n_k\}$ of $\mathbb{Z}^+$ such that $m_k > n_k + v, n_k \geq k$ and:

$$\gamma \leq \liminf_{k\to\infty} \delta(p_{n_k}, p_{m_k}) \leq \limsup_{k\to\infty} \delta(p_{n_k}, p_{m_k}) \leq s\gamma,$$

$$\frac{\gamma}{s} \leq \liminf_{k\to\infty} \delta(p_{n_k}, p_{m_k+1}) \leq \limsup_{k\to\infty} \delta(p_{n_k}, p_{m_k+1}) \leq s\gamma,$$

$$\frac{\gamma}{s} \leq \liminf_{k\to\infty} \delta(p_{n_k-1}, p_{m_k}) \leq \limsup_{k\to\infty} \delta(p_{n_k-1}, p_{m_k}) \leq s\gamma,$$

$$\frac{\gamma}{s^2} \leq \liminf_{k\to\infty} \delta(p_{n_k-1}, p_{m_k+1}) \leq \limsup_{k\to\infty} \delta(p_{n_k-1}, p_{m_k+1}) \leq s^2\gamma,$$

Since S is a triangular α-admissible, by Lemma 2.8 $\alpha(p_{n_k-1}, p_{m_k}) \geq 1$.

If $\frac{1}{2s}\delta(p_{n_k-1}, p_{n_k}) > \delta(p_{n_k-1}, p_{m_k})$, on taking the lower limit as $k \to \infty$, we get $0 \geq \frac{\gamma}{s}$. This is a contradiction.

Hence, putting $p = p_{n_k-1}$ and $q = p_{m_k}$ in Eq. (21), we have:

$$\begin{aligned} \frac{1}{2s}\delta\left(p_{n_k-1}, Sp_{n_k-1}\right) \leq \delta\left(p_{n_k-1}, p_{m_k}\right) \Rightarrow \xi(\delta(p_{n_k}, p_{m_k+1})) &\leq \xi(\alpha(p_{n_k-1}, p_{m_k})\delta(p_{n_k}, p_{m_k+1}) \\ &\leq \mathcal{F}_G(\xi(L(p_{n_k-1}, p_{m_k})), \lambda(L(p_{n_k-1}, p_{m_k})), \theta(L(p_{n_k-1}, p_{m_k}))), \end{aligned} \tag{34}$$

where

$$\begin{aligned} L(p_{n_k-1}, p_{m_k}) &= \max\{\delta(p_{n_k-1}, p_{m_k}), \delta(p_{n_k-1}, Sp_{n_k-1}), \delta(p_{m_k}, Sp_{m_k})\} \\ &= \max\{\delta(p_{n_k-1}, p_{m_k}), \delta(p_{n_k-1}, p_{n_k}), \delta(p_{m_k}, p_{m_k+1})\}. \end{aligned}$$

Thus,

$$\limsup_{k\to\infty} L\left(p_{n_k-1}, p_{m_k}\right) \leq s\gamma. \tag{35}$$

Now, taking the upper limit as $k \to \infty$ in Eq. (34) and using Lemma 3.5, we obtain:

$$\xi(s\gamma) \leq \mathcal{F}_G(\xi(s\gamma), \lambda(s\gamma), \theta(s\gamma)) \leq \max\{\xi(s\gamma), \lambda(s\gamma)\} = \xi(s\gamma), \tag{36}$$

This implies that:

$\mathcal{F}_G(\xi(s\gamma), \lambda(s\gamma), \theta(s\gamma)) = \xi(s\gamma)$.

That means, $\gamma = 0$, a contradiction. Hence $\{p_n\}$ is Cauchy.
Since (M, δ) is a complete $b_v(s)$-metric space, there exists a point $p*$ such that

$$\lim_{n\to\infty} \delta(p_{n+h}, p*) = 0, \text{ for } h = 1,2,3,\dots, v. \tag{37}$$

Since S is continuous, we have

$$\lim_{n\to\infty} \delta(p_{n+h}, Sp*) = \lim_{n\to\infty} \delta(Sp_{n+h-1}, Sp*) = 0. \tag{38}$$

Since limit is unique, from Eqs. (37) and (38), we have $Sp* = p*$. Hence S admits a fixed point.

At last, we show that $p*$ is the only fixed point of S. Assume that $p*, q* \in F(S)$ provided that $p*$ and $q*$ are distinct. Hence $\alpha(p*, q*) \geq 1$. Now, putting $p = p*$ and $q = q*$ in (21), we have:

$$\begin{aligned}\frac{1}{2s}\delta(p*, Sp*) = 0 \leq \delta(p*, q*) \Rightarrow \xi(\delta(p*, q*)) &= \xi(\delta(Sp*, Sq*)) \\ &\leq \xi(\alpha(p*, q*)\delta(Sp*, Sq*)) \\ &\leq \mathcal{F}_G(\xi(L(p*, q*)), \lambda(L(p*, q*)), \theta(L(p*, q*))) \\ &< max\{\xi(L(p*, q*)), \lambda(L(p*, q*))\} \\ &= \xi(L(p*, q*))\end{aligned}$$

for each $p*, q* \in M$, where

$$L(p*, q*) = \max\{\delta(p*, q*), \delta(p*, Sp*), \delta(q*, Sq*) = \delta(p*, q*).$$

$h = 1$, we have:

$$\delta(p_n, p_{n+1}) \leq s[\delta(p_n, p*) + \delta(p*, p_{n+1})] < s[\tfrac{1}{2s}\delta(p_n, p_{n+1}) + \tfrac{1}{2s}\delta(p_n, p_{n+1})]$$
$= \delta(p_n + p_{n+1})$, a contradiction.

Similarly, for $h \geq 2$, we obtain:

$$\delta(p_n, p_{n+h}) \leq s[\delta(p_n, p*) + \delta(p*, p_{n+h})] < s[\frac{1}{2s}\delta(p_n, p_{n+1}) + \frac{1}{2s}\delta(p_n, p_{n+1})]$$
$= \delta(p_n + p_{n+1})$, a contradiction since $\delta(p_n, p_{n+h}) > \delta(p_n + p_{n+1})$. Hence for $n \in \mathbb{Z}^+$ and $h = 1,2,3,\dots v$, either:

$$\tfrac{1}{2s}\delta(p_n, p_{n+1}) \leq \delta(p_n, p*) \quad \text{or} \quad \tfrac{1}{2s}\delta(p_n, p_{n+1}) \leq \delta(p_{n+h}, p*).$$

Suppose that $\frac{1}{2s}\delta(p_n, p_{n+1}) \leq \delta(p_n, p*)$ and $Sp* \neq p*$. Since $p_n \to 0$ as $n \to \infty$, then there is a subsequence $\{p_{n_k}\}$ of $\{p_n\}$ such that $\alpha(p_{n_k}, p*) \geq 1$. for all $k \in \mathbb{Z}^+$. Hence putting $p = p_{n_k}$ and $q = p*$ in Eq. (21), we have:

$$\begin{aligned}&\frac{1}{2s}\delta(p_{n_k}, Sp_{n_k}) \leq \delta(p_{n_k}, p*) \Rightarrow \xi(\delta(p_{n_k+1}, Sp*)) = \xi(\delta(Sp_{n_k}, Sp*)) \leq \xi(\alpha(p_{n_k}, p*)\delta(Sp_{n_k}, Sp*)) \\ &\leq \mathscr{F}_G\left(\xi\left(L(p_{n_k}, p*)\right), \lambda\left(L(p_{n_k}, p*)\right), \theta\left(L(p_{n_k}, p*)\right)\right),\end{aligned} \tag{39}$$

where

$$L(p_{n_k}, p*) = \max\{\delta(p_{n_k}, p*), \delta(p_{n_k}, Sp_{n_k}), \delta(p*, Sp* = \max\{\delta(p_{n_k}, p*), \delta(p_{n_k}, p_{n_k+1}), \delta(p*, Sp*)\}.$$

Thus,

$$\lim_{k\to\infty}\sup L(p_{n_k}, p*) = 0. \tag{40}$$

Now, taking the upper limit as $k \to \infty$ in (39), we obtain:
$\xi(\delta(p*, Sp*)) \le \mathscr{F}_G(\xi(0), \lambda(0), \theta(0)) \le \max\{\xi(0), \lambda(0)\} = \xi(0) = 0.$ (41)

This implies that:

$(\delta(p*, Sp*)) = 0.$

That means, $Sp* = p*$. Hence $p*$ is a fixed point of S.

The proof for uniqueness is similar to the proof of Theorem 3.7. The next example is to verify the validity of Theorem 3.7.

Example 3.9 Let $M = [0, \infty)$. We define $\alpha, \delta: M \times M \to \mathbb{R}_+$ by:

$$\delta(p,q) = \begin{cases} 0 & \text{if } p = q, \\ p & \text{if } p \ne 0 \text{ and } q = 0, \\ q & \text{if } p = 0 \text{ and } q \ne 0, \\ 2(p+q) + \frac{1}{2} & \text{if } p \ne 0 \text{ and } q \ne 0. \end{cases}$$

and $\alpha(p,q) = \begin{cases} 1 & \text{provided that } p, q \in [0,1], \\ 0 & \text{otherwise.} \end{cases}$

δ is a complete $b_2(2)$-metric space with $s = 2$.
$S: M \to M$ and $\mathcal{F}_G: \mathbb{R}_+^3 \to \mathbb{R}$ are defined as

$$S(p) = \begin{cases} 0 & \text{if } p \in [0,1], \\ \frac{3}{2}p - \frac{3}{2} & \text{if } p \ge 1, \end{cases} \quad \text{and} \quad \mathcal{F}_G(u,v,w) = v - \frac{u}{1+w}.$$

We define functions $\xi, \lambda, \theta: \mathbb{R}_+ \to \mathbb{R}_+$ by:

$$\xi(t) = \frac{3}{2}t, \quad \lambda(t) = \begin{cases} t & \text{if } t < 1 \\ \frac{1}{2}t + \frac{3}{4} & \text{if } t \ge 1 \end{cases} \quad \text{and} \quad \theta(t) = \frac{1}{3}t.$$

Case 1: $p, q \in [0,1]$ such that $p = 0, q \ne 0$. In this case,

$$\frac{1}{2s}\delta(0, S0) = 0 < q = \delta(0,q)$$

$\Rightarrow \xi(\delta(S0, Sq) \le \xi(\alpha(0,q)\delta(S0,Sq)) = \xi(0) = 0.$ (42)

Now,
$\mathcal{F}_G(\xi(L(0,q)), \lambda(L(0,q)), \theta(L(0,q))) = \lambda(L(0,q)) - \frac{\xi(L(0,q))}{1+\theta(L(0,q))} < \lambda(L(0,q)),$ (43)

where,

$L(0,q) = \max\{\delta(0,q), \delta(0,S0), \delta(q,Sq)\} = \max\{q, 0, q\} = q.$

Hence from Eqs.(42) and (43), we obtain:

$0 = \xi(0) \leq \mathcal{F}_G(\xi(q), \lambda(q), \theta(q) < \lambda(q) < \xi(q) = \frac{3}{2}q.$
Hence Eq. (21) hold.

Case 2: $p, q \in [0,1]$ such that $q = 0, p \neq 0$. In this case,

$$\frac{1}{2s}\delta(p, Sp) = \frac{1}{2s}p < p = \delta(p, 0)$$

$$\Rightarrow \xi(\delta(Sp, S0) \leq \xi(\alpha(p, 0)\delta(Sp, S0)) = \xi(0) = 0. \tag{44}$$

Now,

$$\mathcal{F}_G(\xi(L(p, 0)), \lambda(L(p, 0)), \theta(L(p, 0)) = \lambda(L(p, 0)) - \frac{\xi(L(p,0))}{1+\theta(L(p,0))} < \lambda(L(p, 0)), \tag{45}$$

where,

$$L(p, 0) = \max\{\delta(p, 0), \delta(p, Sp), \delta(0, S0)\} = \max\{p, p, 0\} = p$$

Hence from Eqs. (44) and (45), we obtain:

$$0 = \xi(0) \leq \mathcal{F}_G(\xi(p), \lambda(p), \theta(p)) < \lambda(p) < \xi(p) = \frac{3}{2}p.$$

Hence Eq. (21) holds.

Case 3: Let $p, q \in [0,1]$ such that $p \neq 0$ and $q \neq 0$. In this case,

$$\frac{1}{2s}\delta(p, Sp) = \frac{1}{2s}\delta(p, 0) < \delta(p, 0) < \delta(p, q)$$

$$\Rightarrow \xi(\delta(Sp, Sq) \leq \xi(\alpha(p, q)\delta(Sp, Sq)) = \xi(0) = 0 \tag{46}$$

Now,

$$\mathcal{F}_G(\xi(L(p, q)), \lambda(L(p, q)), \theta(L(p, q))) = \lambda(L(p, q)) - \frac{\xi(L(p,q))}{1+\theta(L(p,q))} < \lambda(L(p, q)), \tag{47}$$

where,

$$L(p, q) = \max\{\delta(p, q), \delta(p, Sq), \delta(q, Sq)\} = \max\{2[p + q] + \frac{1}{2}, p, q\} = 2[p + q] + \frac{1}{2}.$$

Thus by Eqs. (46) and (47), trivially Eq. (21) holds.

Therefore, S is an α-$\mathcal{F}_G(\xi, \lambda, \theta)$-generalized Suzuki contraction. There exists $p_0 = 0$ such that $\alpha(0, S0) \geq 1$ and $\alpha(S0,0) \geq 1$. Hence S admits a fixed point. The fixed points of S are $p = 0$ and $p = 3$.

Conclusions

In this paper, we have proved the existence of a fixed point for α-$\mathcal{F}_G(\xi, \lambda, \theta)$-generalized Suzuki contractions. Also, we have provided examples for α-$\mathcal{F}_G(\xi, \lambda, \theta)$-generalized Suzuki contractions, C_G-class function, and $b_v(s)$-metric spaces. Since every C-class function is C_G-class function but the converse does not hold, the result obtained in C_G-class function generalizes many fixed point results obtained in C-class functions.

Open problem

Is it possible to generalize C-class function by considering the domain $\mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+$ and so on ?

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