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Investigation of Transverse Vibration Characteristics of Cracked Axially Moving Functionally Graded Beam Under Thermal Load

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Abstract

This article presents a new analysis of the vibration characteristics of open-edge cracks for graded moving beams under thermal load. The material property gradient is based on the distribution of the power law in the direction of the beam thickness. The vibration equation is obtained depending on the precept of Hamilton and resolved by the extension of Galerkin's approach. A rotational spring is used to represent the cracking in the beam. The effects of the axial velocity, gradient index, thermal load, and cracking parameters on vibration characteristics are observed. Furthermore, the mode shapes of the simple support cracked moving graded beam are determined. The results show that the rise in the axial velocity, the crack depth, and the material property index led to a decrease in the natural frequencies, which indicates the results obtained.

Keywords: Dynamic stability, Cracks, Functionally graded material, Beam, Axially moving

Introduction

In nature and engineering practice, the vibration phenomenon of the axial motion system is very common, and it is widely present in military, aerospace, mechanical, electronic, and civil engineering. For example, power transmission belts, magnetic tapes, textile fibers, band saws, paper, composite laminates used in aerospace engineering, etc., can be simplified to beam models for study. Many authors have done much research on the linearity, nonlinear vibration, and stability of axially moving beams.

Based on Euler's theory, the vibration equation for an axially moving beam on a viscoelastic foundation and subjected to an external transverse excitation was derived by Zhang *et al.* [1]. The semi-analytically based on the complex modal method was utilized to solve the vibration equation. The results show that, when the moving speed reaches the lowest divergence speed, the first natural frequency vanishes, also, as the damping factor and stiffness increase, the response amplitude decreases. Moslemi *et al.* [2] derived the equation of the moving homogeneous beam according to Euler's model. The harmonic balance method was used to solve the forced vibration equation. Some factors affecting the system behavior were studied, including axial velocity, excitation frequency, and magnitude of the external force. The results indicated that the Hopf bifurcation boundaries were decreased when axial speed increased. Chen and Chen [3] demonstrated the effect of the axial compression force on the vibration characteristics of a moving homogeneous Timoshenko beam. The vibration equations were solved using Galerkin and differential quadrature techniques. The effect of a moving speed and compressive load factor on the dynamic response was debated. Ding *et al.* [4] studied the nonlinear coupled vibration of an axially moving beam at a supercritical speed based on the discrete Fourier transform, and compared the results with the Galerkin method. Yao *et al.* [5] combined multi-time scale and Galerkin truncation method to study the nonlinear dynamic behavior of axial motion with parametric excitation under multi-pulse excitation. Yang *et al.* [6] applied Galerkin and complex modal methods to study the transverse vibration and stability of axially moving sandwich beams. Zhu and Chung [7] studied the vibrations and stability of axially moving Rayleigh beams. They examined the gyroscopic properties of the system and found that as the beam's rotational inertia increased, the stability of the system decreased. Chen and Yang [8] studied the parametric stability of axially moving viscoelastic beams with oscillating axial velocities and hybrid supports. The natural frequencies, modal functions, and critical velocities are determined for various system parameters have been determined. Ghayesh and Amabili [9] studied the forced nonlinear dynamics of axial moving beams numerically and extracted the steady-state response and system bifurcation points. Bağdatli and Uslu [10] derived the vibration equation of a moving beam exposed to non-ideal conditions relying on the extension of Hamilton. They found that the non-ideal boundary conditions' impacts on natural clamped beam frequencies are more important than that of simple support beams. Also, the axial velocity rise causes the natural frequencies of simply supported and clamped-clamped beams to drop. The bending solutions of the Timoshenko beam (TB) made from FGM in terms of the homogenous Euler-Bernoulli (EB) structure were obtained analytically [11]. Ding *et al.* [12] explained the free vibration of an axially moving beam with free, pinned, clamped end conditions. The dynamic stiffness approach was utilized to resolve the motion equations derived based on Euler-Bernoulli and Timoshenko models.

In all the articles reviewed above, the materials used in the structure were made of homogeneous materials. In recent years, the functionally graded beams with axial motion have attracted the attention of many researchers because the graded materials have a wide range of applications in complex engineering industries compared to homogeneous materials, and also possess excellent advantages such as better corrosion resistance, lower stress concentration, and lower fracture toughness and temperature [13,14]. Hence, the application of functionally gradient (FG) materials in the moving beam can lead to prominent results. For example, the vibrations of flexible axial moving beams made of graded materials based on the finite element method have been studied by Piovan and Sampaio [15]. Their results showed that the use of metal as the main component of the beam compared to ceramic leads to less frequency oscillation in the system. Yao *et al.* [16] studied the transverse free vibration of a graded axial moving beam using Timoshenko's beam theory. They studied the effect of various parameters such as axial velocity and power law index on natural frequencies. The stability of the graded axial moving beam at a time-dependent rate was investigated [17]. The direct multiscale method was used to obtain the stability ranges of the system. The results showed that the stability threshold of the system decreases and increases with increasing axial velocity and support stiffness parameters respectively. Ji *et al.* [18] established the motion equation of a moving graded Euler nanobeam whose properties follow count on the distribution of the power law through the direction of the thickness and adopting Hamilton's principle in driving this equation. Natural frequencies were found by employing the method of the complex mode. Shariati *et al.* [19] found the vibration equation of the viscoelastic axially moving graded beam in which properties change axially relying on exponential law distribution according to Euler and Rayleigh models. Galerkin's method was utilized to solve the vibration equation and find natural frequencies. The results explained that a low rotary inertia factor leads to making the structure more stable.

On the other hand, crack is a major factor in the failure of various engineering structures. The presence of cracks in the geometric structure leads to a decrease in the rigidity of the structure and thus affects the vibration characteristics of the structure [20]. The crack affects the vibration characteristics depending on certain factors such as the crack depth, the crack position along the structure, and the number of cracks in the structure [21]. Therefore, many studies have been conducted to study the dynamic behavior of engineering structure parts subjected to cracks, whether these parts are made of homogeneous materials or made of functionally graded materials. The influence of Winkler-type elastic foundation and constrained elastically springs on the dynamic behavior of the cracked Timoshenko beam model was analyzed by Rosa and Lippiello [22]. The equation of motion is derived based on the Timoshenko beam theory and auxiliary functions were employed. An analytical method was used to determine the vibration frequency and mode shapes. Liu *et al.* [23] applied the exact method to study the vibrations of a cracked non-uniform beam with different ends using the transfer matrix approach. The crack was in the form of a massless torsional spring with sectional flexibility. The characteristic equation is a function of the eigenvalue, the position of the crack, and its sectional flexibility. Natural frequencies, as well as mode shapes, for cracked simply supporting Bernoulli-Euler beam, were obtained using Rayleigh's method as analytical estimation [24]. They found that the vibration frequency that can be obtained by Rayleigh's method is more accurate in comparison with a numerical solution for 1 or 2 cracks in the beam Yan and Yang [25] addressed the vibration of the cracked graded Euler beam due to the impact of the longitudinal moving load and axial compressive load. The variation of the volume fraction of a graded material has been expressed exponentially along the beam thickness. The vibration equations were analyzed by series expansion modal. Daneshmehr *et al.* [26] adopted the differential quadrature approach to analyze the free vibration of a cracked graded beam subjected to torsion-bending loading derived from Hamilton's precept. They observed that the presence of the crack gives the beam a local elasticity and thus tends to reduce the beam stiffness. Banerjee *et al.* [27] studied the transverse vibration of a cracked graded beam modeled by the Timoshenko theory. The governing equations were solved using frequency contours and response surface models with a genetic algorithm. Lien *et al.* [28] addressed free vibration of a graded Timoshenko beam with multiple cracks. They adopted Hamilton's extended as the basis for deriving the kinematic equations for that system. They utilized the dynamic stiffness method to solve the equations of vibration and find natural frequencies. Cunedioglu and Shabani [29] derived the vibration equations of cracked FG stepped cantilever Timoshenko beam by Hamilton's precept and then solved by using the finite element method. The density and young's modulus varied through the direction of thickness relying on the power and exponential laws of the volume fraction. Khiem *et al.* [30] established the vibration equation of a cracked FG Timoshenko beam model. The transfer matrix technique has been used to solve the equation of motion. The results demonstrated that the increase of crack depth and the property gradient index cause reduction in the natural frequencies for clamped-clamped and simply support end supports. A theoretical investigation of open edge cracks of the FG Euler-Bernoulli beam for three boundary conditions is presented by Yang and Chen [31]. A rotational spring model was used to simulate the crack in the beam. The material properties were varied with exponential distributions along with the beam thickness. The influences of the number of cracks, material properties, and end support on the vibration frequency and buckling characteristics of FGM beams with cracks were demonstrated.

According to the authors, the vibrational analysis of cracked graded beams with an axial motion under uniform thermal temperature has not been studied so far. Also, the effect of using functionally graded materials, axial thermal forces, and open crack on vibrational behavior and dynamic stability of moving systems has not been reported so far. The main purpose of this paper is to evaluate the use of functionally graded materials and also to investigate the effects of the axial velocity, gradient index, thermal load, and cracking parameters on improving the performance of structures that are simultaneously moving axially. A program code was developed and implemented in a MATLAB environment. First, in section 2, the physical model of the system is extracted. Then, in section 3, the vibration equation of uncracked and cracked moving FGM beam was derived then in the same section the Galerkin discretization technique is used to derive the reduced-order equations. Next, Numerical Analysis and discussion are addressed in section 4. And to ensure the correctness of the method used in the present study, the results are compared with the results in the literature. Then, the effect of key parameters on the dynamic characteristics of the system is discussed. Finally, some conclusions of the present study will be presented.

Physical model

Consider the cracked axially moving functionally graded beam shown in **Figure 1**. The distance between the two supports is L, the cross-sectional width is b, and the thickness is h. Establish a coordinate system as shown, the origin is on the mid-plane of the beam, the x-axis is along the length of the beam, and the z-axis is along the thickness. The crack parameters are represented as crack depth and position by symbols a and x_c, respectively [32].

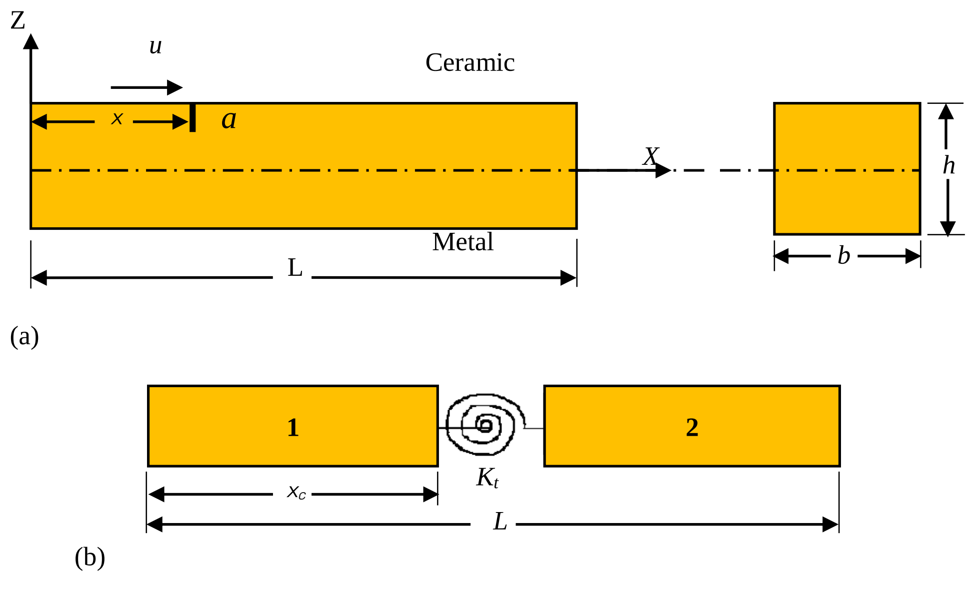

Figure 1 (a) A cracked moving FG beam and (b) The model of the crack as a torsional spring.

If the upper and lower surfaces of the beam are ceramic and metal, respectively, the elastic modulus *E*, density ρ, and thermal expansion α of the material are continuously changed along the thickness direction according to a power function, then [14];

$$\begin{cases} E(z) = (E_c - E_m)\left(\dfrac{z}{h} + \dfrac{1}{2}\right)^k + E_m \\ \rho(z) = (\rho_c - \rho_m)\left(\dfrac{z}{h} + \dfrac{1}{2}\right)^k + \rho_m \\ \alpha(z) = (\alpha_c - \alpha_m)\left(\dfrac{z}{h} + \dfrac{1}{2}\right)^k + \alpha_m \end{cases}, \frac{-h}{2} \leq z \leq \frac{h}{2} \tag{1}$$

in which, the superscript *k* ($k \geq 0$) is the gradient index; the subscripts *c* and *m* stand for ceramics and metals, respectively. From Eq. (1), the elastic modulus changes of the beam along the thickness direction can be obtained, as shown in **Figure 2**.

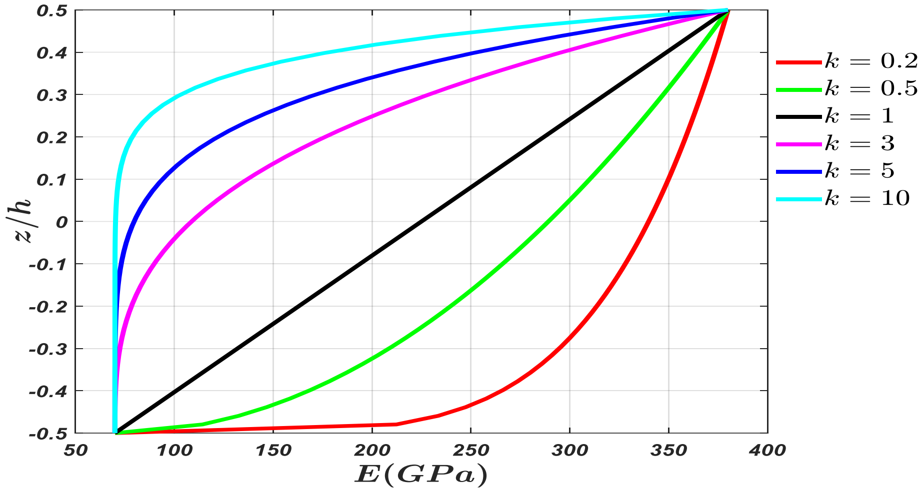

Figure 2 Variations of young's modulus and mass density with respect to the beam thickness.

Vibration equation of uncracked beam

According to Euler beam theory, the displacement field of the beam is written as follows;

$$u(x,z,t) = -z\frac{\partial w(x,t)}{\partial x},\ w(x,z,t) = w(x,t) \tag{2}$$

in which: $u(x,z,t)$ and $w(x,z,t)$ are the axial displacement and lateral position, respectively, t is time. Normal strain ε_{xx} and normal stress σ_{xx} can be expressed as;

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = -z\frac{\partial^2 w}{\partial x^2}, \sigma_x = E(z)\varepsilon_{xx} = -E(z)z\frac{\partial^2 w}{\partial x^2} \tag{3}$$

The Hamilton principle for energy-variational can be written as;

$$\delta\int_{t1}^{t2}\left(U_s + W_{ex} - E_k\right)dt = 0 \tag{4}$$

in which: U_s is the strain energy; W_{ex} is the external force due to thermal force; E_k is the kinetic energy, and their expressions are;

$$U_s = \frac{1}{2}\int_V \sigma_{xx}\varepsilon_{xx} dV = \frac{1}{2}\int_V E(z)z^2\left(\frac{\partial^2 w}{\partial x^2}\right)^2 dV = \frac{1}{2}(EI_{eq})\int_0^L \left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx \tag{5}$$

where; $(EI_{eq}) = \int_A E(z)z^2 dA = E_m I\gamma$ is the bending stiffness of the beam; A is the cross-sectional area of the beam.

The external work variation corresponding to the axial force due to a change in the temperature is calculated by Ebrahimi and Salari [33];

$$\delta W_{ex} = \int_0^L N_T \frac{\partial w}{\partial x}\delta\left(\frac{\partial w}{\partial x}\right) \tag{6}$$

The axial force N_T can be defined as;

$$N_T = \int_A E(z)\alpha(z)\Delta T dA \tag{7}$$

where ΔT is the temperature change, and for the uniform temperature rise takes the following form;

$$\Delta T = T - T_o \tag{8}$$

in which T and T_o are the desired and initial temperatures, respectively. Note that the other linear and nonlinear temperature distributions along the thickness are not considered in this study.

The kinetic energy of the axially moving intact FG beam considering rotary inertia is expressed as;

$$\begin{aligned} E_k &= \int_V \frac{\rho(z)}{2}\left[\left(\frac{\partial w}{\partial t} + v\frac{\partial w}{\partial x}\right)^2 + \left(-z\frac{\partial^2 w}{\partial x \partial t} + v\right)^2\right] dV = \\ &\frac{\rho_{To}}{2}\int_0^L\left[\left(\frac{\partial w}{\partial t} + v\frac{\partial w}{\partial x}\right)^2 + v^2\right]dx - \rho_{T1}\int_0^L\left(v\frac{\partial^2 w}{\partial x \partial t}\right)dx + \frac{\rho_{T2}}{2}\int_0^L\left(\frac{\partial^2 w}{\partial x \partial t}\right)^2 dx \end{aligned} \tag{9}$$

in which; $\rho_{T0} = \int_A \rho(z) dA = \rho_m A\alpha_1$; $\rho_{T1} = \int_A \rho(z) z dA = \rho_m A\alpha_2$; $\rho_{T2} = \int_A \rho(z) z^2 dA = \rho_m I\alpha_3$;

substituting Eqs. (5) - (6) and (9) into Eq. (4) and then applying integration by parts gives to the vibration equation of intact moving FGM beam;

$$(EI)_{eq}\frac{\partial^4 w}{\partial x^4} - \rho_{T2}\frac{\partial^4 w}{\partial x^2 \partial t^2} + (\rho_{T0}v^2 + N_T)\frac{\partial^2 w}{\partial x^2} + \rho_{T0}\frac{\partial^2 w}{\partial t^2} + 2\rho_{T0}v\frac{\partial^2 w}{\partial x \partial t} = 0 \tag{10}$$

Eq. (10) is the equation of motion for a moving FGM intact beam in a thermal environment. If $k = 0$ and $\Delta T = 0$, the dynamic equation, Eq. (10), is reduced to the classical moving Euler–Bernoulli beam model. For the moving FG beam supported at both ends, the boundary conditions are written as;

$$\begin{aligned} &w(0) = M(0) = 0: \quad \text{at } x = 0 \\ &w(L) = M(L) = 0: \quad \text{at } x = L \end{aligned} \tag{11}$$

To derive the dimensionless differential equations, the following parameters are introduced;

$$\zeta = \frac{x}{L}, \eta = \frac{w}{L}, \tau = \frac{t}{L^2}\sqrt{\frac{E_m I}{\rho_m A}}, u = vL\sqrt{\frac{\rho_m A}{E_m I}}, \bar{N}_T = \frac{N_T L^2}{E_m I}$$
$$\lambda = \frac{L}{h}, PP = \frac{\gamma_3}{\alpha_1}, QQ = \frac{\alpha_3}{(12 * \lambda^2 * \alpha_1)}, DD = \frac{(\alpha_1 * u^2 + \bar{N}_T * \gamma_1)}{\alpha_1} \tag{12}$$

where E_m and ρ_m are the values of modulus of elasticity and density for a full metal beam, respectively (i.e., $k = \infty$). Substituting Eq. (12) into Eq. (10), one can obtain the equation of motion in the dimensionless form;

$$PP\frac{\partial^4 \eta}{\partial \zeta^4} + DD\frac{\partial^2 \eta}{\partial \zeta^2} + 2v\frac{\partial^2 \eta}{\partial \zeta \partial \tau} + (1 - QQ\frac{\partial^2 \eta}{\partial \zeta^2})\frac{\partial^2 \eta}{\partial \tau^2} = 0 \tag{13}$$

substituting Eq. (12) into Eq. (11), the dimensionless form of boundary conditions can be expressed as;

$$\eta(0,\tau) = \eta(1,\tau) = 0, \quad \frac{\partial^2 \eta(0,\tau)}{\partial \zeta^2} = \frac{\partial^2 \eta(1,\tau)}{\partial \zeta^2} = 0 \tag{14}$$

Vibration equation of cracked beam

To examine the crack influence on the vibration parameters of a graded beam, the crack model has to be established by adopting on fracture mechanics theorem. The crack is modeled as a torsional spring as seen in **Figure 1(b)**, the bending stiffness of the torsional spring can be expressed as [34,35];

$$K_t = \frac{1}{G`} \tag{15}$$

where $G`$ is the flexibility caused due to crack and can be derived by [35];

$$\frac{(1-v^2)S^2}{E(a)} = \frac{M^2}{2}\frac{dG`}{da} \tag{16}$$

where S is the stress intensity factor (SIF), M is the bending moment at the crack, and $E(a)$ is the modulus of elasticity at the crack tip, the stress intensity factor can be written as[35];

$$S = 6M\sqrt{\frac{\pi a . f(z)}{bh^2}} \tag{17}$$

by substituting Eq. (17) into Eq. (16), the flexibility can be obtained as;

$$G' = \int_0^a \frac{72\pi(1-v^2)a . f(z)}{bE(a)h^2} da \tag{18}$$

After implementing the integral G' and substituting it into Eq. (15), the compliance of the crack of a homogenous beam can be written as [36];

$$C = \frac{EI}{K_t} = 6\pi(1-v^2)h . f(a/h) \tag{19}$$

For a graded beam, the compliance of the crack can be written as [30];

$$C = \frac{I_o}{K_t} = 6\pi(1-v^2)h . f(a/h)\gamma_3 \tag{20}$$

where *C* is dependent on the gradient index and cracks depth ratio. The correct crack function $f(a/h)$ can be expressed as [30,37];

$$f(a/h) = 0.6272(a/h)^2 - 1.04533(a/h)^3 + 4.5948(a/h)^4 - 9.9736(a/h)^5 \\ + 20.2948(a/h)^6 - 33.0351(a/h)^7 + 47.1063(a/h)^8 - 40.7556(a/h)^9 + 19.6(a/h)^{10} \tag{21}$$

where α is the crack depth, and α/*h* is the crack depth ratio.

For cracked beam, the beam is divided into two parts as shown in **Figure 1(b)**. The first part is integrated from 0 to x_c while the second part is integrated from x_c to *L*. Thus, the governing equations of a cracked graded Euler beam with axial motion in the dimensionless form are;

$$PP\frac{\partial^4 \eta_1}{\partial \zeta^4} + DD\frac{\partial^2 \eta_1}{\partial \zeta^2} + 2v\frac{\partial^2 \eta_1}{\partial \zeta \partial \tau} + (1 - QQ\frac{\partial^2 \eta_1}{\partial \zeta^2})\frac{\partial^2 \eta_1}{\partial \tau^2} = 0 \qquad \text{at } 0 \le \zeta \le \zeta_c$$

$$PP\frac{\partial^4 \eta_2}{\partial \zeta^4} + DD\frac{\partial^2 \eta_2}{\partial \zeta^2} + 2v\frac{\partial^2 \eta_2}{\partial \zeta \partial \tau} + (1 - QQ\frac{\partial^2 \eta_2}{\partial \zeta^2})\frac{\partial^2 \eta_2}{\partial \tau^2} = 0 \qquad \text{at } x_c \le \zeta \le 1 \tag{22}$$

Solution method

To solve the vibration equations (22-a, and 22-b), a Galerkin's technique was used. The general form of Galerkin's equation is given as [38];

$$\eta(\xi, T) = \sum_{r=1}^{n} \varphi_r(\xi) q_r(T) \tag{23}$$

where $\varphi_r(\xi)$ and $q_r(T)$ represents the shape function and generalized coordinates, respectively.

The mode shape function of the cracked FG beam can be expressed as the sum of the formation function of the un-cracked FG beam and the polynomial of (ξ) as [39];

$$\varphi_{r1}(\xi) = \varphi(\xi) + A_o + A_1\xi + A_2\xi^2 + A_3\xi^3 \qquad \text{for } 0 \le \xi \le \xi_c$$

$$\varphi_{r2}(\xi) = \varphi(\xi) + B_o + B_1\xi + B_2\xi^2 + B_3\xi^3 \qquad \text{for } \xi_c \le \xi \le 1 \tag{24}$$

The continuity conditions and compatibility of the cracked FG beam are given by;

$$\begin{aligned} &\varphi_{i1}(\xi_c) = \varphi_{i2}(\xi_c) \\ &\varphi''_{i1}(\xi_c) = \varphi''_{i2}(\xi_c) \\ &\varphi'''_{i1}(\xi_c) = \varphi'''_{i2}(\xi_c) \\ &\varphi'_{i2}(\xi_c) - \varphi'_{i1}(\xi_c) = C\varphi''_{i2}(\xi_c) \end{aligned}$$

Substituting the derivatives of the mode shape functions (24) into the compatibility and continuity conditions (25) and the boundary conditions of simply supported (14), which yields;

$$\begin{aligned} &A_o = 0; A_2 = B_2 = 0; A_3 = B_3 = 0 \\ &B_o = C\xi_c(n\pi)^2 \sin(n\pi\xi_c) \\ &B_1 = -B_o = C\xi_c(n\pi)^2 \sin(n\pi\xi_c) \\ &A_1 = B_1 + c(n\pi)^2 \sin(n\pi\xi_c) = C(n\pi)^2 \sin(n\pi\xi_c)(1 - \xi_c) \end{aligned} \tag{25}$$

Substituting Eq. (25) into Eq. (24), the mode shape function of the S-S FG beam is obtained as [39];

$$
\begin{aligned}
&(0 \le \xi \ge \xi_c) \\
&\varphi_{i1} = \varphi(\xi) - \left(\frac{\xi_c - 1}{\xi_c}\right)\left[C\xi_c (n\pi)^2 \sin(n\pi\xi_c)\right]\xi \\
&(\xi_c \le \xi \ge 1) \\
&\varphi_{i2} = \varphi(\xi) + (1-\xi)\left(C\xi_c (n\pi)^2 \sin(n\pi\xi_c)\right)
\end{aligned}
\tag{26}
$$

By substituting Eq. (26) into Eq. (22), and integrating these equations after multiplying by φ_s , the resultant vibration equation in matrix form is given as;

$$\left[M\right]\ddot{q} + \left[C\right]\dot{q} + \left[K\right]q = 0 \tag{27}$$

where $\left[M\right]$ is the mass matrix, $\left[C\right]$ is the damping matrix and $\left[K\right]$ is the stiffness matrix of the cracked axially moving FGM beam, and their elements are given as follows;

$$M_{r,s} = \int_0^{\xi_c} \left(\varphi_{r1}(\xi) - QQ\varphi''_{r1}(\xi)\right)\varphi_{s1}(\xi)d\xi + \int_{\xi_c}^{1} \left(\varphi_{r2}(\xi) - QQ\varphi''_{r2}(\xi)\right)\varphi_{s2}(\xi)d\xi \tag{28}$$

$$C_{r,s} = \int_0^{\xi_c} \left(2u\varphi'_{r1}(\xi)\right)\varphi_{s1}(\xi)d\xi + \int_{\xi_c}^{1} \left(2u\varphi'_{r2}(\xi)\right)\varphi_{s2}(\xi)d\xi \tag{29}$$

$$
\begin{aligned}
K_{r,s} &= \int_0^{\xi_c} \left[PP\varphi_{r1}{}^{(4)}(\xi)\varphi_{s1}(\xi) + DDu^2\varphi''_{r1}(\xi)\varphi_{s1}(\xi)\right]d\xi \\
&+ \int_{\xi_c}^{1} \left[PP\varphi_{r2}{}^{(4)}(\xi)\varphi_{s2}(\xi) + DDu^2\varphi''_{r2}(\xi)\varphi_{s2}(\xi)\right]d\xi
\end{aligned}
\tag{30}
$$

The second-order differential Eq. (27) is reduced to the first-degree differential equation [40];

$$D\dot{Z}(T) + BZ(T) = 0 \tag{31}$$

$$\text{where } D = \begin{bmatrix} M & 0 \\ 0 & C \end{bmatrix},\ B = \begin{bmatrix} -M & 0 \\ 0 & K \end{bmatrix},\ Z(T) = \begin{bmatrix} q(T) \\ \dot{q}(T) \end{bmatrix} \tag{32}$$

Assuming $Z(T) = Qe^{i\omega T}$ yields the following eigenvalue problem;

$$YQ - i\omega J = 0 \tag{33}$$

where J indicates the identity matrix and $Y = -D^{-1}B$. Moreover, ω is the complex-valued natural frequency of the cracked axially moving graded Euler beam model considering rotary inertia and can be specified in terms of various main factors such as axial velocity, property gradient index, parameters of density and elastic modulus, crack position, and crack depth ratio.

Numerical analysis and discussion

The procedure depicted above for the Galerkin technique has been implemented in a MATLAB version (R2015a) program and the natural frequencies are computed. The complex frequency of moving graded beam with and without cracks in thermal environments are discussed in this section. In the following numerical examples, the dimensions used in numerical results for the FG beam are given as the beam width b = 0.1 m, the beam height h = 0.1 m, and the beam length L = 1 m [36].

The FGM plate used in this analysis is composed of aluminum and alumina materials. It is assumed that the FGM plate is made of aluminum metal ($k = \infty$) and alumina ceramics (k = 0) and the material properties are shown in **Table 1**.

Table 1 Material properties of FGM beam [40].

Material	Elasticity modulus	Density	Poison's ratio
Alumina (Al_2O_3)	380 *GPa*	$3800\,kg/m^3$	0.23
Aluminum (Al)	70 *GPa*	$2707\,kg/m^3$	0.23

Validation

To show the accuracy of the formulations, a comparative study of the results is performed with those available in the literature. Assuming that the beam is composed of homogeneous ceramics (k = 0) and metal ($k = \infty$), the axial velocity u = 0, the initial temperature $\Delta T = 0$, so that the first 3-order natural frequencies for (u = 0, $E_{ratio} = E_c/E_m = 4$, $\rho_{ratio} = \rho_c/\rho_m = 1$) were compared with the results obtained by Li *et al.* [11] of the simply supported gradient beams is shown in **Table 2**. From **Table 2**, it can be seen that the solution in this paper is relatively consistent with the solution in the existing literature, which shows that the method in this paper is effective and feasible.

Table 2 Validity of the results of present work for SS FG beam at u = 0.

k	$\mathrm{Im}(\omega)_1$		$\mathrm{Im}(\omega)_2$	
0	Galerkin	Ref. [11]	Galerkin	Ref. [11]
0.1	19.738	19.738	78.944	78.943
0.2	18.825	18.825	75.290	75.290
0.5	18.042	18.042	72.160	72.160
1	16.294	16.294	65.167	65.167
2	16.638	14.638	58.546	58.546
5	13.333	13.333	53.324	53.324
10	12.491	12.491	49.961	49.961

Table 3 presented another validation for the result of the present study to that found in the literature. The frequency ratio for cracked to un-cracked FG beam was compared to the results of reference [30] at a/h = 0.2, x_c/L = 0.4, k = 1.9 as tabulated in **Table 3**. According to this table, the results of the present study are a reasonable agreement with the results of Yang and Chen [31].

Table 3 Comparison of fundamental frequency ratio of a cracked graded beam with Yang and Chen [31].

L/h	**10**		**20**	
E_2/E_1	0.2	5	0.2	5
The present study (EBT)	0.951	0.993	0.974	0.997
Yang and Chen [31]	0.985	0.998	0.992	0.999

Parametric analysis

The above 2 examples verify the reliability and effectiveness of the calculation method in this paper. The following analysis analyzes the cracked moving FGM beam in the thermal environment on simply supported ends. The influence of the material composition index k, the axial movement speed u, crack depth and positions, and other factors on the free vibration frequency of the axially moving FGM cracked beam are presented.

The influence of moving speed

The variation curves of the non-dimensional vibration frequency of the first three modes of the moving intact and cracked FGM beam $\Delta T = 50$, $L/h = 10$ with different gradient indexes (i.e. k = 0, k = 2, and k = 5) are analyzed in **Figures 3** and **4**, respectively. **Figures 3(a)** - **3(c)** show the $\mathrm{Re}(\omega)$ and $\mathrm{Im}(\omega)$ parts of the eigenvalues of the aluminum/alumina material intact FGM beam as a function of the axial velocity. For k = 0, **Figure 3(a)** shows that when the axial movement velocity u is less than the critical velocity u = 6.06, the real part $\mathrm{Re}(\omega)$ of the first 3-order mode eigenvalues of the FGM beam gradually decreases with the increase of the axial movement speed, and the vibration frequency $\mathrm{Im}(\omega)$ is zero; when the axial movement speed is equal to the critical speed u = 6.06, the vibration frequency $\mathrm{Im}(\omega)$ of the first modal eigenvalue is reduced to zero, and the real part $\mathrm{Re}(\omega)$ of the second and third modal eigenvalue is not zero. The beam begins to diverge and lose stability; when the axial movement speed continues to increase to u =12.2, the first and second modes begin to stabilize again. As u > 12.72, the imaginary components of the first and second modes merge and keep positive, while their real components include positive and negative values, which indicates the beam undergoes flutter instability when the first mode couples second mode. At this time, the axial speed u =12.72 is the first flutter critical speed. When the volume fraction index increased to k =2, the critical velocity and combination for first and second modes values are reduced in comparison to **Figure 3(a)** for k = 0 as shown in **Figure 3(b)** and **Table 4**. Also, in this Figure, when the speed is 15.09, the vibration frequency $\mathrm{Im}(\omega)$ of the second mode and third mode starts coincident, the FGM beam has secondary flutter instability at this time. At this time, the axial velocity u = 15.09 is the second flutter critical speed, and its corresponding vibration frequency of 50.08 is the secondary flutter critical. When increasing the volume fraction index to k = 5, the first mode divergence, first mode and second mode combination and combined between second mode and third mode decrease as shown in **Table 4** and **Figure 3(c)**. This is due to the verity that the content of Al in the FGM beam increases whiles the content of Alumina decreases with increasing exponent k, and young's modulus of Alumina is much larger than that of Al.

The variation of the first 3 modes of real and imaginary parts for cracked moving FG beam for $\Delta T = 50$, $L/h = 10$, $x_c = 0.1$, $a/h = 0.2$ with different gradient index (i.e. k = 0, k = 2, and k = 5) is shown in **Figures 4(a)** - **4(c)**. From these figures and **Table 4**, it is seen that when increasing in moving speed, the imaginary frequency decreases. This is because higher moving speed weakens the structure's stiffness. Also, when Comparing **Figure 3** with **Figure 4**, all the coupled-mode flutters disappear. The convergence in modes, not coupled-mode flutters is occurring. The existence of crack contributes to decreasing in the critical velocity and then the divergence instability as shown in **Table 4**.

Table 4 Critical velocities for *different volume fraction index k.*

Case	Critical velocity	Volume fraction index k		
		k = 0	k = 2	k = 5
Intact moving FG beam	u_d	6.06	4.74	4.38
	u_{f1}	12.72	10.14	9.36
	u_{f2}	-	15.96	14.88
Cracked-moving FG beam	u_d	5.94	4.68	4.32

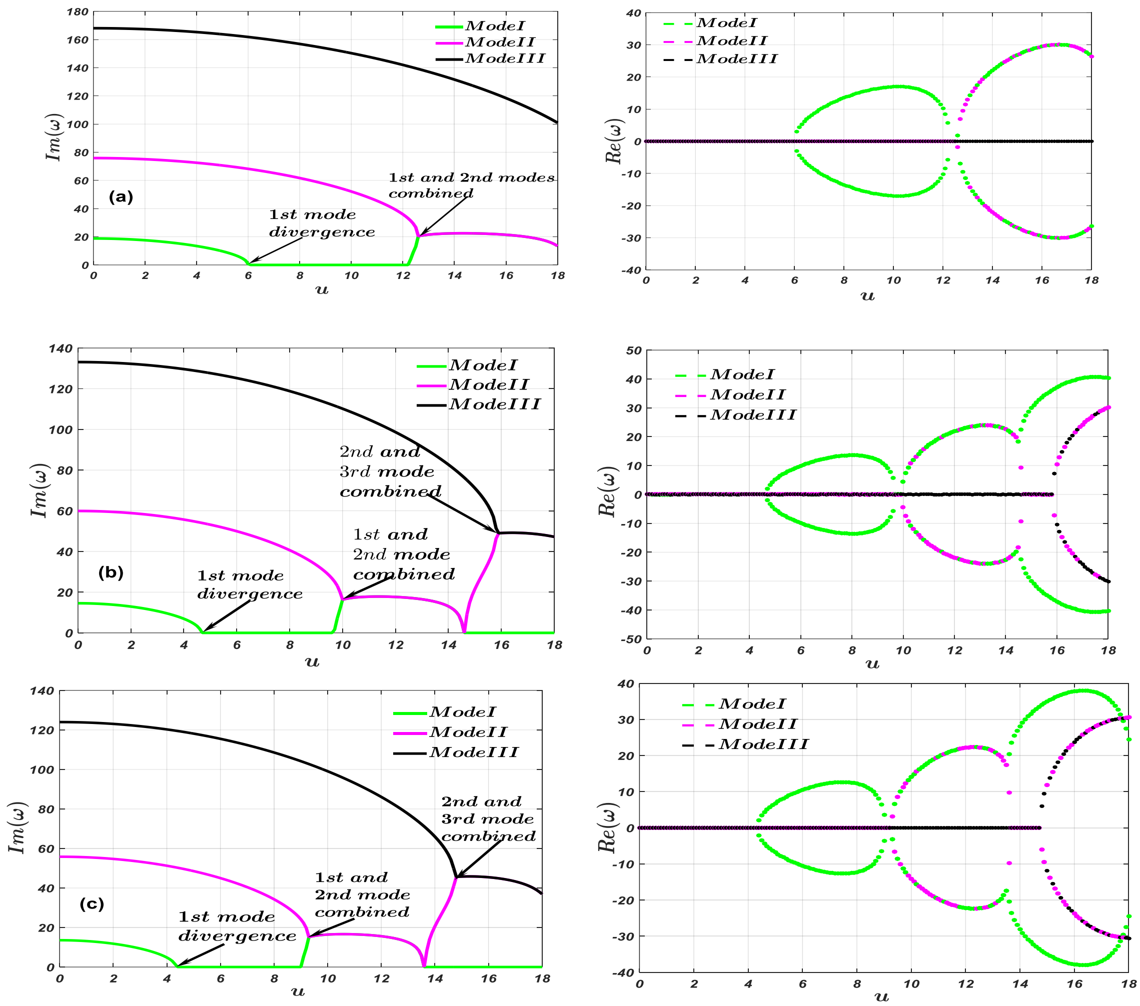

Figure 3 Effect of axial velocity on real and imaginary natural frequencies of S-S intact FG beam at $\Delta T = 50$, $L/h = 10$ depending on the gradient index (a) k = 0, (b) k = 2, and (c) k = 5.

Effect of thermal load

The effect of the power-law exponent on the first imaginary frequency of uncracked and cracked moving FGM beam with different temperature uniform rise (ΔT) is illustrated in this subsection. **Figures 5(a) - 5(c)** showed that the dimensionless natural frequencies affect by crack location, temperature uniform rise, and the property gradient index (k) of moving FGM beam with simply supported end conditions. From the results, it can be seen that the natural frequencies of the FGM beam are inversely proportional to crack location, temperature uniform rise, and the gradient index. The reason for the decrease in the frequencies is the fact that an increase in the gradient index means a decrease in the ceramic material relative to the metal material inside the beam, and the fact that the metal has a modulus of elasticity less than the elastic modulus of ceramics makes the beam more flexible (the beam stiffness decreases), which leads to a decrease in the natural frequencies. Also, increasing crack depth and location weak the beam stiffness and then decreases the vibration frequency.

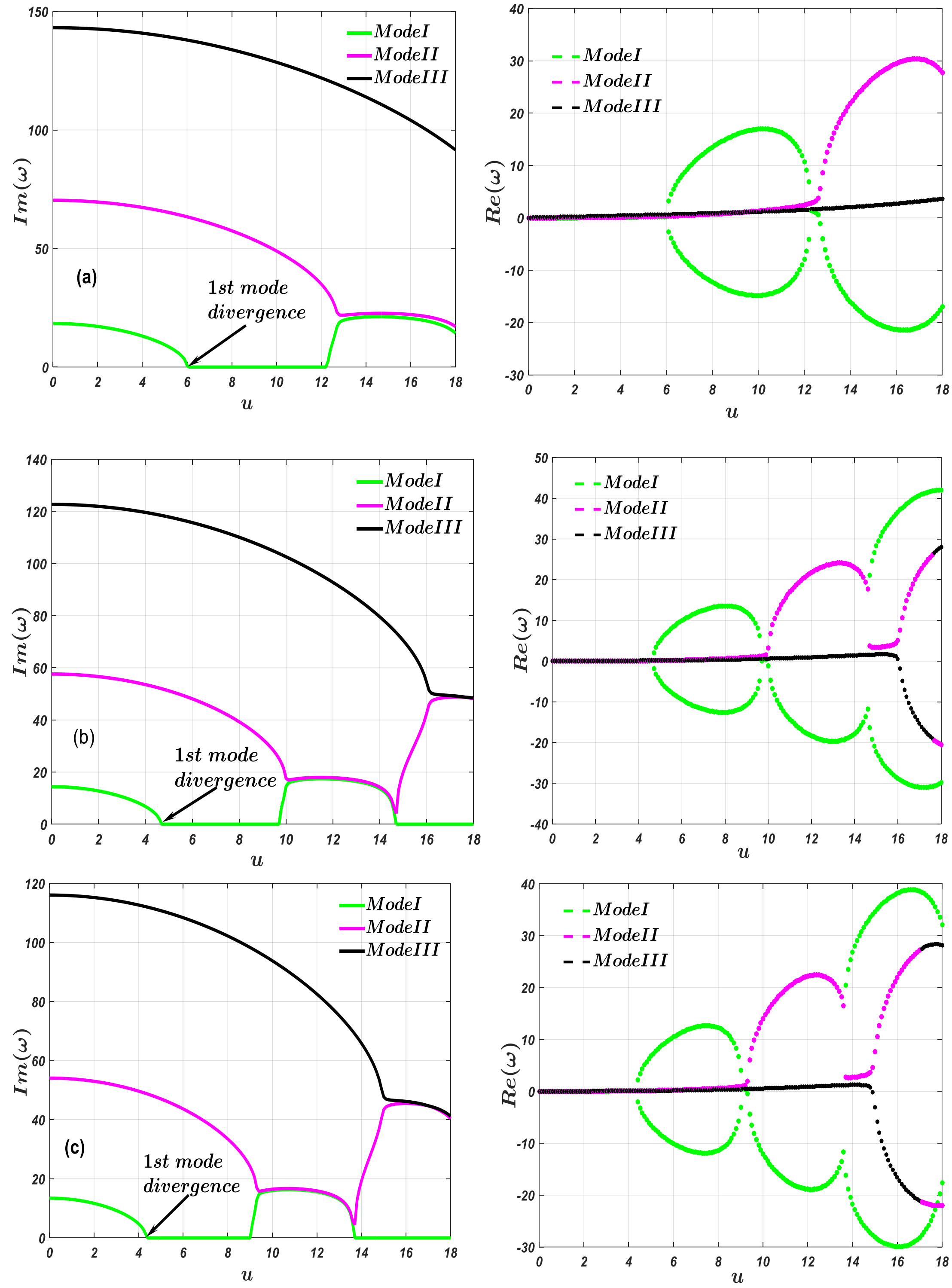

Figure 4 Effect of axial velocity on Real and imaginary natural frequencies of cracked FG beam at $\Delta T = 50$, $L/h = 10$, $x_c = 0.1$, $a/h = 0.2$ (a) $k = 0$, (b) $k = 2$, and (c) $k = 5$.

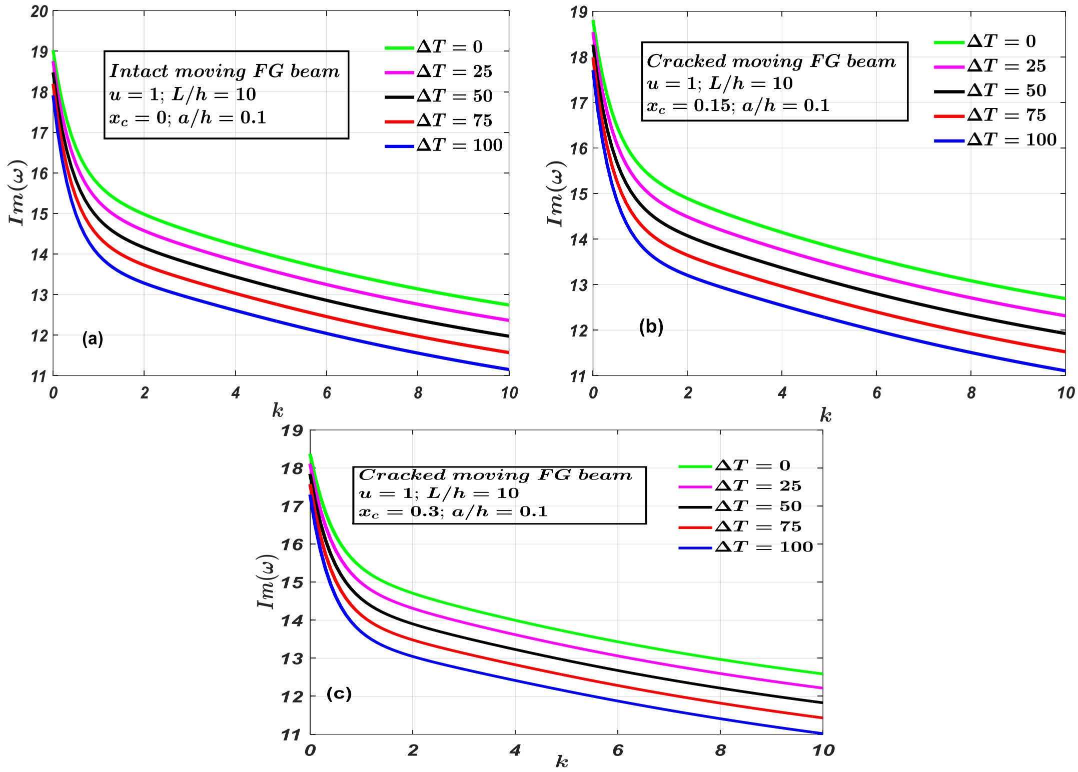

Figure 5 The variation of the first dimensionless frequency versus gradient index for different temperature rises of $\Delta T = 0; \Delta T = 25; \Delta T = 50; \Delta T = 75; \Delta T = 100$: a) $x_c = 0$, b), $x_c = 0{,}15$ and c) $x_c = 0.3$.

On the other hand, the first and second natural frequencies against the crack position along the beam with various temperature rise for different crack depth ratio is illustrated in **Figures 6(a) - 6(b)** and **Figures 7(a) - 7(b)**, respectively. From this figure, it is shown that the first and second frequency decrease with the temperature rise increase. Also, it is shown that the natural frequencies decrease when the crack depth ratio increases. Increasing the crack depth ratio results in a decrease in beam stiffness and thus a decrease in the natural frequencies of the FG beam.

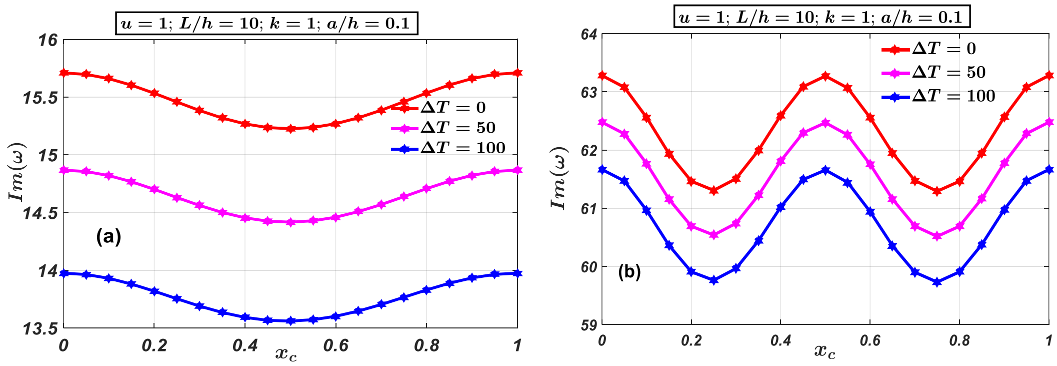

Figure 6 The 1st and 2nd order natural frequencies of moving cracked FGM beam with different temperature rise (ΔT) at $a/h = 0.1$ (a) 1st order frequency and (b) 2nd order frequency.

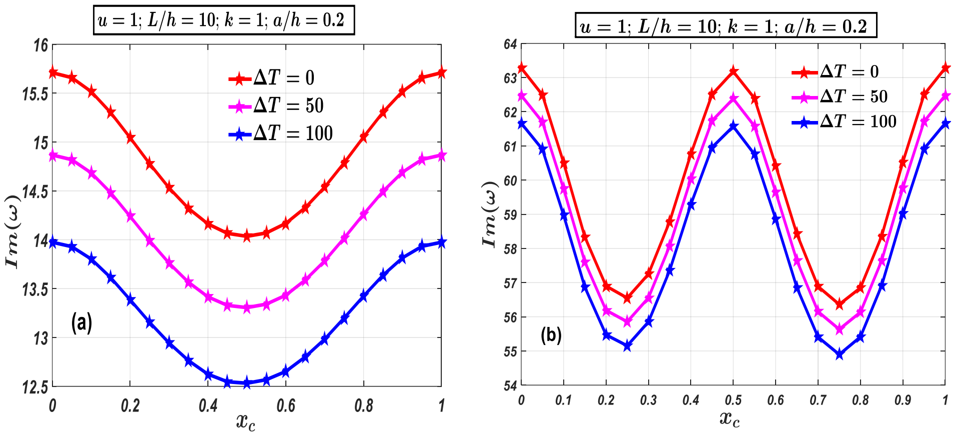

Figure 7 The 1st and 2nd order natural frequencies of moving cracked FGM beam with different temperature rise (ΔT) at $a/h = 0.2$ (a) 1st order frequency and (b) 2nd order frequency.

Conclusions

In this study, the free vibration behavior of moving FGM cracked simply supported Euler- Bernoulli beam exposed to thermal fields is investigated. The vibration frequency was derived based on Hamilton's precept and then solved using Galerkin's approach to find the vibration frequency under different parameters. The influences of different parameters on the natural frequencies of moving cracked FGM beams have been studied. The following conclusions have been drawn.

1) Increasing in gradient index and axially moving of the FGM beam leads to a drop in natural frequencies of intact and cracked axially moving graded beam.

2) Increases in crack depth ratio and crack position leads to a decrease in the natural frequencies of the moving FG beam.

3) Vibration frequencies are reduced by the presence of a crack in the FG beam and the symmetry of the vibration mode shapes is distorted depending on the crack intensity and crack location. The mid-distance crack location causes the natural frequencies to drop more than in other locations.

4) The vibration frequencies of the moving FG beam decrease due to the increase in temperature, and when the presence of a crack is combined with a temperature rise. The vibration frequencies decrease more.

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