Neutrosophic Orbit Topological Spaces

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Abstract

Neutrosophy is a flourishing arena which conceptualizes the notion of true, falsity and indeterminancy attributes of an event. In the study of dynamical systems, an orbit is a collection of points related by the evolution function of the dynamical system. Hence in this paper we focus on introducing the concept of neutrosophic orbit topological space denoted as \((X, \tau_{NO})\). Also, some of the important characteristics of neutrosophic orbit open sets are discussed with suitable examples.

Keywords: Neutrosophic orbit open set, Neutrosophic orbit topology, Neutrosophic orbit topological spaces

Introduction

Fuzzy concept has invaded almost all branches of Mathematics since its introduction of the concept of fuzzy set by Zadeh [14]. Fuzzy sets have applications in many fields. The idea of fuzzy set is welcomed because it handles uncertainty and vagueness which Cantorian set could not address. However, in reality, it may not always be true that the degree of non-membership of an element in a fuzzy set is equal to 1 minus the membership degree because there may be some hesitation degree. Therefore, a generalization of fuzzy sets was introduced by Atanassov [1] as intuitionistic fuzzy sets (IFS) which incorporated the degree of hesitation called hesitation margin (and is defined as 1 minus the sum of membership and non-membership degrees respectively). As a generalization of intuitionistic fuzzy sets neutrosophic set was formulated by Smarandache [8-10] originally gave the definition of a neutrosophic set and neutrosophic logic. The neutrosophic logic is a formal frame trying to measure the truth, indeterminacy and falsehood. In 2012 Salama and Alblowi [11,12] introduced the concept of neutrosophic topological spaces (NTOP). The concept of orbit function in general metric space was introduced by Devaney [2]. The orbit in mathematics has an important role in the study of dynamical systems. The concept of fuzzy orbit open sets under the mapping \(f : X \to X\) in a fuzzy topological space \((X, \tau)\) was introduced by Malathi and Uma[4]. The concept of fuzzy orbit topological spaces was introduced by Majeed and El-Sheikh[5]. The concept of intuitionistic fuzzy orbit set and intuitionistic fuzzy orbit topological space was introduced by Priscilla and Irudayam [3]. The concept of neutrosophic orbit set was introduced by Madhumathi et al. [6]. The purpose of this paper is to study the collection of all neutrosophic orbit open sets under the mapping \(f : X \to X\). We introduce the necessary conditions on the mapping \(f\) in order to obtain a fixed orbit of a neutrosophic set (i.e., \(f(\mu) = \mu\)) for any neutrosophic orbit open set \(\mu\) under the mapping \(f\). Also, some properties of neutrosophic orbit open sets related with union (intersection) of these sets are introduced. We also prove that the family of all neutrosophic orbit open sets constructs a new neutrosophic topological space. This new space is called neutrosophic orbit topological space \((X, \tau_{NO})\). Furthermore, the concept of neutrosophic orbit interior (closure) is defined. Finally, the category of neutrosophic orbit topological spaces (NOTOP) is defined. And we show this category is isomorphic to a subcategory of the category of NTOP.
Preliminaries

1 Definition [9] Let $X$ be a nonempty set. A neutrosophic set (NS for short) $A$ is an object having the form $A = <x, A^0, A^1, A^2>$ where $A^0, A^1, A^2$ represent the degree of membership, the degree of indeterminacy and the degree of non-membership respectively of each element $x \in X$ to the set $A$.

2 Definition [9] Let $X$ be a nonempty set, $A = <x, A^0, A^1, A^2>$ and $B = <x, B^0, B^1, B^2>$ be neutrosophic sets on $X$, and let $\{A_i : i \in J\}$ be an arbitrary family of neutrosophic sets in $X$, where $A_i = <x, A^0_i, A^1_i, A^2_i>$.

(i) $A \subseteq B$ if and only if $A^0 \leq B^0$, $A^1 \geq B^1$ and $A^2 \geq B^2$.

(ii) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

(iii) $A^0 = B^0$.

(iv) $A \cap B = <x, A^0 \wedge B^0, A^1 \vee B^1, A^2 >$.

(v) $A \cup B = <x, A^0 \vee B^0, A^1 \wedge B^1, A^2 >$.

(vi) $\bigcap A_i = <x, \bigwedge_i A^0_i, \bigvee_i A^1_i, \bigwedge_i A^2_i>$.

(vii) $\bigcup A_i = <x, \bigvee_i A^0_i, \bigwedge_i A^1_i, \bigvee_i A^2_i>$.

(viii) $A \setminus B = A \setminus B$.

(ix) $0_n = <x, 0, 1, 0>, 1_n = <x, 1, 0, 0>$.

3 Definition [11] A neutrosophic topology (NT for short) on a nonempty set $X$ is a family $\tau$ of neutrosophic open sets in $X$ satisfying the following axioms:

(i) $0_n, 1_n \in \tau$.

(ii) $G \cup G_2 \in \tau$ for any $G, G_2 \in \tau$.

(iii) $\bigvee G \in \tau$ for any arbitrary family $\{G_i : i \in \mathcal{I}\} \subseteq \tau$.

In this case the pair $(X, \tau)$ is called a neutrosophic topological space (NTS for short) and any neutrosophic set in $\tau$ is called a neutrosophic open set (NOS for short) in $X$. The complement of a neutrosophic open set $A$ is called a neutrosophic closed set (NCS for short) in $X$.

4 Definition [11] Let $(X, \tau)$ be a neutrosophic topological space and $A = <x, A^0, A^1, A^2>$ be a set in $X$. Then the closure and interior of $A$ are defined by

$\text{Ncl}(A) = \{K : K$ is a neutrosophic closed set in $X$ and $A \subseteq K\}$,

$\text{Nint}(A) = \{G : G$ is a neutrosophic open set in $X$ and $G \subseteq A\}$.

It can be also shown that $\text{Ncl}(A)$ is a neutrosophic closed set and $\text{Nint}(A)$ is a neutrosophic open set in $X$, and $A$ is a neutrosophic closed set in $X$ iff $\text{Ncl}(A) = A$; and $A$ is a neutrosophic open set in $X$ iff $\text{Nint}(A) = A$.

5 Definition [2] Orbit of a point $x$ in $X$ under the mapping $f$ is $O_f(x) = \{x, f(x), f^2(x), \ldots\}$

6 Definition [6] Let $X$ be a nonempty set and $f : X \rightarrow X$ be any mapping. Let $\alpha$ be any neutrosophic set in $X$. The neutrosophic orbit $O_f(\alpha)$ of $\alpha$ under the mapping $f$ is defined as $O_f(\alpha) = \{\alpha, f^1(\alpha), f^2(\alpha), \ldots, f^n(\alpha)\}$, where $O_f(\alpha) = \{\alpha, f^1(\alpha), f^2(\alpha), \ldots, f^n(\alpha)\}$ for $\alpha \in X$ and $n \in \mathbb{Z}$.

7 Definition [6] Let $X$ be a nonempty set and let $f : X \rightarrow X$ be any mapping. The neutrosophic orbit set of $\alpha$ under the mapping $f$ is defined as $\text{NO}_f(\alpha) = \{\alpha, O_f(\alpha), O_{f^2}(\alpha), \ldots\}$ for $\alpha \in X$, where $O_f(\alpha) = \{\alpha, f^1(\alpha), f^2(\alpha), \ldots, f^n(\alpha)\}$, $O_{f^2}(\alpha) = \{\alpha, f^1(\alpha), f^2(\alpha), \ldots, f^{2n}(\alpha)\}$.

8 Definition [6] Let $(X, \tau)$ be a neutrosophic topological space. Let $f : X \rightarrow X$ be any mapping. The neutrosophic orbit set of $\alpha$ under the mapping $f$ which is in neutrosophic topology $\tau$ is called neutrosophic orbit open set under the mapping $f$. Its complement is called a neutrosophic orbit closed set under the mapping $f$.

9 Example Let $X = \{a, b, c\} = Y$. Define $\tau = \{0_n, 1_n, \alpha, \gamma\}$ where $\alpha^T, \gamma^T : X \rightarrow [-0.1, 1^+]$

\begin{align*}
\alpha^T, \gamma^T : X & \rightarrow [-0.1, 1^+] \\
\alpha^T(a) & = 0.3, \alpha^T(b) = 0.6, \alpha^T(c) = 0.1
\end{align*}

are defined as

\begin{align*}
\gamma^T(a) & = 0.1, \gamma^T(b) = 0.8, \gamma^T(c) = 0.7
\end{align*}

\begin{align*}
\alpha^F, \gamma^F : X & \rightarrow [-0.1, 1^+] \\
\alpha^F(a) & = 0.3, \alpha^F(b) = 0.6, \alpha^F(c) = 0.1
\end{align*}

are defined as

\begin{align*}
\alpha^F(a) & = 0.3, \alpha^F(b) = 0.6, \alpha^F(c) = 0.1
\end{align*}

$\gamma^T(a) = 0.1, \gamma^T(b) = 0.8, \gamma^T(c) = 0.7$.
Define \( f : X \to X \) as \( f(a)=c, f(b)=a, f(c)=b \). The neutrosophic orbit set of \( \alpha \) under the mapping \( f \) is defined as \( \text{NO}(\alpha) = \alpha \cap f(\alpha) \cap f^2(\alpha) \cap \cdots \cap f^n(\alpha) \), \( \text{NO}(\alpha) = \gamma \). Then \( \gamma \) is a neutrosophic orbit open set under the mapping \( f \).

From the above definition it is clear that every neutrosophic orbit open set under the mapping \( f \) is an open neutrosophic set in \( X \). But the converse is not true, in this example the neutrosophic set \( \alpha \) is an open neutrosophic set, however it is not neutrosophic orbit open set under the mapping \( f \), because there is not exists a neutrosophic set \( \theta \in f^X \) such that \( \text{NO}(\theta) = \alpha \).

**10 Definition** [13] A mapping \( f : (X, \tau) \to (Y, \tau) \) is called neutrosophic continuous if the inverse image of every closed set in \( Y \) is neutrosophic closed in \( X \).

### Some properties of neutrosophic orbit open sets

In our work we consider \( X \) as a nonempty countable set, we give the conditions on a mapping \( f: X \to X \), to obtain a fixed neutrosophic orbit open set (i.e., \( f(\mu) = \mu \)) for any neutrosophic orbit open set \( \mu \), and study some properties of these sets.

**Theorem 3.1:** Let \((X, \tau)\) be a NTOP and \( f: X \to X \) be any bijective mapping. Then \( f(\mu) = \mu \) for any neutrosophic orbit open set \( \mu \) under the mapping.

**Proof:**

Let \((X, \tau)\) be a NTOP and \( f: X \to X \) be a bijective mapping. Then we have 3 cases:

**Case 1:**

If \( f(x_i) = x_j \), \( x_i \in X \) and \( i, j \in I \). Suppose \( X = \{x_1, x_2\} \) and \( f: X \to X \) defined as \( f(x_1) = x_2, f(x_2) = x_1 \). Let \( \mu \) be a neutrosophic orbit open set under the mapping \( f \).

Then there exists a neutrosophic set \( \lambda \in \mathbb{I}^* \) such that \( \text{NO}(\lambda) = \lambda \cap f(\lambda) \cap f^2(\lambda) \cap \cdots \cap f^n(\lambda) = \mu \).

Let \( \lambda = \{(x_1, u_1, v_1, w_1), (x_2, u_2, v_2, w_2)\} \). Then \( \lambda \cap f(\lambda) = \{(x_1, u_1, v_1, w_1), (x_2, u_2, v_2, w_2)\} \).

Thus \( \text{NO}(\lambda) = \{\lambda, \mu\} \) and \( \mu \) be a neutrosophic orbit open set under the mapping \( f \).

In general, if \( X = \{x_1, x_2, \ldots\} \) and \( \mu \) be a neutrosophic orbit open set under the mapping \( f \), then there exists a neutrosophic set \( \lambda = \{(x_1, u_1, v_1, w_1), \ldots\} \) such that \( \text{NO}(\lambda) = \{\lambda, \mu\} \).

Now for each \( x_i \in X \), we have

\[
f(\mu)(x_i) = \begin{cases} 
\mu(x_i), & \text{if } f^{-1}(x_i) \neq \emptyset \\
(0,1,1), & \text{if } f^{-1}(x_i) = \emptyset 
\end{cases}
\]

From the hypothesis and the definition of \( f \), we get \( f(\mu)(x_i) = \mu(x_i) = r, s, t \) for all \( x_i \in X \). Hence \( f(\mu) = \mu \).

**Case 2:**

If \( f(x_i) = x_j \), \( x_i, x_j \in X \) and \( i, j \in I \). In this case the least number of elements in \( X \) must be 3 elements. So, suppose that \( X = \{x_1, x_2, x_3\} \), then from the hypothesis and the definition of \( f \), the mapping \( f: X \to X \) can be defined as \( f(x_1) = x_2, f(x_2) = x_3, f(x_3) = x_1 \).

Thus \( \text{NO}(\lambda) = \{\lambda, \mu\} \) and \( \mu \) be a neutrosophic orbit open set under the mapping \( f \).

Then there exists a neutrosophic set \( \lambda \in \mathbb{I}^* \) such that \( \text{NO}(\lambda) = \lambda \cap f(\lambda) \cap f^2(\lambda) \cap \cdots \cap f^n(\lambda) = \mu \).

Let \( \lambda = \{(x_1, u_1, v_1, w_1), (x_2, u_2, v_2, w_2), (x_3, u_3, v_3, w_3)\} \). Then \( \text{NO}(\lambda) = \{\lambda, \mu\} \) and \( \mu \) be a neutrosophic orbit open set under the mapping \( f \).

From the definition of \( f \), we get

\[
f(\lambda) = \{(x_1, u_1, v_1, w_1), (x_2, u_2, v_2, w_2), (x_3, u_3, v_3, w_3)\},
\]

\[
f^2(\lambda) = \{(x_1, u_1, v_1, w_1), (x_2, u_2, v_2, w_2), (x_3, u_3, v_3, w_3)\},
\]

\[
\text{NO}(\lambda) = \{\lambda, \mu\}.
\]

Define the neutrosophic orbit open sets as

\[
\text{NO}(\lambda) = \{\lambda, \mu\}.
\]

From the above definition it is clear that every neutrosophic orbit open set under the mapping \( f \) is an open neutrosophic set in \( X \). But the converse is not true, in this example the neutrosophic set \( \alpha \) is an open neutrosophic set, however it is not neutrosophic orbit open set under the mapping \( f \), because there is not exists a neutrosophic set \( \theta \in f^X \) such that \( \text{NO}(\theta) = \alpha \).
(x₃, [min{u₃, u₂}, max{v₃, v₂}, max{w₃, w₂}])
= µ

In general, if X = {x₁, x₂, ...,} and µ be a neutrosophic orbit open set under the mapping f, then there exists a neutrosophic set
λ = {(x₁, u₁, v₁, w₁), (x₂, u₂, v₂, w₂), (x₃, u₃, v₃, w₃), ...} = {(xᵢ, uᵢ, vᵢ, wᵢ); xᵢ ∈ X, uᵢ, vᵢ, wᵢ ∈ l, i ∈ IA}

Such that NOₙ(λ) = µ. This implies

NOₙ(λ) = \{ f(xᵢ) = xⱼ, i = j, i ∈ IA \}
= \{ \{ f(xᵢ) = xⱼ, i ≠ j, i ∈ IA \} \}

= µ

Now for each xᵢ ∈ X, we have
f(µ)(xᵢ) = µ(xᵢ) = \begin{cases} µ(xᵢ) & \text{if } f^⁻¹(xᵢ) ≠ ∅ \\ (0,1,1) & \text{if } f^⁻¹(xᵢ) = ∅ \end{cases}

From the hypothesis and the definition of f, we get for all xᵢ ∈ X:

f(µ)(xᵢ) = \begin{cases} (uᵢ, vᵢ, wᵢ) & \text{if } i = j \\ (r, s, t) & \text{if } i ≠ j \end{cases}

Hence f(µ) = µ.

Case 3:
If f is the identity mapping. In this case, every open neutrosophic set in X is neutrosophic orbit open set under the mapping f. Then f(µ) = µ for every neutrosophic set µ ∈ Iₓ. Thus the proof is obtained.

Theorem 3.2:
Let (X, τ) be a NTOP and f: X → X be any constant mapping. Then f(µ) = µ for any neutrosophic orbit open set µ under the mapping f.

Proof:
Let (X, τ) be a neutrosophic topological space.
Let µ be a neutrosophic orbit open set under the mapping f.
Then, from Definition, there exists a neutrosophic set λ = {(xᵢ, uᵢ, vᵢ, wᵢ); xᵢ ∈ X, uᵢ, vᵢ, wᵢ ∈ l, i ∈ IA} such that NOₙ(λ) = µ.

Since f is constant mapping, this implies there exists a fixed element xₖ ∈ X such that f(xᵢ) = xₖ for all xᵢ ∈ X and i ∈ IA.

Now form, the definition of f(λ) for all xᵢ ∈ X, we have

f(λ)(xᵢ) = \begin{cases} \lambda(xᵢ) & \text{if } f^⁻¹(xᵢ) ≠ ∅ \\ (0,1,1) & \text{otherwise} \end{cases}

Thus

f(λ)(xᵢ) = \begin{cases} \{ sup_{IA}(λ(xᵢ)), inf_{IA}(λ(xᵢ)), sup_{IA}(λ(xᵢ)) \} & \text{if } xᵢ = xₖ, \\ (0,1,1) & \text{if } xᵢ ≠ xₖ \end{cases}

Therefore, f(λ) = \{ \{ xₖ, sup_{IA}(λ(xᵢ)), inf_{IA}(λ(xᵢ)), sup_{IA}(λ(xᵢ)) \}, \\ \text{otherwise} \}

This means f(λ) is a neutrosophic point in X with support xₖ and degree sup_{IA}(λ(xᵢ)), degree inf_{IA}(λ(xᵢ)), degree inf_{IA}(λ(xᵢ)).

By the same way we have

f^²(λ) = \{ \{ xₖ, sup_{IA}(λ(xᵢ)), inf_{IA}(λ(xᵢ)), inf_{IA}(λ(xᵢ)) \}, \\ \text{otherwise} \}

f^³(λ) = \{ \{ xₖ, sup_{IA}(λ(xᵢ)), inf_{IA}(λ(xᵢ)), inf_{IA}(λ(xᵢ)) \}, \\ \text{otherwise} \}

For more clearing we have the following:

λ = \{ (x₁, u₁, v₁, w₁), (x₂, u₂, v₂, w₂), ..., (xₖ, uₖ, vₖ, wₖ), ... \}

f(λ) = \{ (x₁, 0,1,1), (x₂, 0,1,1), ..., (xₖ, sup_{IA}(λ(xᵢ)), inf_{IA}(λ(xᵢ)), inf_{IA}(λ(xᵢ))) \}, \\ \text{otherwise} \}

f^²(λ) = \{ (x₁, 0,1,1), (x₂, 0,1,1), ..., (xₖ, sup_{IA}(λ(xᵢ)), inf_{IA}(λ(xᵢ)), inf_{IA}(λ(xᵢ))) \}, \\ \text{otherwise} \}

f^³(λ) = \{ (x₁, 0,1,1), (x₂, 0,1,1), ..., (xₖ, sup_{IA}(λ(xᵢ)), inf_{IA}(λ(xᵢ)), inf_{IA}(λ(xᵢ))) \}, \\ \text{otherwise} \}

...
Thus \( NO_f(\lambda) = \lambda \cap f(\lambda) \cap f^2(\lambda) \ldots \)
\[ NO_f(\lambda) = \left\{ \begin{array}{ll}
(x_{k}, 0,1,1) & \text{if } i \neq k \\
(x_{k}, \min\{u_{k}, \sup_{i \in \mathcal{A}}(\lambda(x_{i}))\}, \max\{v_{k}, \inf_{i \in \mathcal{A}}(\lambda(x_{i}))\}, \max\{w_{k}, \inf_{i \in \mathcal{A}}(\lambda(x_{i}))\}) & \text{if } i = k
\end{array} \right. \]

This yield \( NO_f(\lambda) = \mu \) is a neutrosophic point in \( X \) with support \( x \) and degree min \( \{u_{k}, \sup_{i \in \mathcal{A}}(\lambda(x_{i}))\}, \max\{v_{k}, \inf_{i \in \mathcal{A}}(\lambda(x_{i}))\}, \max\{w_{k}, \inf_{i \in \mathcal{A}}(\lambda(x_{i}))\} \).

Hence, from the definition of \( f \), we get \( f(\mu) = \mu \).

**Remark 3.3:**
The condition to be \( f : X \rightarrow X \) is bijective or constant is necessary condition to obtain fixed neutrosophic orbit open sets for any neutrosophic orbit open set \( \mu \) under the mapping \( f \).

For more explain, we give an example for a neutrosophic topological space \((X, \tau)\) and \( f : X \rightarrow X \) not bijective, we show that \( (\mu) \neq \mu \) for some neutrosophic orbit open set \( \mu \) under the mapping \( f \).

**Example 3.4:**
Let \( X = \{x_{1}, x_{2}, x_{3}, x_{4}, x_{5}\} \). Define \( \tau = \{\emptyset, T, \mu\} \) where \( \mu \in I^{X} \) defined as \( \mu = \{(x_{1}, (0,1,3)), (x_{2}, (0,1,0.6,0.4)), (x_{3}, (0,0.8,1)), (x_{4}, (0,0.6,0.2,0.4)), (x_{5}, (0,0.6,0.2,0.4))\} \).
Define \( f : X \rightarrow X \) as \( f(x_{1}) = x_{2}, f(x_{2}) = x_{3}, f(x_{3}) = x_{4}, f(x_{4}) = x_{5} \). It is clear that \( f \) is not bijective mapping.(i.e., \( f \) is not one to one and not onto). Let \( \lambda \in I^{X} \) defined as follows:
\( \lambda = \{(x_{1}, 0.1,0.3,0.9), (x_{2}, 0.2,0.4,0.8), (x_{3}, 0.7,0.1,0.3), (x_{4}, 0.6,0.2,0.4), (x_{5}, 0.7,0.1,0.3)\} \).
Then the neutrosophic orbit of \( \lambda \) are \( NO_f(\lambda) = \lambda \cap f(\lambda) \cap f^2(\lambda) \ldots = \mu \) which is
\( f(\lambda) = \{(x_{1}, 0.1,0.3,0.9), (x_{2}, 0.2,0.4,0.8), (x_{3}, 0.7,0.1,0.3), (x_{4}, 0.6,0.2,0.4)\} \)
\( f^2(\lambda) = \{(x_{1}, 0.1,1), (x_{2}, 0.2,0.4,0.8), (x_{3}, 0.7,0.1,0.3), (x_{4}, 0.6,0.2,0.4), (x_{5}, 0.7,0.1,0.3)\} \)
\( f^3(\lambda) = \{(x_{1}, 0.1,1), (x_{2}, 0.1,0.3,0.9), (x_{3}, 0.2,0.4,0.8), (x_{4}, 0.7,0.1,0.3), (x_{5}, 0.6,0.2,0.4)\} \)
Therefore, the neutrosophic orbit set of \( \lambda \) is \( NO_f(\lambda) = \lambda \cap f(\lambda) \cap f^2(\lambda) \ldots = \mu \).
\( NO_f(\lambda) = \{(x_{1}, [\inf, 0.1,0.0,0 \ldots], \sup[0.3,1,1, \ldots], \sup[0.9,1,1, \ldots]), (x_{2}, [\inf, 0.2,0.1,0.2,0.1, \ldots], \sup[0.4,0.3,0.4 \ldots], \sup[0.8,0.9,0.8 \ldots]), (x_{3}, [\inf, 0.1,0.0,0 \ldots], \sup[0.8,0.4,0.3,0.4 \ldots], \sup[1.0,8,0.9,0.8 \ldots]), (x_{4}, [\inf, 0.6,0.7,0.6,0.7 \ldots], \sup[0.2,0.1,0.2,0.1, \ldots], \sup[0.4,0.3,0.4,0.3 \ldots]), (x_{2}, [\inf, 0.7,0.6,0.7,0.6 \ldots], \sup[0.1,0.2,0.1,0.2, \ldots], \sup[0.3,0.4,0.3,0.4 \ldots]), \}

Thus, the open neutrosophic set \( \mu \) is neutrosophic orbit open set under the mapping \( f \). But \( f(\mu) \neq \mu \).
From Theorem 3.1 and 3.2 we obtain the following result

**Result 3.5:**
Let \( (X, \tau) \) be a NTOP and \( f : X \rightarrow X \) be any mapping such that either \( f \) is bijective mapping or \( f \) is constant mapping and \( \mu \) is a neutrosophic orbit open set under the mapping \( f \), then \( f(\mu) = \mu \).
In our work, we consider the mapping \( f : X \rightarrow X \) that satisfies the conditions this Result.

**Proposition 3.6:**
Let \( (X, \tau) \) be a NTOP and \( f : X \rightarrow X \) be any mapping. If \( \mu \) is a neutrosophic orbit open set under the mapping \( f \), then \( NO_f(\mu) = \mu \).

Proof:
The proof follows directly from the definition of \( NO_f(\mu) \) and Result 3.5. i.e., \( NO_f(\mu) = \mu \cap f(\mu) \cap f^2(\mu) \cap \ldots \)
From Result 3.5, we have \( f(\mu) = \mu \), this implies \( f^2(\mu) = f(f(\mu)) = \mu, f^3(\mu) = f(f^2(\mu)) = \mu, \ldots \) Hence, \( NO_f(\mu) = \mu \).

**Theorem 3.7:**
Let \( (X, \tau) \) be a neutrosophic topological space and \( f : X \rightarrow X \) be a mapping.
If \( \mu_1 \) and \( \mu_2 \) are neutrosophic orbit open sets under the mapping \( f \), then \( NO_f(\mu_1 \cap \mu_2) = NO_f(\mu_1) \cap NO_f(\mu_2) \).

Proof:
First we prove the theorem if \( f \) is bijective mapping. From Theorem 3.1, we have 3 cases. We prove the theorem in case 1.
The proof of theorem in case 2 is similar to case 1, and the prove of theorem in case 3 is easy.

Case 1:
Suppose that $f$ is bijective mapping and $f(x_i) = x_j, x_i, x_j \in X$ and $i \neq j$ for all $i, j \in \Lambda$.

Let $\mu_1$ and $\mu_2$ are neutrosophic orbit open sets under the mapping $f$.

Then, there exist $\lambda_1, \lambda_2, x_i, x_j \in X$ defined as $\lambda_1 = \{(x_i, u_i, v_i, w_i); x_i, x_j \in X, u_i, v_i, w_i \in I, i \in \Lambda\}$ and $\lambda_2 = \{(x_i, t_i); x_i \in X, r_i, s_i, t_i \in I, i \in \Lambda\}$ such that $NO_f(\lambda_1) = \mu_1$ and $NO_f(\lambda_2) = \mu_2$.

From Theorem 3.1 case 1, we have $NO_f(\lambda_1) = \{(x_i, (u_i, v, w)); u = \inf\{u_i, i \in \Lambda\}, v = \sup\{v_i, i \in \Lambda\}\}$, $w = \sup\{w_i, i \in \Lambda\}$

And $NO_f(\lambda_2) = \{(x_i, (r_i, s, t); r = \inf\{r_i, i \in \Lambda\}, s = \sup\{s_i, i \in \Lambda\}\}$, $t = \sup\{t_i, i \in \Lambda\} = \mu_2$.

Thus, $\mu_1 \cap \mu_2 = \{(x_i, [\min\{u, r\}], \max\{v, s\}, \max\{w, t\}); x_i \in X, i \in \Lambda\}$.

Let $a = \min\{u, r\}, b = \max\{v, s\}, c = \max\{w, t\}$.

Now for all $x_i \in X, j \in \Lambda$.

$f(\mu_1 \cap \mu_2)(x_i) = \bigcup_{f(x_i) = x_j} (\mu_1 \cap \mu_2)(x_j)$ if $f^{-1}(x_j) \neq \emptyset$

$f(\mu_1 \cap \mu_2)(x_i) = (0, 1, 1)$ if $f^{-1}(x_j) = \emptyset$

Hence $f(\mu_1 \cap \mu_2) = \mu_1 \cap \mu_2$.

This implies $f^2(\mu_1 \cap \mu_2) = \mu_1 \cap \mu_2, f^3(\mu_1 \cap \mu_2) = \mu_1 \cap \mu_2$ ...

Therefore, from the definition of $NO_f(\mu_1 \cap \mu_2)$ and Theorem 3.1 we get $NO_f(\mu_1 \cap \mu_2) = \mu_1 \cap \mu_2 = NO_f(\lambda_1) \cap NO_f(\lambda_2)$.

Case 2:
Suppose that $f$ is bijective mapping and $f(x_i) = x_j, x_i, x_j \in X$ and $i = j$ for all $i, j \in \Lambda$.

Let $\mu_1$ and $\mu_2$ are neutrosophic orbit open sets under the mapping $f$.

Then, there exist $\lambda_1, \lambda_2, x_i, x_j \in X$ defined as $\lambda_1 = \{(x_i, u_i, v_i, w_i); x_i \in X, u_i, v_i, w_i \in I, i \in \Lambda\}$ and $\lambda_2 = \{(r_i, s_i, t_i); x_i \in X, r_i, s_i, t_i \in I, i \in \Lambda\}$ such that $NO_f(\lambda_1) = \mu_1$ and $NO_f(\lambda_2) = \mu_2$.

From Theorem 3.1 case 2, we have $NO_f(\lambda_1) = \{(x_i, (u_i, v, w)); f(x_i) = x_j, i = j, (x_i, u = \inf\{u_i, i \in \Lambda\}, v = \sup\{v_i, i \in \Lambda\}, w = \sup\{w_i, i \in \mu_2\})\}

And $NO_f(\lambda_2) = \{(x_i, (r_i, s, t); f(x_i) = x_j, i = j, (x_i, r = \inf\{r_i, i \in \Lambda\}, s = \sup\{s_i, i \in \Lambda\}, t = \sup\{t_i, i \in \Lambda\})\}$.

Thus $\mu_1 \cap \mu_2 = \{(x_i, [\min\{u, r\}], \max\{v, s\}, \max\{w, t\}); f(x_i) = x_j, i = j\}$

Let $k = \min\{m_i, n_i\}, t = \max\{m_i, n_i\}$.

Now for all $x_i \in X, j \in \Lambda$.

$f(\mu_1 \cap \mu_2)(x_i) = \bigcup_{f(x_i) = x_j} (\mu_1 \cap \mu_2)(x_j)$ if $f^{-1}(x_j) \neq \emptyset$

$f(\mu_1 \cap \mu_2)(x_i) = (0, 1, 1)$ if $f^{-1}(x_j) = \emptyset$

Hence $f(\mu_1 \cap \mu_2) = \mu_1 \cap \mu_2$.

This implies $f^2(\mu_1 \cap \mu_2) = \mu_1 \cap \mu_2, f^3(\mu_1 \cap \mu_2) = \mu_1 \cap \mu_2$ ...

Therefore, from the definition of $NO_f(\mu_1 \cap \mu_2)$ and Theorem 3.1 we get $NO_f(\mu_1 \cap \mu_2) = \mu_1 \cap \mu_2 = NO_f(\lambda_1) \cap NO_f(\lambda_2)$.

Case 3:
Now, if $f$ is constant mapping, let $\mu_1$ and $\mu_2$ are neutrosophic orbit open sets under the mapping $f$.

Then, there exist $\lambda_1, \lambda_2, x_i, x_j \in I^X$ defined as $\lambda_1 = \{(x_i, u_i, v_i, w_i); x_i \in X, u_i, v_i, w_i \in I, i \in \Lambda\}$ and $\lambda_2 = \{(x_i, t_i); x_i \in X, r_i, s_i, t_i \in I, i \in \Lambda\}$ such that $NO_f(\lambda_1) = \mu_1$ and $NO_f(\lambda_2) = \mu_2$.

From Theorem 3.2, we have $NO_f(\lambda_1) = \{(x_i, 0, 1, 1)\}$ if $i \neq k$

$\{(x_i, \min\{u_i\}, \sup\{v_i, s_i\}, \max\{w_i, t_i\})\} = \mu_1$

$NO_f(\lambda_2) = \{(x_i, 0, 0, 0)\}$ if $i \neq k$
Thus
\[
\mu_1 \cap \mu_2 = \left\{ \begin{array}{ll}
(x, \min\{\min\{(u_k, \sup_{i \in A}(\lambda_1(x_i)))\}, \min\{(r_k, \sup_{i \in A}(\lambda_2(x_i)))\}], \\
\max\{\max\{(s_k, \inf_{i \in A}(\lambda_1(x_i)))\}, \max\{(t_k, \inf_{i \in A}(\lambda_2(x_i)))\}] & \text{if } i \neq k
\end{array} \right.
\]
This means that \(\mu_1 \cap \mu_2\) is a neutrosophic point in \(X\) with support \(x_k\) and degree

\[
\min\{\min\{(u_k, \sup_{i \in A}(\lambda_1(x_i)))\}, \min\{(r_k, \sup_{i \in A}(\lambda_2(x_i)))\}].
\]

Hence from the definition of \(f\) we get \((\mu_1 \cap \mu_2) = \mu_1 \cap \mu_2\). This implies \(f^2(\mu_1 \cap \mu_2) = \mu_1 \cap \mu_2\).

Therefore, from the definition of \(NO_f(\mu_1 \cap \mu_2)\) and Theorem 3.2 we get

\(NO_f(\mu_1 \cap \mu_2) = NO_f(\mu_1) \cap NO_f(\mu_2)\).

Hence the proof.

**Theorem 3.8:**
Let \((X, r)\) be a NTOP and \(f: X \rightarrow X\) be a mapping. Let \(\{\mu_\alpha\}_{\alpha \in \Delta}\) be any family of neutrosophic orbit open sets under the mapping \(f\), then \(NO_f(U_{\alpha \in \Delta} \mu_\alpha) = U_{\alpha \in \Delta} NO_f(\mu_\alpha)\).

Proof:
The outline of proofing this theorem is proceeds in a way similar to Theorem 3.7. As in Theorem 3.7, we consider 3 cases:

Case 1:
Suppose that \(f\) is bijective mapping and \(f(x_i) = x_j; x_i, x_j \in X\) and \(i \neq j\) for all \(i, j \in A\).

Let \(\{\mu_\alpha\}_{\alpha \in \Delta}\) be any family of neutrosophic orbit open sets under the mapping \(f\), then there exists \(\lambda_\alpha \in I^X, \alpha \in \Delta\) defined as
\[
\lambda_\alpha = \{\left(\lambda_i, u_{i\alpha}, v_{i\alpha}, w_{i\alpha}\right); x_i \in X, u_{i\alpha}, v_{i\alpha}, w_{i\alpha} \in I, i \in A\}\text{ such that } NO_f(\lambda_\alpha) = \mu_\alpha \text{ for all } \alpha \in \Delta.
\]
From Theorem 3.1 case 1, we have
\[
NO_f(\lambda_\alpha) = \{\lambda_i, u_{i\alpha}, v_{i\alpha}, w_{i\alpha}\}; u_{i\alpha} = \inf\{u_{i\alpha}, i \in A\}, v_{i\alpha} = \sup\{v_{i\alpha}, i \in A\}, w_{i\alpha} = \sup\{w_{i\alpha}, i \in A\}\} = \mu_\alpha
\]
Thus \((U_{\alpha \in \Delta} \mu_\alpha) = \{\lambda_i, [\sup_{i \in A}(u_{i\alpha})], \inf_{i \in A}(v_{i\alpha}), \inf_{i \in A}(w_{i\alpha}); x_i \in X, i \in A\}.

Let \(a = \sup_{i \in A}(u_{i\alpha}), b = \inf_{i \in A}(v_{i\alpha}), c = \inf_{i \in A}(w_{i\alpha}).\) Now for all \(x_i \in X, i \in A\).

\[
f(U_{\alpha \in \Delta} \mu_\alpha)(x_i) = \begin{cases}
U_{f(x_i)=x_j}(U_{f(x_i)=x_j} \mu_\alpha)(x_i) \\
(0,1,1)
\end{cases}
\]
if \(f^{-1}(x_i) \neq \emptyset\)

\[
= (a,b,c)
\]
Hence \(f(U_{\alpha \in \Delta} \mu_\alpha) = (U_{\alpha \in \Delta} \mu_\alpha)\). This implies \(f^2(U_{\alpha \in \Delta} \mu_\alpha) = (U_{\alpha \in \Delta} \mu_\alpha), f^3(U_{\alpha \in \Delta} \mu_\alpha) = (U_{\alpha \in \Delta} \mu_\alpha), \ldots\)

Therefore from the definition of \(NO_f(U_{\alpha \in \Delta} \mu_\alpha)\) and Theorem 3.1 we get
\[
NO_f(U_{\alpha \in \Delta} \mu_\alpha) = (U_{\alpha \in \Delta} \mu_\alpha) = U_{\alpha \in \Delta} NO_f(\mu_\alpha)
\]

Case 2:
Suppose that \(f\) is bijective mapping and \(f(x_i) = x_j; x_i, x_j \in X\) and \(i = j\) for all \(i, j \in A\).

Let \(\{\mu_\alpha\}_{\alpha \in \Delta}\) be any family of neutrosophic orbit open sets under the mapping \(f\), then there exists \(\lambda_\alpha \in I^X, \alpha \in \Delta\) defined as
\[
\lambda_\alpha = \{\left(\lambda_i, u_{i\alpha}, v_{i\alpha}, w_{i\alpha}\right); x_i \in X, u_{i\alpha}, v_{i\alpha}, w_{i\alpha} \in I, i \in A\}\text{ such that } NO_f(\lambda_\alpha) = \mu_\alpha \text{ for all } \alpha \in \Delta.
\]
From Theorem 3.1 case 2 we have
\[
NO_f(\lambda_\alpha) = \{\lambda_i, u_{i\alpha}, v_{i\alpha}, w_{i\alpha}\}; f(x_i) = x_j, i = j, \{\inf_{i \in A}(u_{i\alpha}), \sup_{i \in A}(v_{i\alpha}), \sup_{i \in A}(w_{i\alpha})\}
\]
\[
\text{put } u_{i\alpha} = \inf_{i \in A}(u_{i\alpha}), v_{i\alpha} = \sup_{i \in A}(v_{i\alpha}), w_{i\alpha} = \sup_{i \in A}(w_{i\alpha}), \text{it follows:}
\]
\[
(U_{\alpha \in \Delta} \mu_\alpha) = \{\lambda_i, [\sup_{i \in A}(u_{i\alpha})], \inf_{i \in A}(v_{i\alpha}), \inf_{i \in A}(w_{i\alpha}); f(x_i) = x_i, i = j, \lambda_i, u_{i\alpha}, v_{i\alpha}, w_{i\alpha} \in I, i \in A\}
\]
\[
\text{let } k = \inf_{i \in A}(1 - (m_{i\alpha}, n_{i\alpha})), t = \inf_{i \in A}(\inf_{i \in A}(m_{i\alpha}), \sup_{i \in A}(n_{i\alpha}), \text{ and } q = \inf_{i \in A}(1 - \{m_{i\alpha}, n_{i\alpha}\})
\]
\[
\text{Now for all } x_i \in X, t \in A
\]
\[
f(U_{\alpha \in \Delta} \mu_\alpha)(x_i) = \begin{cases}
U_{f(x_i)=x_j}(U_{f(x_i)=x_j} \mu_\alpha)(x_i) \\
(0,1,1)
\end{cases}
\]
if \(f^{-1}(x_i) \neq \emptyset\)
Let \((\sup_{\alpha \in \Delta}(u_{\alpha i}), \inf_{\alpha \in \Delta}(v_{\alpha i}), \inf_{\alpha \in \Delta}(w_{\alpha i}))\) if \(f(x_i) = x_j, i = j,
\sup_{\alpha \in \Delta}(u_{\alpha i}), \inf_{\alpha \in \Delta}(v_{\alpha i}), \inf_{\alpha \in \Delta}(w_{\alpha i})\) if \(f(x_i) = x_j, i \neq j\)
Hence \(f(U_{\alpha \in \Delta} M_a) = (U_{\alpha \in \Delta} M_a)\). This implies \(f^2(U_{\alpha \in \Delta} M_a) = (U_{\alpha \in \Delta} M_a), f^3(U_{\alpha \in \Delta} M_a) = (U_{\alpha \in \Delta} M_a)\), ...
Therefore from the definition of \(NO_f(U_{\alpha \in \Delta} M_a)\) and Theorem 3.1 we get \(NO_f(U_{\alpha \in \Delta} M_a) = (U_{\alpha \in \Delta} NO_f(M_a))\)

**Case 3:**
Now if \(f\) is any constant mapping,
Let \(\{\mu_a\}_{\alpha \in \Delta}\) be any family of neutrosophic orbit open sets under the mapping \(f\), then there exists \(\lambda_a \in I^\alpha, \alpha \in \Delta\) defined as \(\lambda_a = \{(x_i, u_{\alpha i}, v_{\alpha i}, w_{\alpha i}) ; x_i \in X, u_{\alpha i}, v_{\alpha i}, w_{\alpha i} \in I, i \in \Delta\}\) such that \(NO_f(\lambda_a) = \mu_a\) for all \(\alpha \in \Delta\).
From Theorem 3.2 we get,
\(NO_f(\lambda_a) = (x_j, 0,1,1)\) if \(i \neq k\)
\(= \{(x_k, \sup_{\alpha \in \Delta}(\min(u_{\alpha k}, \sup_{\alpha \in \Delta}(\lambda_a(x_i)))), \inf_{\alpha \in \Delta}(\max(v_{\alpha k}, \inf_{\alpha \in \Delta}(\lambda_a(x_i)))), \inf_{\alpha \in \Delta}(\max(w_{\alpha k}, \inf_{\alpha \in \Delta}(\lambda_a(x_i))))\}\) if \(i = k\)

This means \((U_{\alpha \in \Delta} M_a)\) is a neutrosophic point in \(X\) with support \(x_k\) and degree \(\sup_{\alpha \in \Delta}(\min((m_{\alpha k}), \sup_{\alpha \in \Delta}(\lambda_a(x_i))))\)
Hence from the definition of \(f\) we get \(f(U_{\alpha \in \Delta} M_a) = (U_{\alpha \in \Delta} M_a)\). This implies \(f^2(U_{\alpha \in \Delta} M_a) = (U_{\alpha \in \Delta} M_a), f^3(U_{\alpha \in \Delta} M_a) = (U_{\alpha \in \Delta} M_a)\), ...
Therefore from the definition of \(NO_f(U_{\alpha \in \Delta} M_a)\) and Theorem 3.2 we get \(NO_f(U_{\alpha \in \Delta} M_a) = (U_{\alpha \in \Delta} NO_f(M_a))\)
Hence the proof.

**Neutrosophic orbit topological spaces**

In this section we show that the family of all neutrosophic orbit open sets under the mapping \(f\) constrict a neutrosophic orbit topology on \(X\), denoted by \(\tau_{NO}\) which is coarser than \(\tau\).

**Theorem 4.1:**
Let \((X, \tau)\) be a NTS and \(f : X \to X\) be a mapping. Let \(\tau_{NO}\) denote to the family of all neutrosophic orbit open sets under the mapping \(f\). Then, \(\tau_{NO}\) is a neutrosophic topology on \(X\) coarser than \(\tau\).

**Proof:**
We must show \(\tau_{NO}\) satisfies the 3 axioms of the definition of neutrosophic topology. It is clear that \(\overline{0}\) and \(\overline{1}\) are neutrosophic orbit open sets because there exist \(\lambda = \overline{0}\) and \(\nu = \overline{1}\) such that \(NO_f(\lambda) = \overline{0} \in \tau\) and \(NO_f(\nu) = \overline{1} \in \tau\).
Hence the definition of \(f\) we get \(f(U_{\alpha \in \Delta} M_a) = (U_{\alpha \in \Delta} M_a)\). This implies \(f^2(U_{\alpha \in \Delta} M_a) = (U_{\alpha \in \Delta} M_a), f^3(U_{\alpha \in \Delta} M_a) = (U_{\alpha \in \Delta} M_a)\), ...
Therefore from the definition of \(NO_f(U_{\alpha \in \Delta} M_a)\) and Theorem 3.2 we get \(NO_f(U_{\alpha \in \Delta} M_a) = (U_{\alpha \in \Delta} NO_f(M_a))\)
Hence the proof.

Let \((X, \tau)\) be a NTONP and \(f : X \to X\) be a mapping. The pair \((X, \tau_{NO})\) is called neutrosophic orbit topological space (NOT) associated with \((X, \tau)\) if it satisfies the following axioms

(i) \(\overline{0} \in \tau_{NO}\) and \(\overline{1} \in \tau_{NO}\)
(ii) \(G_1 \cap G_2 \in \tau_{NO}\) for any \(G_1, G_2 \in \tau_{NO}\)
(iii) $\cup G_i \in \tau_{NO}$, for any arbitrary family $\{G_i, G_i \in \tau_{NO}, i \in I\}$

**Example 4.3:**

1. For any nonempty countable set $X$, $\tau_{NO}^0 = \{\emptyset, \overline{X}\}$ is a neutrosophic orbit topology on $X$, and is called the indiscrete neutrosophic orbit topology.
2. For any nonempty countable set $X$, if $f: X \to X$ is the identity mapping, then $\tau_{NO} = \tau$

Next the notion of neutrosophic orbit closure (resp. interior) of a neutrosophic set is introduced.

**Definition 4.4:**

Let $(X, \tau_{NO})$ be a neutrosophic orbit topological space and $\lambda \in I^X$. The neutrosophic orbit closure of $\lambda$, denoted by $cl_{NO}(\lambda)$, is the intersection of all neutrosophic orbit closed supersets under the mapping $f$ of $\lambda$ i.e.,

$$cl_{NO}(\lambda) = \cap \{\rho \in I^X / \rho \supseteq \lambda, \overline{\rho} \in \tau_{NO}\}$$

And, the neutrosophic orbit interior of $\lambda$, denoted by $int_{NO}(\lambda)$, is the union of all neutrosophic orbit open subsets under the mapping $f$ of $\lambda$ i.e.,

$$int_{NO}(\lambda) = \cup \{\rho \in I^X / \rho \subseteq \lambda, \rho \in \tau_{NO}\}$$

Clearly, $cl_{NO}(\lambda)$ (resp., $int_{NO}(\lambda)$) is the smallest (resp., largest) neutrosophic orbit closed (resp., open) set under the mapping $f$ which contains (resp., contained in) $\lambda$.

**Proposition 4.5:**

Let $(X, \tau_{NO})$ be a neutrosophic orbit topological space and $\lambda \in I^X$. Then $int_{NO}(\lambda) \subseteq int(\lambda) \subseteq \lambda \subseteq cl(\lambda) \subseteq cl_{NO}(\lambda)$

Proof:

The proof follows directly from the fact that every neutrosophic orbit closed (resp., open) set under the mapping $f$ is closed (resp., open) neutrosophic set.

**Proposition 4.6:**

Let $(X, \tau_{NO})$ be a neutrosophic orbit topological space and $\lambda, \mu \in I^X$. Then

1. $cl_{NO}(\emptyset) = \emptyset$ and $cl_{NO}(\overline{X}) = \overline{X}$.
2. $\lambda \subseteq cl_{NO}(\lambda)$
3. $cl_{NO}(\lambda \cup \mu) = cl_{NO}(\lambda) \cup cl_{NO}(\mu)$
4. If $\lambda \subseteq \mu$, then $cl_{NO}(\lambda) \subseteq cl_{NO}(\mu)$
5. $cl_{NO}(cl_{NO}(\lambda)) = cl_{NO}(\lambda)$
6. If $\lambda$ is neutrosophic orbit closed set under the mapping $f$, then $\lambda = cl_{NO}(\lambda)$.

Proof: Straightforward

**Proposition 4.7:**

Let $(X, \tau_{NO})$ be a neutrosophic orbit topological space and $\lambda, \mu \in I^X$. Then

1. $int_{NO}(\emptyset) = \emptyset$ and $int_{NO}(\overline{X}) = \overline{X}$.
2. $int_{NO}(\emptyset) \subseteq \lambda$.
3. $int_{NO}(\lambda \cup \mu) = int_{NO}(\lambda) \cup int_{NO}(\mu)$
4. If $\lambda \subseteq \mu$, then $int_{NO}(\lambda) \subseteq int_{NO}(\mu)$
5. $int_{NO}(int_{NO}(\lambda)) = int_{NO}(\lambda)$
6. If $\lambda$ is neutrosophic orbit open set under the mapping $f$, then $\lambda = int_{NO}(\lambda)$.

Proof: Straightforward

**Theorem 4.8:**

Let $(X, \tau_{NO})$ be a neutrosophic orbit topological space and $\lambda \in I^X$. Then,

1. $\overline{X} - int_{NO}(\lambda) = cl_{NO}(\overline{X} - \lambda)$.
2. $\overline{X} - cl_{NO}(\lambda) = int_{NO}(\overline{X} - \lambda)$.

Proof:

We prove that part 1 and by the similar way one can prove part 2. From Proposition 4.7 part 2 $int_{NO}(\lambda) \subseteq \lambda$ so by taking the complement we have $\overline{\lambda} - \lambda \subseteq \overline{\lambda} - int_{NO}(\lambda)$. Since $\overline{\lambda} - int_{NO}(\lambda)$ is a neutrosophic orbit closed set and by Proposition 4.6 part 4, $cl_{NO}(\overline{\lambda} - \lambda) \subseteq cl_{NO}(\overline{\lambda} - int_{NO}(\lambda)) = \overline{\lambda} - int_{NO}(\lambda)$. Hence $cl_{NO}(\overline{\lambda} - \lambda) \subseteq \overline{\lambda} - int_{NO}(\lambda)$. 


Conversely, by Proposition 4.6 part 2, $\overline{T} - \lambda \subseteq cl_{\Omega}(\overline{T} - \lambda)$. By taking the complement $\overline{T} - cl_{\Omega}(\overline{T} - \lambda) \subseteq \lambda$. Since $cl_{\Omega}(\overline{T} - \lambda)$ is a neutrosophic orbit closed set. Then $\overline{T} - cl_{\Omega}(\overline{T} - \lambda)$ is a neutrosophic orbit open set and by Proposition 4.7 part 6, we have $\overline{T} - cl_{\Omega}(\overline{T} - \lambda) \subseteq int_{\Omega}(\lambda)$, again by taking the complement we obtain, $\overline{T} - int_{\Omega}(\lambda) \subseteq cl_{\Omega}(\overline{T} - \lambda)$.

**Theorem 5.5:**
But $f$ is not neutrosophic continuous with respect to $\tau$ and $\tau'$ with respect to $\tau$.

**Proof:**
Let $\nu \in I$.

**Example 5.4:**

**Remark 5.3:**
Set mapping. Thus, we get to the required result.

**First, from Theorem 4.9,** the composition of neutrosophic continuous mappings between NOTOP is also neutrosophic continuous, hence the composition of morphisms is well defined and associative. Second, to each object $(X, \tau_{\text{NO}})$ in NOTOP define the identity morphism $1_{(X, \tau_{\text{NO}})}: (X, \tau_{\text{NO}}) \to (X, \tau_{\text{NO}})$ by the idtity set mapping. Thus, we get to the required result.

**Remark 5.5:**
NOTOP is not a subcategory of NOTOP, because if $f$ is intuitionistic fuzzy continuous from $(X, \tau_{\text{NO}})$ to $(Y, \tau'_{\text{NO}})$, then $f$ need not to be neutrosophic continuous from $(X, \tau_{\text{NO}})$ to $(Y, \tau'_{\text{NO}})$.

**Example 5.4:**
Let $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1, y_2, y_3\}$. Define $\tau = \{0, 1, \lambda\}$ and $\tau' = \{0, 1, \mu_1, \mu_2\}$ where $\mu_1 \in \{0, 1, \lambda\}$ and $\mu_2 \in \{0, 1, \lambda\}$ such that $\lambda = \{(x_1, 0.2, 0.4, 0.8), (x_2, 0.3, 0.5, 0.7), (x_3, 0.3, 0.5, 0.7)\}$, $\mu_1 = \{(y_1, 0.2, 0.4, 0.8), (y_2, 0.3, 0.5, 0.7), (y_3, 0.3, 0.5, 0.7)\}$ and $\mu_2 = \{(y_1, 0.2, 0.4, 0.8), (y_2, 0.3, 0.5, 0.7), (y_3, 0.3, 0.5, 0.7)\}$ clearly $(X, \tau, (Y, \tau))$ are NOTOP.

**Define:**
$f_1: X \to Y$, $f_1(x_1) = y_1$, $f_1(x_2) = y_2$, $f_1(x_3) = y_3$, $f_2: X \to Y$, $f_2(x_1) = x_1$, $f_2(x_2) = x_2$, $f_2(x_3) = x_3$, $f_3: X \to Y$, $f_3(x_1) = x_1$, $f_3(x_2) = x_2$, $f_3(x_3) = x_3$.

Then $\tau_{\text{NO}} = \{0, 1, \lambda\}$ and $\tau'_{\text{NO}} = \{0, 1, \mu_1, \mu_2\}$, it is clear that $f_3$ is neutrosophic continuous with respect to $\tau_{\text{NO}}$ and $\tau'_{\text{NO}}$.

**But f is not neutrosophic continuous with respect to $\tau_{\text{NO}}$ and $\tau'_{\text{NO}}$ since $\mu_2$ is an open neutrosophic set in $X$, however $f^{-1}(\mu_2)$ is not neutrosophic open set in X.

**Theorem 5.5:**
NOTOP isomorphic to a subcategory of NOTOP.

**Proof:**
Let NOTOP be a collection $\{X, I_X\}$ of objects in NOTOP, such that $I_X$ is the indiscrete neutrosophic topology on X. For any pair of objects $(X, I_X), (Y, I_Y)$ of NOTOP, we take $\text{Mor}(X, I_X), (Y, I_Y)$ (in NOTOP) as the set of morphisms in NOTOP. Then it is clear NOTOP is a subcategory of NOTOP. Now define $F$: NOTOP $\to$ NOTOP by $F((X, I_X)) = (X, I_X)$ and for each morphism $f: (X, I_X) \to (Y, I_Y)$ define $F(f) = f: (X, I_X) \to (Y, I_Y)$. It can be verified that $F$ is indeed a bijective functor. Thus NOTOP isomorphic to NOTOP.
Remark 5.6: From the above theorem, we can say that NOTOP is embedded in NTOP as a subcategory.

Conclusions

In this paper, we study the collection of neutrosophic orbit open sets under the mapping \( f: X \to X \). We give the necessary conditions on the mapping \( f \) in order to obtain a fixed orbit of a neutrosophic set for any neutrosophic orbit open set under the mapping \( f \). As a main result, we prove the family of all neutrosophic orbit open sets constructs a neutrosophic orbit topological space. In addition, the category of NOTOP and neutrosophic continuous mappings NOTOP is defined. And we show this category is isomorphic to a subcategory of the category of NTOP.

References